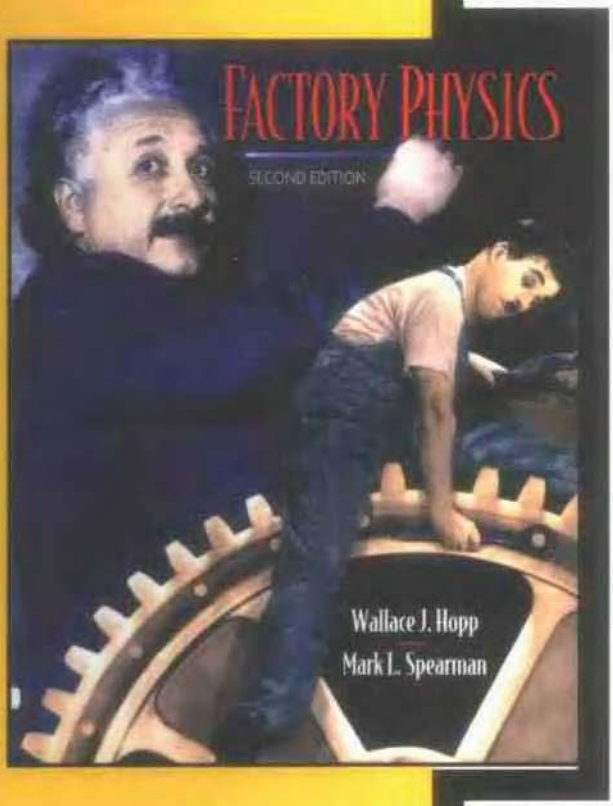
国外大学优秀教材——工业工程系列（影印版）

Wallace J. Hopp　Mark L. Spearman

工厂物理学

——制造企业管理基础

（第2版）



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该书的作者是美国西北大学的Wallace J.Hopp教授和佐治亚理工学院的Mark L.Spearman教授，是生产运作管理领域的知名学者，他们运用自己深厚的物理学中方法论的背景，在多年实践经验和理论研究的基础上，深刻分析与阐述了作业管理中的内在规律，以独特的视角与思维方式对发生在制造企业中的现象和本质进行了透彻的分析和系统的总结，以类似于物理学中定律定理的方式给出了准确的定性描述或定量计算公式。书中不仅对生产管理的发展历史和现状、取得的成就和问题等进行了精辟的总结和分析，而且紧密跟踪当前最先进的方法和技术，并预测了今后的发展趋势。该书不同于一般的教科书，一方面涉猎范围极宽，广泛介绍了生产领域的概念、方法、技术及实践效果；另一方面对重点问题进行了极为深入细致的研究，探究了事物的本质，提出了独到的见解。

该书的起点较高，适合作为“生产系统”和“运作管理”方面的研究生课程的主教材。对本科生教学，可以作为“生产运作管理”、“生产计划与控制”、“设施规划与物流分析”、“质量管理”等课程的主要参考书。

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Factory Physics

Foundations of Manufacturing Management

SECOND EDITION

工厂物理学

——制造企业管理基础

（第2版）

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Northwestern University
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Georgia Institute of Technology

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Forward

This textbook series is published at a very opportunity time when the discipline of industrial engineering is experiencing a phenomenal growth in China academia and with its increased interests in the utilization of the concepts, methods and tools of industrial engineering in the workplace. Effective utilization of these industrial engineering approaches in the workplace should result in increased productivity, quality of work, satisfaction and profitability to the cooperation.

The books in this series should be most suitable to junior and senior undergraduate students and first year graduate students, and to those in industry who need to solve problems on the design, operation and management of industrial systems.

Gavriel Salvendy

Department of Industrial Engineering, Tsinghua University

School of Industrial Engineering, Purdue University

April, 2002

前　言

本教材系列的出版正值中国学术界工业工程学科经历巨大发展、实际工作中对工业工程的概念、方法和工具的使用兴趣日渐浓厚之时。在实际工作中有效地应用工业工程的手段将无疑会提高生产率、工作质量、合作的满意度和效果。

该系列中的书籍对工业工程的本科生、研究生和工业界中需要解决工程系统设计、运作和管理诸方面问题的人士最为适用。

加弗瑞尔·沙尔文迪
清华大学工业工程系
普渡大学工业工程学院（美国）
2002 年 4 月

P R E F A C E

Origins of Factory Physics

In 1988 we were working as consultants at the IBM raw card plant in Austin, Texas, helping to devise more effective production control procedures. Each time we suggested a particular course of action, our clients would, quite reasonably, ask us to explain *why* such a thing would work. Being professors, we responded by immediately launching into theoretical lectures, replete with outlandish metaphors and impromptu graphs. After several semicoherent presentations, our sponsor, Jack Fisher, suggested we organize the essentials of what we were saying into a formal one-day course.

We did our best to put together a structured description of basic plant behavior. While doing this, we realized that certain very fundamental relations—for example, the relation between throughput and WIP, and several other basic results of Part II of this book—were not well known and were not covered in any standard operations management text. Our six offerings of the course at IBM were well received by audiences ranging from machine operators to mid-level managers. During one class, a participant observed, "Why, this is like physics of the factory!" Since both of us have bachelor's degrees in physics and keep a soft spot in our hearts for the subject, the name stuck. Factory physics was born.

Buoyed by the success of the IBM course, we developed a two-day industry course on short-cycle manufacturing, using factory physics as the organizing framework. Our focus on cycle time reduction forced us to strengthen the link between fundamental relations and practical improvement policies. Teaching to managers and engineers from a variety of industries helped us extend our coverage to more general production environments.

