

# Assessing Measurement System Variation

## Example 1 Fuel Injector Nozzle Diameters

### Problem

A manufacturer of fuel injector nozzles has installed a new digital measuring system. Investigators want to determine how well the new system measures the nozzles.

### Data collection

Nine nozzles were randomly sampled across all major sources of process variation (machine, time, shift, job change) to be representative of those typically produced. The nozzles were coded to identify measurements from specific nozzles.

The first operator measured each of the 9 nozzles in random order. Then, the 9 nozzles were randomized again and given to the second operator for measurement. This process was repeated twice for each operator, for a total of 36 measurements.

**Note** | For good measurement system analyses, it is critical to ensure that parts are randomly sampled and are measured in random order.

The specification for the nozzle diameters is  $9012 \pm 4$  microns (the tolerance is 8 microns).

### Tools

- Gage R&R Study (Crossed)

### Data set

NOZZLE.MPJ

Variable	Description
Nozzle	Fuel injector nozzle being measured
Operator	Operator who took the measurement
Run Order	Original run order of the experiment
Diameter	Measured diameter of nozzle (microns)

# Measurement systems analysis

## What is measurement systems analysis

Measurement systems analysis assesses the properties of a measurement system to ensure adequacy for a given application. When measuring the output from a process, consider two sources of variation:

- Part-to-part variation
- Measurement system variation

If measurement system variation is large compared to part-to-part variation, the measurements may not provide useful information.

## When to use measurement systems analysis

A measurement system must adequately discriminate between parts for effective monitoring of a process. Before collecting data from your process (e.g. to analyze process control or capability), use measurement system analysis to confirm that the measurement system is measuring consistently and accurately and can adequately discriminate between parts.

## Why use measurement systems analysis

Use measurement systems analysis when you want to answer the following types of questions:

- Can the measurement system adequately discriminate between different parts?
- Is the measurement system stable over time?
- Is the measurement system accurate throughout the range of parts?

For example,

- Can a viscometer adequately discriminate between the viscosity of several paint samples?
- Does a scale need to be periodically recalibrated to accurately weigh filled bags of potato chips?
- Does a thermometer accurately measure the temperature for all heat settings used in the process?

## Gage R&R study (crossed)

### What is a gage R&R study (crossed)

A crossed gage R&R study is an experiment that is used to estimate how much of the total process variation is due to the measurement system. Total process variation consists of part-to-part variation plus measurement system variation. Measurement system variation can be further broken down into:

- Repeatability—variation due to the measuring *device*, or the variation observed when the *same* operator measures the same part repeatedly with the same device
- Reproducibility—variation due to the measuring *system*, or the variation observed when *different* operators measure the same part using the same device.

To estimate repeatability, each operator measures each part at least twice. To estimate reproducibility, at least two operators must measure the parts. It is important for operators to measure the parts in random order, and the selected parts should represent the possible range of measurements.

### When to use a gage R&R study (crossed)

- Use gage R&R to evaluate or qualify a measurement system prior to using it for process monitoring or process improvement activities.
- The crossed analysis is typically used when each part in the study can be measured multiple times.

### Why use a gage R&R study (crossed)

Use this study to compare the measurement system variation to total process variation and/or tolerance. If the measurement system variation is a large proportion of the total variation, the system may not be capable of distinguishing between parts.

A crossed gage R&R study can answer questions such as:

- Is the variability of a measurement system small compared with the manufacturing process variability?
- Is the variability of a measurement system small compared with the process specification limits?
- How much variability in a measurement system is caused by differences between operators?
- Is a measurement system capable of discriminating between different parts?

For example,

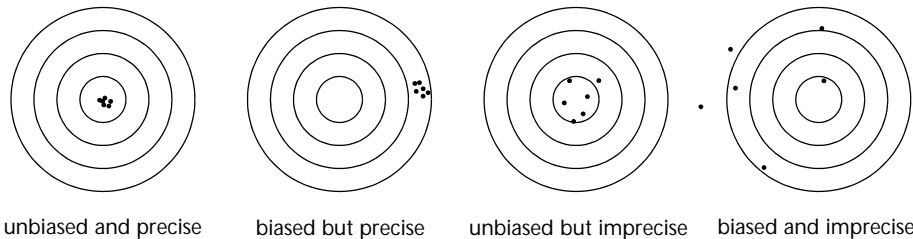
- How much of the variability in the measured diameters of a bearing is caused by the caliper?
- How much of the variability in the measured diameters of a bearing is caused by the operator?
- Is the measurement system capable of discriminating between bearings of different size?