In 1990, Northwestern University launched the Master of Management in Manufacturing (MMM) program, for which we were asked to design and teach courses in management science and operations management. By this time we had enough confidence in factory physics to forgo traditional problem-based and anecdote-based approaches to these subjects. Instead, we concentrated on building intuition about basic manufacturing behavior as a means for identifying areas of leverage and comparing alternate control policies. For completeness and historical perspective, we added coverage of conventional topics, which became the basis for Part I of this book. We received enthusiastic support from the MMM students for the factory physics approach. Also, because they had substantial and varied industry experience, they constructively challenged our ideas and helped us sharpen our presentation.

In 1993, after having taught the MMM courses and the industry short course several times, we began writing out our approach in book form. This proved to be a slow process because it revealed a number of gaps between our presentation of concepts and their

implementation in practice. Several times we had to step back and draw upon our own research and that of many others, to develop practical discussions of key manufacturing management problem areas. This became Part III of this book.

Factory physics has grown a great deal since the days of our terse tutorials at IBM and will undoubtedly continue to expand and mature. Indeed, this second edition contains several new developments and changes of presentation from the first edition. But while details will change, we are confident that the fundamental insight behind factory physics—that there are principles governing the behavior of manufacturing systems, and understanding them can improve management practice—will remain the same.

Intended Audience

Factory Physics is intended for three principal academic audiences:

1. *Manufacturing management students* in a core manufacturing operations course.
2. *MBA students* in a second operations management course following a general survey course.
3. *BS and MS industrial engineering students* in a production control course.

We also hope that practicing manufacturing managers will find this book a useful training reference and source of practical ideas.

How to Use this Book

After a brief introductory chapter, the book is organized into three parts: Part I, The Lessons of History; Part II, Factory Physics; and Part III, Principles in Practice. In our own teaching, we generally cover Parts I, II, and III in order, but vary the selection of specific topics depending on the course. Regardless of the audience, we try to cover Part II completely, as it represents the core of the factory physics approach. Because it makes extensive use of pull production systems, we make sure to cover Chapter 4 on "The JIT Revolution" prior to beginning Part II. Finally, to provide an integrated framework for carrying the factory physics concepts into the real world, we regard Chapter 13, "A Pull Planning Framework," as extremely important. Beyond this, the individual instructor can select historical topics from Part I, applied topics from Part III, or additional topics from supplementary readings to meet the needs of a specific audience.

The instructor is also faced with the choice of how much mathematical depth to use. To assist readers who want general concepts with minimal mathematics, we have set off certain sections as *Technical Notes*. These sections, which are labeled and indented in the text, present justification, examples, or methodologies that rely on mathematics (although nothing higher than simple calculus). These sections can be skipped completely without loss of continuity.

In teaching this material to both engineering and management students, we have found, not surprisingly, that management students are less interested in the mathematical aspects of factory physics than are engineering students. However, we have not found management students to be averse to mathematics; it is math without a concrete purpose to which they object. When faced with quantitative developments of core manufacturing ideas, these students not only are capable of grasping the math, but also are able to appreciate the practical consequences of the theory.

New to the Second Edition

The basic structure of the second edition is the same as that of the first. Aside from moving Chapter 12 on Total Quality Manufacturing from Part III to Part II, where it has been adapted to highlight the importance of quality to the science of factory physics, the basic content and placement of the chapters are unchanged. However, a number of enhancements have been made, including the following:

- *More problems.* The number of exercises at the end of each chapter has been increased to offer the reader a wider range of practice problems.
- *More examples.* Almost all models are motivated with a practical application before the development of any mathematics. Frequently, these applications are then used as examples to illustrate how the model is used.
- *Web support.* Powerpoint presentations, case materials, spreadsheets, derivations, and a solutions manual are now available on the Web. These are constantly being updated as more material becomes available. Go to http://www.mhhe.com/pom under Text Support for our web site.
- *Inventory management.* The development of inventory models in Chapter 2 has been enhanced to frame historical results in terms of modern theory and to provide the reader with the most sophisticated tools available. Excel spreadsheets and inventory function add-ins are available over the Web to facilitate the more complex inventory calculations.
- *Enterprise resources planning.* Chapters 3 and 5 describe how materials requirements planning (MRP) has evolved into enterprise resources planning (ERP) and gives an outline of a typical ERP structure. We also describe why ERP is not the final solution to the production planning problem.
- *People in production systems.* Chapter 7 now includes some laws concerning the behavior of production lines in which personnel capacity is an important constraint along with equipment capacity.
- *Variability pooling.* Chapter 8 introduces the fundamental idea that variability from independent sources can be reduced by combining the sources. This basic idea is used throughout the book to understand disparate practices, such as how safety stock can be reduced by stocking generic parts, how finished goods inventories can be reduced by "assembling to order," and how elements of push and pull can be combined in the same system.
- *Systems with blocking.* Chapter 8 now includes analytic models for evaluating performance of lines with finite, as well as infinite, buffers between stations. Such models can be used to represent kanban systems or systems with physical limitations of interstation inventory. A spreadsheet for examining the tradeoffs of additional WIP buffers, decreasing variability, and increasing capacity is available on the Web.
- *Sharper variability results.* Several of the laws in Chapter 9, The Corrupting Influence of Variability, have been restated in clearer terms; and some important new laws, corollaries, and definitions have been introduced. The result is a more complete science of how variability degrades performance in a production system.
- *Optimal batch sizes.* Chapters 9 and 15 extend the factory physics analysis of the effects of batching to a normative method for setting batch sizes to minimize cycle times in multiproduct systems with setups and discuss implications for production scheduling.

- *General CONWIP line models.* Chapter 10 now includes an analytic procedure for computing the throughput of a CONWIP line with general processing times. Previously, only the case with balanced exponential stations (the practical worst case) was analyzed explicitly. These new models are easy to implement in a spreadsheet (available on the Web) and are useful for examining inventory, capacity, and variability tradeoffs in CONWIP lines.
- *Quality control charts.* The quality discussion of Chapter 12 now includes an overview of statistical process control (SPC).
- *Forecasting.* The section on forecasting has been expanded into a separate section of Chapter 13. The treatment of time series models has been moved into this section from an appendix and now includes discussion of forecasting under conditions of seasonal demand.
- *Capacitated material requirements planning.* The MRP-C methodology for scheduling production releases with explicit consideration of capacity constraints has been extended to consider material availability constraints as well.
- *Supply chain management.* The treatment of inventory management is extended to the contemporary subject of supply chain management. Chapter 17 now deals with this important subject from the perspective of multiechelon inventory systems. It also discusses the "bullwhip effect" as a means for understanding some of the complexities involved in managing and designing supply chains.

W.J.H.
M.L.S.

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Factory Physics Principles

Law (Little's Law):

$$\text{WIP} = \text{TH} \times \text{CT}$$

Law (Best-Case Performance): *The minimum cycle time for a given WIP level w is given by*

$$\text{CT}_{\text{best}} = \begin{cases} T_0 & \text{if } w \leq W_0 \\ \dfrac{w}{r_b} & \text{otherwise} \end{cases}$$

The maximum throughput for a given WIP level w is given by

$$\text{TH}_{\text{best}} = \begin{cases} \dfrac{w}{T_0} & \text{if } w \leq W_0 \\ r_b & \text{otherwise} \end{cases}$$

Law (Worst-Case Performance): *The worst-case cycle time for a given WIP level w is given by*

$$\text{CT}_{\text{worst}} = wT_0$$

The worst-case throughput for a given WIP level w is given by

$$\text{TH}_{\text{worst}} = \frac{1}{T_0}$$

Definition (Practical Worst-Case Performance): *The practical worst-case (PWC) cycle time for a given WIP level w is given by*

$$\text{CT}_{\text{PWC}} = T_0 + \frac{w - 1}{r_b}$$

The PWC throughput for a given WIP level w is given by

$$\text{TH}_{\text{PWC}} = \frac{w}{W_0 + w - 1} r_b$$

Law (Labor Capacity): *The maximum capacity of a line staffed by n cross-trained operators with identical work rates is*

$$\text{TH}_{\text{max}} = \frac{n}{T_0}$$

Law (CONWIP with Flexible Labor): *In a CONWIP line with n identical workers and w jobs, where $w \geq n$, any policy that never idles workers when unblocked jobs are available will achieve a throughput level $\text{TH}(w)$ bounded by*

$$\text{TH}_{\text{CW}}(n) \leq \text{TH}(w) \leq \text{TH}_{\text{CW}}(w)$$

where $\text{TH}_{\text{CW}}(x)$ represents the throughput of a CONWIP line with all machines staffed by workers and x jobs in the system.

Law (Variability): *Increasing variability always degrades the performance of a production system.*

Corollary (Variability Placement): *In a line where releases are independent of completions, variability early in a routing increases cycle time more than equivalent variability later in the routing.*

Law (Variability Buffering): *Variability in a production system will be buffered by some combination of*

1. *Inventory*
2. *Capacity*
3. *Time*

Corollary (Buffer Flexibility): *Flexibility reduces the amount of variability buffering required in a production system.*

Law (Conservation of Material): *In a stable system, over the long run, the rate out of a system will equal the rate in, less any yield loss, plus any parts production within the system.*

Law (Capacity): *In steady state, all plants* will *release work at an average rate that is strictly less than the average capacity.*

Law (Utilization): *If a station increases utilization without making any other changes, average WIP and cycle time will increase in a highly nonlinear fashion.*

Law (Process Batching): *In stations with batch operations or with significant changeover times:*

1. *The minimum process batch size that yields a stable system may be greater than one.*
2. *As process batch size becomes large, cycle time grows proportionally with batch size.*
3. *Cycle time at the station will be minimized for some process batch size, which may be greater than one.*

Law (Move Batching): *Cycle times over a segment of a routing are roughly proportional to the transfer batch sizes used over that segment, provided there is no waiting for the conveyance device.*

Law (Assembly Operations): *The performance of an assembly station is degraded by increasing any of the following:*

1. *Number of components being assembled.*
2. *Variability of component arrivals.*
3. *Lack of coordination between component arrivals.*

Definition (Station Cycle Time): *The average cycle time at a station is made up of the following components:*

$$\begin{aligned}\text{Cycle time} = {} & \text{move time} + \text{queue time} + \text{setup time} + \text{process time} \\ & + \text{wait-to-batch time} + \text{wait-in-batch time} \\ & + \text{wait-to-match time}\end{aligned}$$

Definition (Line Cycle Time): *The average cycle time in a line is equal to the sum of the cycle times at the individual stations, less any time that overlaps two or more stations.*

Law (Rework): *For a given throughput level, rework increases both the mean and standard deviation of the cycle time of a process.*

Law (Lead Time): *The manufacturing lead time for a routing that yields a given service level is an increasing function of both the mean and standard deviation of the cycle time of the routing.*

Law (CONWIP Efficiency): *For a given level of throughput, a push system will have more WIP on average than an equivalent CONWIP system.*

Law (CONWIP Robustness): *A CONWIP system is more robust to errors in WIP level than a pure push system is to errors in release rate.*

Law (Self-Interest): *People, not organizations, are self-optimizing.*

Law (Individuality): *People are different.*

Law (Advocacy): *For almost any program, there exists a champion who can make it work—at least for a while.*

Law (Burnout): *People get burned out.*

Law (Responsibility): *Responsibility without commensurate authority is demoralizing and counterproductive.*

BRIEF CONTENTS

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C H A P T E R

0 FACTORY PHYSICS?

Perfection of means and confusion of goals seem to characterize our age.
Albert Einstein

0.1 The Short Answer

What is factory physics, and why should one study it?