## Measurement system error

Measurement system errors can be classified into two categories: accuracy and precision.

- Accuracy is the difference between the measurement and the part's actual value.
- Precision is the variation seen when you measure the same part repeatedly with the same device.

Within any measurement system, you can have one or both of these problems. For example, a device may measure parts precisely (little variation in the measurements) but not accurately. Or a device may be accurate (the average of the measurements is very close to the master value), but not precise (the measurements have large variance). Or a device may be neither accurate nor precise.



### Accuracy

The accuracy of a measurement system has three components:

- Bias—a measure of the bias in the measurement system; the difference between the observed average measurement and a *master value*
- Linearity—a measure of how the size of the part affects the bias of the measurement system; the difference in the observed bias values through the *expected range* of measurements
- Stability—a measure of how well the system performs over time; the total variation obtained with a particular device, on the same part, when measuring a single characteristic *over time*.

### Precision

Precision, or measurement variation, has two components:

- Repeatability—variation due to the measuring *device*, or the variation observed when the *same* operator measures the same part repeatedly with the same device
- Reproducibility—variation due to the measuring *system*, or the variation observed when *different* operators measure the same part using the same device

## Assessing the measurement system

Use Gage R&R Study (Crossed) to assess:

- How well the measuring system can distinguish between parts
- Whether the operators measure consistently

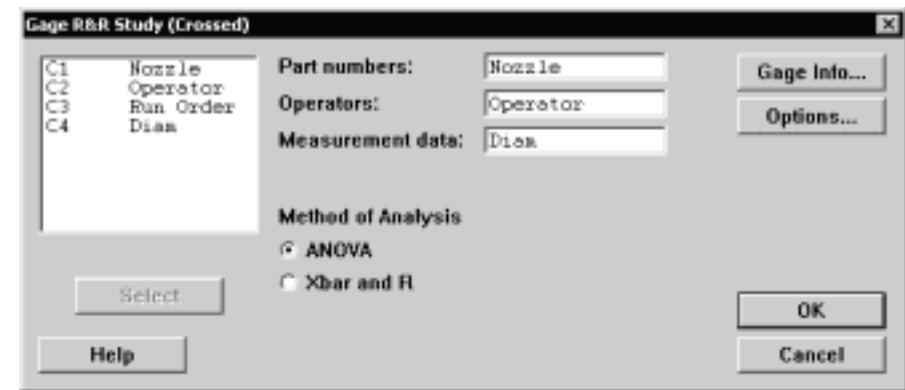
### Tolerance

The specification limits for the nozzle diameters are  $9012 \pm 4$  microns. In other words, the nozzle can vary by as much as 4 microns in either direction. The tolerance is the difference between the specification limits; here,  $9016 - 9008 = 8$  microns.

By entering a value in Process tolerance, you can estimate what proportion of the tolerance is used by the variation in the measurement system.

### Gage R&R Study (Crossed)

- 1 Open the project NOZZLE.MPJ.
- 2 Choose **Stat** ► **Quality Tools** ► **Gage Study** ► **Gage R&R Study (Crossed)**.
- 3 Complete the dialog box as shown below.



- 4 Click **Options**.
- 5 In **Process tolerance**, enter 8.
- 6 Click **OK** in each dialog box.

## Interpreting your results

### Analysis of variance tables

MINITAB uses the analysis of variance (ANOVA) procedure to calculate variance components, which are then used to estimate the percent variation due to the measuring system. The percent variation appears in the Gage R&R table.

The two-way ANOVA table includes terms for the part (Nozzle), operator (Operator), and operator-by-part interaction (Nozzle\*Operator).

If the p-value for the operator-by-part interaction is  $\geq 0.25$ , MINITAB generates a second ANOVA table that omits the interaction term from the model. To alter the default type I error rate of 0.25, click Options and enter a new value (for example, 0.3).

Here, the p-value for Nozzle\*Operator is 0.707. Therefore, MINITAB removes the interaction term from the model and generates a second ANOVA table.