Briefly, **factory physics** is a *systematic description of the underlying behavior of manufacturing systems*. Understanding it enables managers and engineers to work with the natural tendencies of manufacturing systems to

1. Identify opportunities for improving existing systems.
2. Design effective new systems.
3. Make the tradeoffs needed to coordinate policies from disparate areas.

0.2 The Long Answer

The above definition of factory physics is concise, but leaves a great deal unsaid. To provide a more precise description of what this book is all about, we need to describe our focus and scope, define more carefully the meaning and purpose of factory physics, and place these in context by identifying the manufacturing environments on which we will concentrate.

0.2.1 Focus: Manufacturing Management

To answer the question of why one should study factory physics, we must begin by answering the question of why one should study manufacturing at all. After all, one frequently hears that the United States is moving to a service economy, in which the manufacturing sector will represent an ever-shrinking component. On the surface this appears to be true: Manufacturing employed on the order of 50 percent of the workforce in 1950, but only about 20 percent by 1985. To some, this indicates a trend in manufacturing that parallels the experience in agriculture earlier in the century. In 1929, agriculture

employed 29 percent of the workforce; by 1985, it employed only three percent. During this time there was a shift away from low-productivity, low-pay jobs in agriculture and toward higher-productivity, higher-pay jobs in manufacturing, resulting in a dramatic increase in the overall standard of living. Similarly, proponents of this analogy argue, we are currently shifting from a manufacturing-based workforce to an even more productive service-based workforce, and we can expect even higher living standards.

However, as Cohen and Zysman point out in their elegant and well-documented book *Manufacturing Matters: The Myth of the Post-Industrial Economy* (1987), there is a fundamental flaw in this analogy. Agriculture was *automated,* while manufacturing, at least partially, is being moved *offshore*—moved abroad. Although the number of agricultural jobs declined, due to a dramatic increase in productivity, American agricultural output did not decline after 1929. As a result, most of the jobs that are *tightly linked* to agriculture (truckers, vets, crop dusters, tractor repairers, mortgage appraisers, fertilizer sales representatives, blight insurers, agronomists, chemists, food processing workers, etc.) were not lost. When these tightly linked jobs are considered, Cohen and Zysman estimate that the number of jobs currently dependent on agricultural production is not three million, as one would obtain by looking at an SIC (standard industrial classification) count, but rather something on the order of six to eight million. That is, two or three times as many workers are employed in jobs tightly linked to agriculture as are employed directly in agriculture itself.

Cohen and Zysman extend this linkage argument to manufacturing by observing that many jobs normally thought of as being in the service sector (design and engineering services, payroll, inventory and accounting services, financing and insuring, repair and maintenance of plant and machinery, training and recruiting, testing services and labs, industrial waste disposal, engineering support services, trucking of semifinished goods, etc.) depend on manufacturing for their existence. If the number of manufacturing jobs declines due to an increase in productivity, many of these tightly linked jobs will be retained.

But if American manufacturing declines by being moved offshore, many tightly linked jobs will shift overseas as well. There are currently about 21 million people employed directly in manufacturing. Therefore, if a similar multiplier to that estimated by Cohen and Zysman for agriculture applies, there are some 20 to 40 million tightly linked jobs that depend on manufacturing. This implies that over half of the jobs in America are strongly tied to manufacturing. Even without considering the indirect effects (e.g., unemployed or underemployed workers buy fewer pizzas and attend fewer symphonies) of losing a significant portion of the manufacturing jobs in this country, the potential economic consequences of moving manufacturing offshore are enormous.

During the 1980s when we began work on the first edition of this book, there were many signs that American manufacturing was not robust. Productivity growth relative to that in other industrialized countries had slowed dramatically. Shares of domestic firms in several important markets (e.g., automobiles, consumer electronics, machine tools) had declined alarmingly. As a result of rising imports, America had become the world's largest debtor nation, mounting huge trade deficits with other manufacturing powers, such as Japan. The fraction of American patents granted to foreign inventors had doubled over the previous two decades. These and many other trends seemed to indicate that American manufacturing was in real trouble.

The reasons for this decline were complex and controversial, as we will discuss further in Part I. Moreover, in many regards, American manufacturing made a recovery in the 1990s as net income of manufacturers rose almost 65 percent in constant dollars from 1985 to 1994 (Department of Commerce 1997). But one conclusion stands out

as obvious—global competition has intensified greatly since World War II, particularly since the 1980s, due to the recovery of economies devastated by the war. Japanese, European, and Pacific Rim firms have emerged as strong competitors to the once-dominant American manufacturing sector. Because they have more options, customers have become increasingly demanding. It is no longer possible to offer products, as Henry Ford once did, in "any color as long as it's black." Customers expect variety, reasonable price, high quality, comprehensive service, and responsive delivery. Therefore, from now on, in good economic times and bad, only those firms that can keep pace along all these dimensions will survive.

Although speaking of manufacturing as a monolithic whole may continue to make for good political rhetoric, the reality is that the rise or fall of the American manufacturing sector will occur one firm at a time. Certainly a host of general policies, from tax codes to educational initiatives, can help the entire sector somewhat; the ultimate success of each individual firm is fundamentally determined by the *effectiveness of its management*. Hence, quite literally, our economy, and our very way of life in the future, depends on how well American manufacturing managers adapt to the new globally competitive environment and evolve their firms to keep pace.

0.2.2 Scope: Operations

Given that the study of manufacturing is worthwhile, how should we study it? Our focus on management naturally leads us to adopt the high-level orientation of **"big M" manufacturing,** which includes product design, process development, plant design, capacity management, product distribution, plant scheduling, quality control, workforce organization, equipment maintenance, strategic planning, supply chain management, interplant coordination, as well as direct production—**"little m" manufacturing**—functions such as cutting, shaping, grinding, and assembly.

Of course, no single book can possibly cover all big M manufacturing. Even if one could, such a broad survey would necessarily be shallow. To achieve the depth needed to promote real understanding, we must narrow our scope. However, to preserve the "big picture" management view, we cannot restrict it too much; highly detailed treatment of narrow topics (e.g., the physics of metal cutting) would constitute such a narrow viewpoint that, while important, would hardly be suitable for identifying effective management policies. The middle ground, which represents a balance between high-level integration and low-level details, is the operations viewpoint.

In a broad sense, the term **operations** refers to the application of resources (capital, materials, technology, and human skills and knowledge) to the production of goods and services. Clearly, all organizations involve operations. Factories produce physical goods. Hospitals produce surgical and other medical procedures. Banks produce checking account transactions and other financial services. Restaurants produce food and perhaps entertainment. And so on.

The term *operations* also refers to a specific function in an organization, distinct from other functions such as product design, accounting, marketing, finance, human resources, and information systems. Historically, people involved in the operations function are housed in departments with names like production control, manufacturing engineering, industrial engineering, and planning, and are responsible for the activities directly related to the production of goods and services. These typically include plant scheduling, inventory control, quality assurance, workforce scheduling, materials management, equipment maintenance, capacity planning, and whatever else it takes to get product out the door.

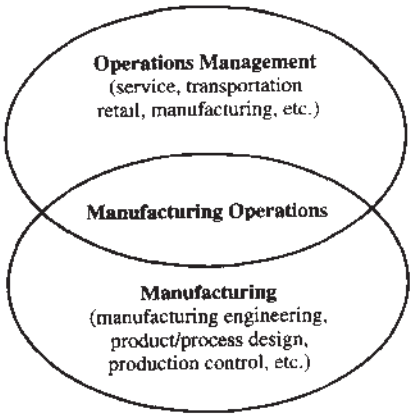
In this book, we view operations in the broad sense rather than as a specific function. We seek to give general managers the insight necessary to sift through myriad details in a production system and identify effective policies. The operations view focuses on the *flow of material* through a plant, and thereby places clear emphasis on most of the key measures by which manufacturing managers are evaluated (throughput, customer service, quality, cost, investment in equipment and materials, labor costs, efficiency, etc.). Furthermore, by avoiding the need for detailed descriptions of products or processes, this view concentrates on *generic manufacturing behavior,* which makes it applicable to a wide range of specific environments.

The operations view provides a unifying thread that runs through all the various big-M manufacturing issues. For instance, operations and product design are linked in that a product's design determines how it must flow through a plant and how difficult it will be to make. Adopting an operations viewpoint in the design process therefore promotes **design for manufacturability.** In the same fashion, operations and strategic planning are closely tied, since strategic decisions determine the number and types of products to be produced, the size of the manufacturing facilities, the degree of vertical integration, and many other factors that affect what goes on inside the plant. Embedding a concern for operations in strategic decision making is essential for ensuring feasible plans. Other manufacturing functions have analogous relationships to operations, and hence can be coordinated with the actual production process by addressing them from an operations viewpoint.

The traditional field in which operations are studied is called **operations management (OM).** However, OM is broader than the scope of this book, since it encompasses operations in service, as well as manufacturing, organizations. Just as our operations scope covers only part of (big M) manufacturing, our manufacturing focus includes only part of operations management. In short, the scope of this book can be envisioned as the intersection between OM and manufacturing, as illustrated in Figure 0.1.

The operations view of manufacturing may seem a somewhat technical perspective for a management book. This is not accidental. Some degree of technicality is required just to accurately describe manufacturing behavior in operations terms. More importantly, however, is the reality that in today's environment, *manufacturing itself is technical.* Intense global competition is relentlessly raising market standards, causing seemingly small details to take on large strategic importance. For example, quality acceptable to customers in the 1970s may have been possible with relatively unsophisticated systems. But to meet customer expectations and comply with standards common

FIGURE 0.1
Manufacturing and operations management



for vendor certification today is virtually impossible without rigorous quality systems in place. Similarly, it was not so long ago when customer service could be ensured by maintaining large inventories. Today, rapid technological change and smaller profit margins make such a strategy uneconomical—literally forcing companies into the tighter control systems necessary to run with low-inventory levels. These shifts are making operations a more integral, and more technical, component of running a manufacturing business.

The trends of the 1990s may make it appear that the importance of operations is a new phenomenon. But, as we will discuss in greater depth in Part I, low-level operations details have *always* had strategic consequences for manufacturing firms. A recent reminder of this fact was the experience of Japan in the 1970s and 1980s. As Chapter 4 describes, Japanese firms, particularly Toyota, were able to carry out a strategy of low-cost, small-lot production only through intense attention to minute details on the factory floor (e.g., die changing, statistical process control, material flow control) over an extended time. The net result was an enormously effective competitive weapon that permitted Toyota to rise from obscurity to a position as a worldwide automotive leader.

Today, the importance of operations to the health, and even viability, of manufacturing firms is greater than ever due to global competition in the following three dimensions:

1. **Cost.** This is the traditional dimension of competition that has always been the domain of operations management. Efficient utilization of labor, material, and equipment is essential to keeping costs competitive. We should note, however, that from the customer standpoint it is **unit cost** (total cost divided by total volume) that matters, implying that both cost reduction and volume enhancement are worthy OM objectives.

2. **Quality.** The 1980s brought widespread recognition in America that quality is a key competitive weapon. Of course, *external* quality, that seen by the customer, has always been a concern in manufacturing. The quality revolution of the 1980s served to focus attention on *internal* quality at each step in the manufacturing process, and its relationship to customer satisfaction. Facets of operations management, such as statistical process control, human factors, and material flow control, have loomed large in this context as components of **total quality management (TQM)** strategies.