### Gage R&R Study - ANOVA Method

#### Two-Way ANOVA Table With Interaction

Source	DF	SS	MS	F	P
Nozzle	8	46.1489	5.76861	769.148	0.000
Operator	1	0.0400	0.04000	5.333	0.050
Nozzle * Operator	8	0.0600	0.00750	0.675	0.707
Repeatability	18	0.2000	0.01111		
Total	35	46.4489			

#### Two-Way ANOVA Table Without Interaction

Source	DF	SS	MS	F	P
Nozzle	8	46.1489	5.76861	576.861	0.000
Operator	1	0.0400	0.04000	4.000	0.056
Repeatability	26	0.2600	0.01000		
Total	35	46.4489			

## Interpreting your results

### Variance components

MINITAB also calculates a column of variance components (VarComp), which are the basis for calculating %Gage R&R using the ANOVA method.

The gage R&R tables show how the total variability is divided among the following sources:

- Total Gage R&R, broken into:
  - Repeatability represents variability from repeated measurements by the same operator.
  - Reproducibility (which can be further divided into operator and operator-by-part components) represents variability when same part is measured by different operators.
- Part-to-Part, the variability in measurements across different parts.

### Why use variance components?

Use variance components to assess the variation contributed by each source of measurement error relative to the total variation

Ideally, differences between parts should account for most of the variability; and variability from repeatability and reproducibility should be very small.

### Gage R&R

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	0.01167	0.80
Repeatability	0.01000	0.69
Reproducibility	0.00167	0.11
Operator	0.00167	0.11
Part-To-Part	1.43965	99.20
Total Variation	1.45132	100.00

## Interpreting your results

### Percent contribution

%Contribution, based on the variance component estimates, is calculated by dividing each value in VarComp by the Total Variation, then multiplying the result by 100.

For example, to calculate the %Contribution for Part-to-Part, divide the VarComp for Part-to-Part by the Total Variation and multiply by 100:

$$(1.43965/1.45132) * 100 \approx 99.20$$

Therefore, 99.2% of the total variation in the measurements is due to the differences between parts. This is considered very good. When %Contribution for Part-to-Part is high, the system is able to distinguish between parts.

### Using variance versus standard deviation

%Contribution, because it is based on the variance, sums to 100%.

MINITAB also displays columns with percentages based on the standard deviation (or square root of variance) of each term. These columns, labeled %StudyVar and %Tolerance, do not sum to 100% in this example. An advantage of using the standard deviation as a measure of the variation is that it has the same units as the part measurements and the tolerance. This allows for meaningful comparisons.

**Note** | MINITAB displays the column %Process when you enter a historical standard deviation in Options.

### Gage R&R

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Total Variation	1.45132	100.00

Source	StdDev (SD)	Study Var (6 * SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	0.10801	0.64807	8.97	8.10
Repeatability	0.10000	0.60000	8.30	7.50
Reproducibility	0.04082	0.24495	3.39	3.06
Operator	0.04082	0.24495	3.39	3.06
Part-To-Part	1.19986	7.19913	99.60	89.99
Total Variation	1.20471	7.22824	100.00	90.35

Number of Distinct Categories = 15



## Interpreting your results

### Percent study variation

Use %StudyVar when you are interested in comparing the measurement system variation to the total variation.

%StudyVar is calculated by dividing each value in StudyVar by Total Variation and multiplying by 100.

%StudyVar for gage R&R is

$$0.64807 / 7.22824 * 100 = 8.97\%.$$

StudyVar is calculated as 6 times the standard deviation for each source.

### Why use 6?

Typically, process variation is defined as 6s (standard deviation as estimate of  $\sigma$ ). When data are normally distributed, approximately 99.73% of the data fall within 6 standard deviations (+/- 3 standard deviations from the mean), and approximately 99% of the data fall within 5.15 standard deviations (+/- 2.575 standard deviations from the mean).

**Note** | The Automotive Industry Action Group (AIAG) recommends the use of 6 in gage R&R studies.

### Gage R&R

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## Interpreting your results

### Percent tolerance

Often, a comparison between the measurement system variation and the tolerance is informative.

If you enter the tolerance, MINITAB calculates %Tolerance, which compares measurement system variation to specifications. It is interpreted as the percentage of the tolerance used up by the measurement system variability.