3. **Speed.** While cost and quality remain critical, the 1990s can be dubbed the *decade of speed*. Rapid development of new products, coupled with quick customer delivery, are pillars of the **time-based competition (TBC)** strategies that have been adopted by leading firms in many industries. Bringing new products to market swiftly requires both performance of development tasks in parallel and the ability to efficiently ramp up production. Responsive delivery, without inefficient excess inventory, requires short manufacturing cycle times, reliable processes, and effective integration of disparate functions (e.g., sales and manufacturing). These issues are central to operations management, and they arise repeatedly throughout this book.

These three dimensions are broadly applicable to most manufacturing industries, but their relative importance obviously varies from one firm to another. A manufacturer of a commodity (baking soda, machine screws, resistors) depends critically on efficiency, since low cost is a condition for survival. A manufacturer of premium goods (luxury automobiles, expensive watches, leatherbound books) relies on quality to retain its market. A manufacturer of a high-technology product (computers, patent-protected pharmaceuticals, high-end consumer electronics) requires speed of introduction to be competitive and to maximally exploit potential profit during the limited economic lifetime of the product. Clearly, the management challenges in these varying environments are different. Since operations are integral to cost, quality, and speed, however, operations management has a key strategic role in each.

0.2.3 Method: Factory Physics

So far, we have determined that the focus of this book is manufacturing management, and the scope is operations. The question now becomes, How can managers use an operations viewpoint to identify a sensible combination of policies that are both effective now and flexible enough to adapt to future needs?

In our opinion, some conventional approaches to manufacturing management fall short:

1. *Management by imitation* is not the answer. Watching the competition can provide a company with a valuable source of benchmarking and may help it to avoid getting stuck in established modes of thinking. But imitation cannot provide the impetus for a truly significant competitive edge. Bold new ideas must come from within, not without.

2. *Management by buzzword* is not the answer. Manufacturing firms have become inundated with a wave of "revolutions" in recent years. MRP, JIT, TQM, BPR, TBC (and even a few without three-letter acronyms) have swept through the manufacturing community accompanied by soaring rhetoric and passionate emotion, but with little concrete detail. As we will observe in Part I, these movements have contained many valuable insights. However, they are very dangerous as management systems because it is far too easy for managers to become attached to catchy slogans and trendy buzzwords and lose sight of the fundamental objectives of the business. The result can be very poor decisions for the long run.

3. *Management by consultant* is, at best, only a partial solution. A good consultant can make an objective evaluation of a firm's policies and provide a source of new ideas. However, as an outsider, the consultant is not in a position to obtain the support of key people so critical to implementing new management systems. Additionally, a consultant can never have the intimate familiarity with the business that an insider has, and is therefore likely to push generic solutions, rather than customized methods that match the specific needs of the firm. No matter how good an off-the-shelf technology (e.g., scheduling tools, optical scanners, AGVs, robots) is, the manufacturing *system* must be ultimately designed in-house, if it is to be effective as a whole.

So, what is the answer? In our view, the answer is not *what to do* about manufacturing problems but rather *how to think* about them. Each manufacturing environment is unique. No single set of procedures can work well under all conditions. Therefore, effective manufacturing managers of the future will have to rely on a solid understanding of their systems to enable them to identify leverage points, creatively leapfrog the competition, and engender an environment of continual improvement. For the student of manufacturing management, this is something of a "good news–bad news" message. The bad news is that manufacturing managers will need to know more about the fundamentals of manufacturing than ever before. The good news is that the manager who has developed these skills will be increasingly valuable in industry.

From an operations viewpoint, there are behavioral tendencies shared by virtually all manufacturing enterprises. We feel that these can be organized into a body of knowledge to serve as a manufacturing manager's knowledge base, just as the field of medicine serves as a physician's knowledge base. In this book, we employ a spirit of rational inquiry to seek a **science of manufacturing** by establishing basic concepts as building blocks, stating fundamental principles as "manufacturing laws," and identifying general insights from specific practices. Our primary goal is to provide the reader with an organized framework from which to evaluate management practices and develop useful intuition about manufacturing systems. Our secondary goal is to encourage others to

push the science of manufacturing even further, developing new and better structures than we can offer at this time.

We use the term **factory physics** to distinguish our long-term emphasis on general principles from the short-term fixation on specific techniques inherent in the buzzword approach. We emphatically stress that factory physics is *not factory magic*. Rather, it is a discipline based on the scientific method that has several features in common with the field of physics:

1. *Problem-solving framework.* Just as there are few easy solutions in physics, there are few in manufacturing management. Physics offers rational approaches for understanding nature. An understanding of basic physics is critical to the engineer in building or designing a complex system. Likewise, factory physics provides a context for understanding manufacturing operations that allows the manufacturing manager or engineer to pose and solve the right problems.

2. *Technical approach.* Physics is generally viewed as a hard, technical subject. But, as we noted, OM is a hard technical subject as well. A presentation of OM without some technical content is like a newspaper description of an engineering feat without any physical description—it sounds interesting but the reader cannot tell how it is actually done. Such an approach might be legitimate as a *survey* of operations management, but is not suited to developing the skills needed by manufacturing managers and engineers.

3. *Role of intuition.* Physicists generally have well-developed intuition about the physical world. Even before writing any mathematical equations to represent a system, a physicist forms a qualitative feel for the important parameters and their relationships. Analogously, to make good decisions, a manager needs sound intuition about system behavior and the consequences of various actions. Thus, while we will spend a fair amount of time developing concepts with mathematical models, our real concern is not the analyses themselves, but rather the general intuition we can draw from them.

In the spirit of factory physics, we can summarize the key skills that will be required by the manager of the future as falling into three distinct categories: **basics, intuition,** and **synthesis.**[1] The relation of these to operations management and their role in this book are as follows:

1. **Basics.** The language and elementary concepts for describing manufacturing systems are essential prerequisites for any manufacturing manager. Although many basics of relevance to the manufacturing manager (e.g., elementary mathematics, statistics, physics of manufacturing processes) are outside the realm of OM and therefore the scope of this text, we do present a number of basic concepts integral to OM, dealing with variability, reliability, behavior of queuing systems, and so on. These are introduced as needed in Part II. We also cull valuable basic concepts from traditional OM practices in the historical survey of Part I.

2. **Intuition.** The single most important skill of a manufacturing manager is intuition regarding the behavior of manufacturing systems. Solid intuition enables a manager to identify leverage points in a plant, evaluate the impacts of proposed changes, and coordinate improvement efforts. We therefore devote the bulk of Part II to developing intuition about key types of manufacturing behavior.

[1] While these categories may be new for a manufacturing book, they are hardly revolutionary. The *Trivium*, which constituted the basis for a liberal education in the Middle Ages and consisted of grammar (the basic rules), logic (rational relationships), and rhetoric (fitting it all together), is virtually identical to our structure.

3. **Synthesis.** Close behind intuition on the list of important skills for a manufacturing manager is the ability to bring together the disparate components of a system into an effective whole. In part, this is related to the ability to understand tradeoffs and focus on critical parameters. But it also depends on the capacity to step back and view the system from a holistic perspective. We discuss a formal method for problem solving based on this view—the **systems approach**—in Chapter 6. A good manufacturing manager also considers improvements based on many different approaches (e.g., process changes, logistics changes, personnel policy changes) and is sensitive to the effects of changes in one area or another. In Part III, we present a production planning hierarchy that integrates potentially disjoint decisions, and we describe the interfaces between different functions. Often, the "biggest bang for the buck" lies at the interfaces, so we highlight them wherever possible throughout Parts II and III.

0.2.4 Perspective: Flow Lines

To use the factory physics method to study manufacturing management from an operations standpoint, we must select a primary perspective through which to view manufacturing systems. Without this, environmental differences will tend to obscure common underlying behavior and make development of a science of manufacturing impossible. The reason is that even when we adopt an operations view and ignore the low-level differences in products and processes, manufacturing environments vary greatly with respect to their **process structure,** that is, the manner in which material moves through the plant. For instance, the continuous flow nature of a chemical plant behaves very differently and hence presents a very different management picture than does the one-at-a-time artisan environment of a custom machine shop. Hayes and Wheelwright (1979) classify manufacturing environments by process structure into four categories (see Figure 0.2) which can be summarized as follows:

1. **Job shops.** Small lots are produced with a high variety of routings through the plant. Flow through the plant is jumbled, setups are common, and the environment has more of an atmosphere of project work than pacing. For example, a commercial printer, where each job has unique requirements, will generally be structured as a job shop.

2. **Disconnected flow lines.** Product batches are produced on a limited number of identifiable routings (i.e., paths through the plant). Although routings are distinct, individual stations within lines are not connected by a paced material handling system, so that inventories can build up between stations. The majority of manufacturing systems in industry resemble the disconnected flow line environment to some extent. For example, a heavy equipment (e.g., tank car) manufacturer will use well-defined assembly lines but, because of the scale and complexity of the processes at each station, generally will not automate and pace movement between stations.

3. **Connected flow lines.** This is the classic moving assembly line made famous by Henry Ford. Product is fabricated and assembled along a rigid routing connected by a paced material handling system. Automobiles, where frames travel along a moving assembly line between stations at which components are attached, are the classic application of the connected flow line. But, despite the familiarity and historic appeal of this type of system, automatic assembly lines are actually much less common than disconnected flow lines in industry.

4. **Continuous flow processes.** Continuous product (food, chemicals, oil, roofing materials, fiberglass insulation, etc.) flows automatically down a fixed routing. Many food processing plants, such as sugar refineries, make use of continuous flow to achieve high efficiency and product uniformity.

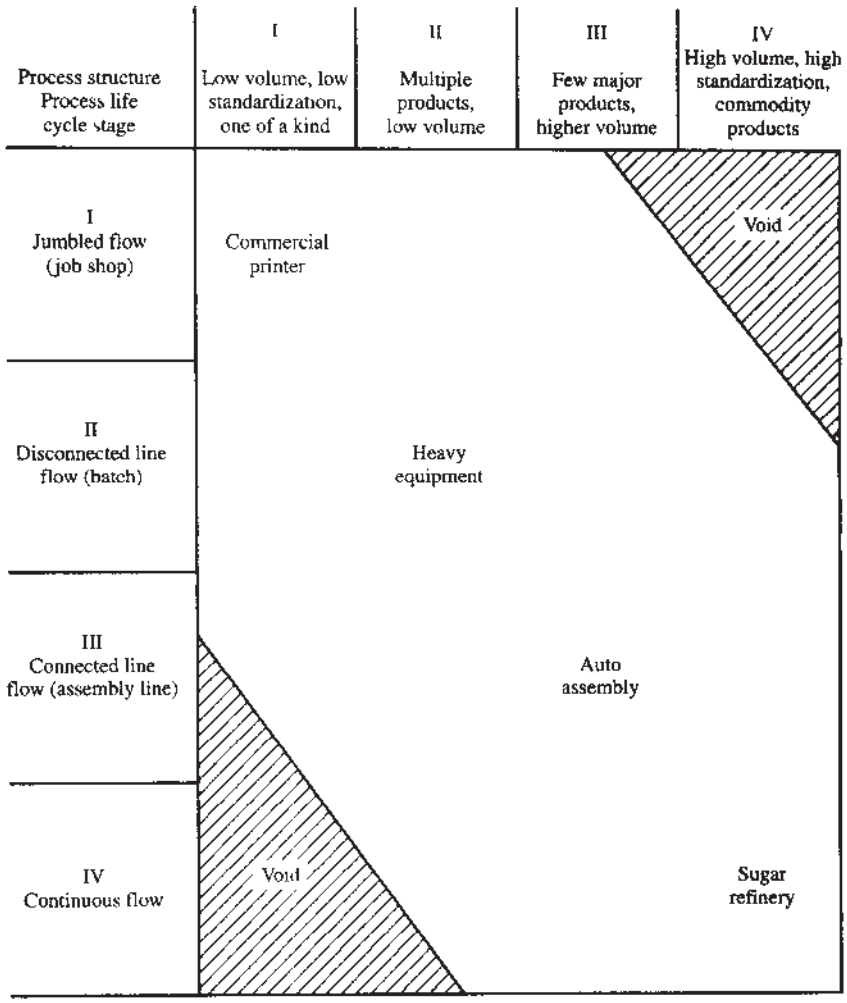


FIGURE 0.2
The product process matrix
(Source: Hayes and Wheelwright 1979)