**Note** | If your measurement has a single-sided specification, you do not have a tolerance range to use in your analysis. Therefore, the %Tolerance column will not appear in the output.

The measurement system variation (6\*SD for Total Gage R&R) is divided by the tolerance. The resulting proportion is multiplied by 100 and reported as %Tolerance.

%Tolerance for gage R&R is

$$0.64807 / 8 * 100 = 8.10\%$$

### Which metric to use?

Use %Tolerance or %StudyVar to evaluate your measuring system depending on its application.

- If the measurement system is used for process improvement (reducing part-to-part variation), %StudyVar is a better estimate of measurement precision.
- If the measurement system is used to evaluate parts relative to specifications, %Tolerance is a more appropriate metric.

### Gage R&R

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Number of Distinct Categories = 15

## Interpreting your results

### Total Gage R&R

Whether you consider %Study Var or %Tolerance, the contribution of the measurement system to the overall variation in this study is less than 10%

Total Gage R&R:

- %Study Var—8.97
- %Tolerance—8.10

Remember that the difference between %Tolerance and %Study Var is the divisor. Because the range for tolerance (8) is wider than the total study variation (7.22824), the percentages for %Tolerance are lower in this example.

### Gage R&R

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## Interpreting your results

### Number of Distinct Categories

The number of distinct categories estimates how many separate groups of parts the system is able to distinguish.

The number of distinct categories that can be reliably observed is calculated by:

$$\frac{S_{\text{part}}}{S_{\text{measuring system}}} \times \sqrt{2}$$

The value is then truncated to the integer.

Number of categories	Means...
<2	The system cannot discriminate between parts.
2	Parts can be divided into high and low groups. This is equivalent to attributes data.
≥5	The system is acceptable (according to the AIAG) and can distinguish between parts.

Here, the number of distinct categories is 15, which indicates the system is very capable of distinguishing between parts.

**Note** | The AIAG recommends the number of distinct categories to be 5 or more. See page 45 of [1].)

### Gage R&R

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## Interpreting your results

### Components of Variation

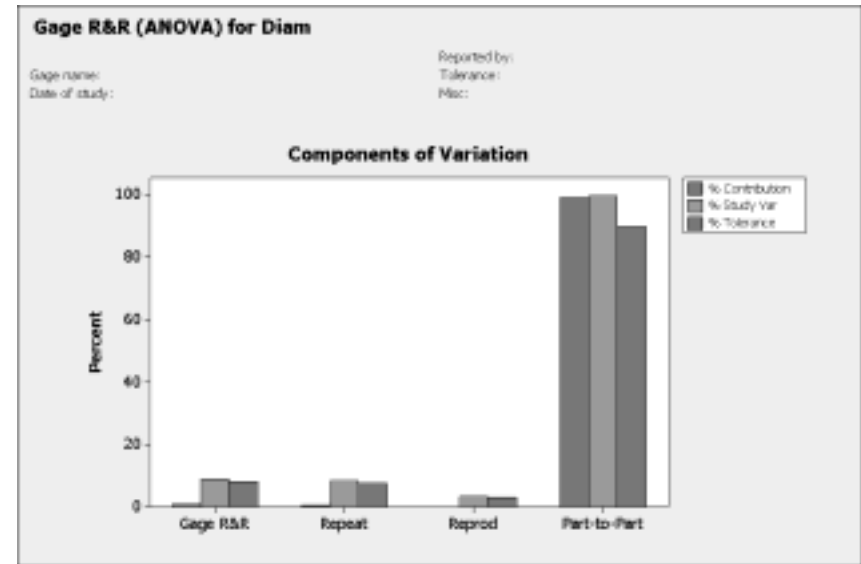
The Components of Variation chart graphically represents the gage R&R table in the Session window output.

Each cluster of bars represents a source of variation. By default, each cluster will have two bars, corresponding to %Contribution and %StudyVar. If you add a tolerance and/or historical standard deviation, bars for %Tolerance and/or %Process appear.

In a good measurement system, the largest component of variation is part-to-part variation. If, instead, large variation is attributed to the measurement system, then corrective action may be needed.

For the nozzle data, the difference in parts accounts for most of the variation.

**Note** For the %Study and %Tolerance measures, the Repeat and Reprod bars may not sum to the Gage R&R bar. This is because these percentages are based on standard deviations, not variances.



## Interpreting your results

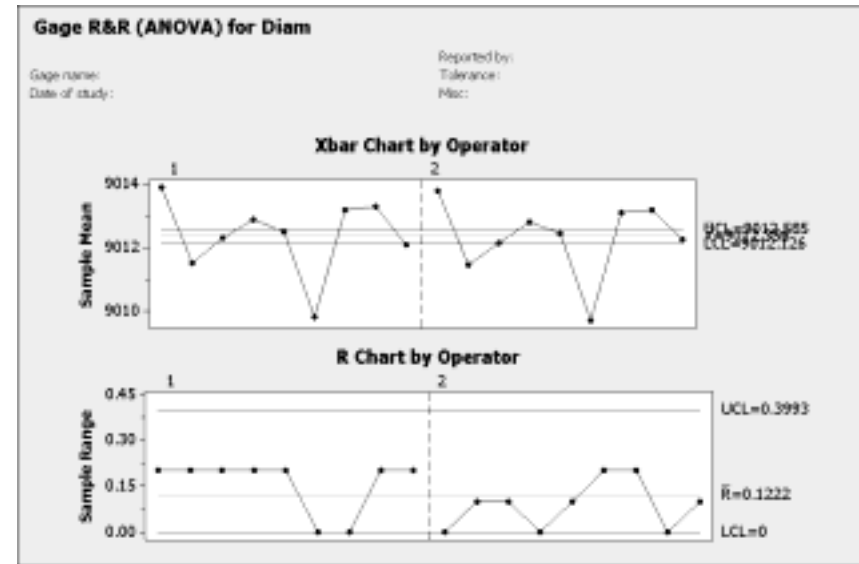
### R chart

The R chart is a control chart of subgroup ranges which graphically displays operator consistency. An R chart consists of:

- Plotted points, which represent, for each operator, the difference between the largest and smallest measurements of each part. If the measurements are the same, the range = 0.
- Because the points are plotted by operator, you can compare the consistency of each operator.
- Center line, which is the grand average of the ranges (average of all the subgroup ranges).
- Control limits (UCL and LCL) for the subgroup ranges. These limits are calculated using the variation within subgroups.

If any of the points on the R-chart go above the upper control limit (UCL), that operator is having difficulty consistently measuring that part or parts. The UCL value takes into account the number of times an operator measures a part. If operators are measuring consistently, these ranges should be small relative to the data and the points should be in statistical control.

**Note** | The R chart is displayed when the number of replicates is less than 9; otherwise, an S chart is displayed.



## Interpreting your results

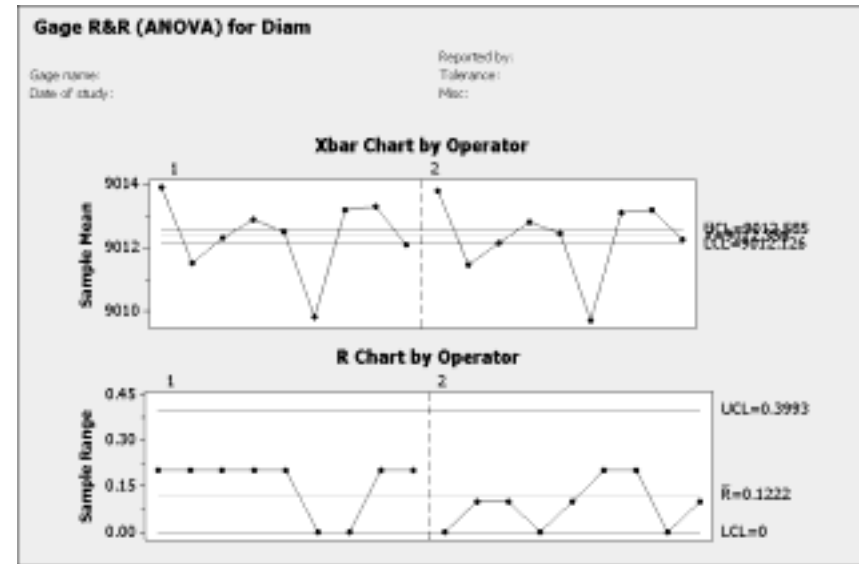
### $\bar{X}$ chart

The  $\bar{X}$  chart compares the part-to-part variation to the repeatability component. The  $\bar{X}$  chart consists of:

- Plotted points, which represent, for each operator, the average measurement of each part.
- Center line, which is the overall average for all part measurements by all operators.
- Control limits (UCL and LCL), which are based on the number of measurements in each average and the repeatability estimate.

This graph should ideally show lack of control because the parts chosen for a gage R&R study should represent the range of feasible parts, and it is desirable to have small repeatability variation compared to the part to part variation.

Lack of statistical control exists when many points are above the upper control limit and/or below the lower control limit. For these data, many points are well beyond the control limits, which indicates that part-to-part variation is much greater than variation caused by the measurement device.



## Interpreting your results

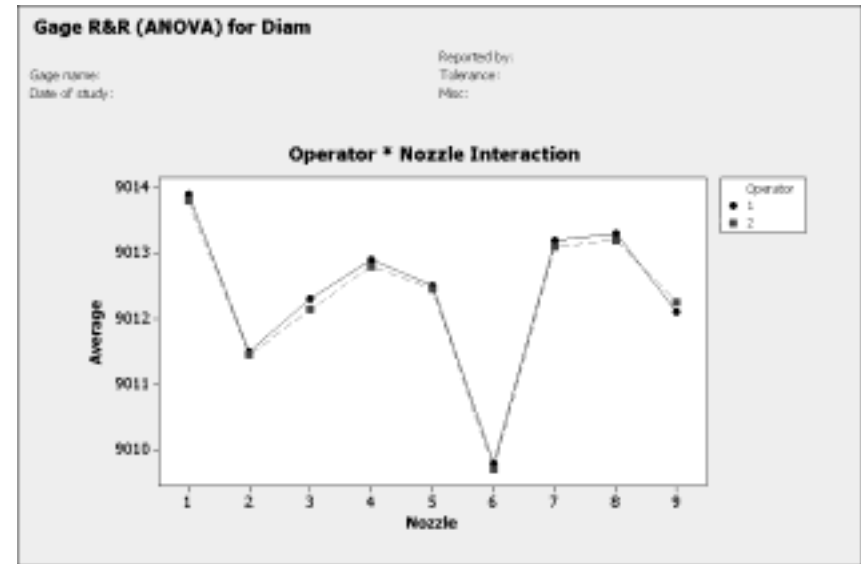
### Operator by part interaction

The Operator\*Nozzle Interaction plot displays the average measurements taken by each operator for each part. Each line connects the averages for a single operator.

Ideally, the lines are coincident and the part averages vary enough so that differences between parts are clear.

Pattern...	Means...
Lines are virtually identical.	Operators are measuring the parts similarly.
One line is consistently higher or lower than the others.	One operator is measuring parts consistently higher or lower than the other operators.
Lines are not parallel or they cross.	An operator's ability to measure a part depends on which part is being measured (an interaction between Operator and Part).

Here, the lines follow one another closely and the differences between parts are clear. The operators seem to be measuring parts similarly.





## Interpreting your results

### Measurements by operator

The By Operator main effects plot can help determine whether the measurements and variability are consistent across operators.

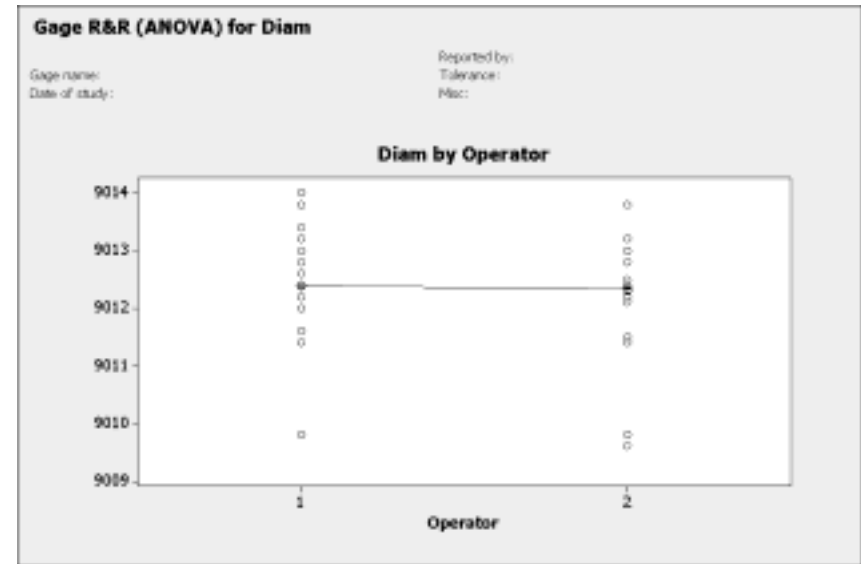
The By Operator graph shows all study measurements arranged by operator. Dots represent the measurements; black circles represent the means. The line connects the average measurements for each operator.

If the line is...	Then...
Parallel to the x-axis	The operators are measuring the parts similarly, on average.
Not parallel to the x-axis	The operators are measuring the parts differently, on average.

You can also use this graph to assess whether the overall variability in part measurements for each operator is the same:

- Is the spread in the measurements similar?
- Is one operator exhibiting more variation in the results than the others?

Here, the operators appear to be measuring the parts consistently, with approximately the same variation.



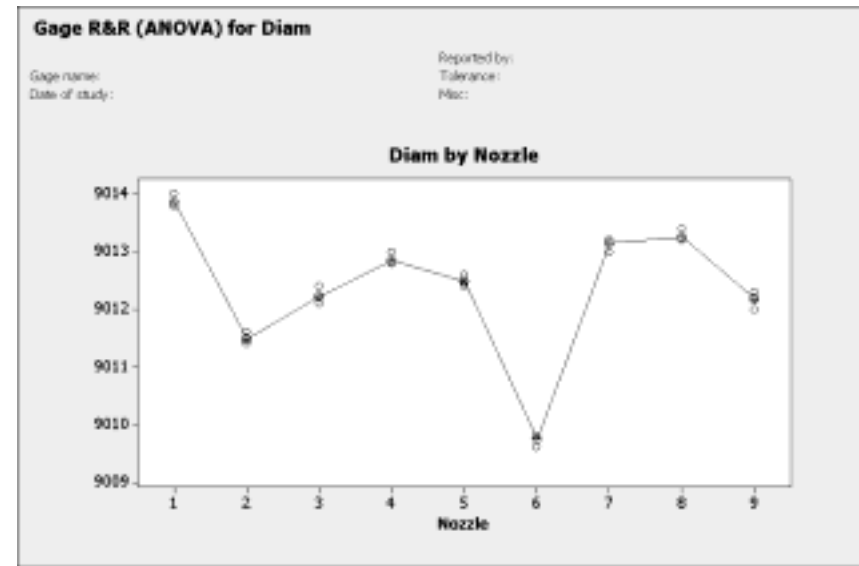
## Interpreting your results

### Measurements by part

The By Nozzle plot shows all the measurements taken in the study, arranged by part. The measurements are represented by empty circles; the means by solid circles. The line connects the average measurements for each part.

Ideally:

- Multiple measurements for individual parts have little variation (the empty circles for each part will be close together).
- Averages will vary enough so that differences between parts are clear.



## Final considerations

### Summary and conclusions

In this example, the measuring system contributes very little variation to the overall variation, as confirmed by both the gage R&R table and graphs.

The variation due to the measuring system, whether as a percent of study variation or as a percent of tolerance, is less than 10%. Based on AIAG guidelines, this is an acceptable system.

### Additional considerations

AIAG guidelines for the gage R&R table follow:

%Tolerance, %StudyVar	%Contribution	System is...
10% or less	1% or less	Acceptable
10% – 30%	1% – 9%	Marginal
30% or greater	9% or greater	Unacceptable

Source: page 77 of [1].

Graph patterns that show low measuring-system variation:

Graph	Pattern
R-bar	Small average range
$\bar{X}$ chart	Narrow control limits and many points out of control
By part	Very similar individual measurements for each part across all operators, and obvious differences between parts
By operator	Straight horizontal line
Operator by part	Overlaid lines

Because Gage R&R (Crossed) studies, like other MSA procedures, are designed experiments, good practices (for example, randomization) are essential to obtain valid results.