These environments are suited to different types of products. Because a job shop provides maximum flexibility, it is well suited to low-volume, highly customized products. However, because a job shop is not very efficient on a unit cost basis, it is unattractive for higher-volume products. Therefore, most discrete parts manufacturing plants make at least partial use of some kind of flow line. The decision of how much to automate and pace the line depends on whether the volume and expected economic life justify the necessary capital investment. In continuous product manufacturing, the analogous decision is how far to move from "bench-top" batch production toward a continuous flow process.

Figure 0.2 presents an often-cited **product process matrix** that relates process structure to product type. The basic message of this figure is that higher volumes tend to go hand in hand with smoother-flow process structures. This suggests that the appropriate manufacturing environment may depend on the stage of the product in its life cycle. Newly introduced products are typically produced in small volumes and are subject to design tinkering during a start-up phase, which makes them well suited to the flexibility provided by a job shop environment. As the product progresses through growth and

maturation phases, volumes justify a shift to a more efficient (disconnected) flow line. If the product matures into a commodity (i.e., instead of declining out of the market), even greater standardization of flow, in an automated assembly line or continuous flow line, may be justified. This evolution can be viewed as traversing the diagonal of the product process matrix in Figure 0.2 from the upper left to the lower right over the life of the product.

While the product process matrix is useful for characterizing differences in process structures and their relationship to product requirements, it presents only part of the picture. If manufacturing strategy were simply a matter of noting the type of product and selecting the appropriate process from such a matrix, we wouldn't need a science of manufacturing (or highly trained manufacturing managers). But, as we have stressed, customers today want it all: variety, low cost, high quality, and quick responsive delivery. A major challenge facing modern manufacturing firms is how to structure the environment so that it attains the speed and low cost of the high-volume flow lines while retaining the flexibility and customization potential of a low-volume job shop, all within an atmosphere of continually improving quality.

In this book, we select as our perspective discrete parts production on disconnected flow lines. We do this in part because such environments are most prevalent in industry. Additionally, the flow line perspective enables us to identify concepts for "unjumbling" flow and improving efficiency in job shop environments. Finally, flow lines provide a logical link between discrete parts production and continuous flow processes, and hence a vehicle for looking to continuous systems as a source of ideas for smoothing flow and improving cost efficiency. Thus, the disconnected flow line perspective serves as the foundation upon which to build a problem-solving framework that is applicable across a broad range of manufacturing environments.

0.3 An Overview of the Book

The remainder of this book is divided into three major parts:

Part I, The Lessons of History, provides a history of manufacturing in America, along with a review of traditional operations management techniques, including inventory control models, material requirements planning (MRP), and just-in-time (JIT). In reviewing each of these, we identify the essential insights that are necessary components of the science of manufacturing. Part I concludes with a critical review of why these historical techniques are, by themselves, inadequate for the future needs of manufacturing.

Part II, Factory Physics, presents the core concepts of the book. We begin with the basic structure of the science of manufacturing and a discussion of the systems approach to problem solving. Then we examine key behavioral tendencies of manufacturing plants, starting with basic relationships between measures (e.g., throughput, inventory, and cycle time) and working up to the subtle influences of variability. We also examine the science behind some popular Japanese techniques by comparing push and pull production systems. For clarity, the main conclusions are stated as "manufacturing laws," although, as we will discuss, some of these laws are true laws that always hold, while others are useful generalities that hold most of the time. We include in Part II a brief discussion of critical human issues in manufacturing systems to emphasize the essential point that manufacturing is more than just machinery and logistics—it is people, too. We also identify key links between logistics and quality, to provide some science behind TQM practices.

Part III, Principles in Practice, treats specific manufacturing management issues in detail. By applying the lessons of Part I and the laws of Part II, we contrast and compare

different approaches to problems commonly encountered in running a manufacturing facility. These include shop floor control, sequencing and scheduling, long-range aggregate planning, workforce planning, capacity management, and coordination of planning and control across levels in a hierarchical system. The focus is on choosing the right measures and controls and providing a framework within which to build solutions. We illustrate problem-solving procedures by providing explicit "how to" instructions for selected problems. The purpose of these detailed solutions is not so much to provide user-ready tools, but rather to help the reader visualize how general concepts of Part II can be applied to specific problems.

This three-part approach roughly parallels the three categories of skills required by manufacturing managers and engineers: basics, intuition, and synthesis. Part I concentrates on basics, by providing a historical perspective and introducing traditional terms and techniques. Part II focuses on intuition, by describing fundamental behavior of manufacturing systems. Part III addresses synthesis, by developing a framework for integrating disparate manufacturing planning problems. A manufacturing professional with mastery of these three skills can identify the essential problems in a factory *and* do something about them.

And now, on to *Factory Physics*.

PART

I THE LESSONS OF HISTORY

Those who cannot remember the past are condemned to repeat it.
George Santayana

CHAPTER

1 MANUFACTURING IN AMERICA

What has been will be again, what has been done will be done again; there is nothing new under the sun.

Ecclesiastes

1.1 Introduction

A fundamental premise of this book is that to manage something effectively, one must first understand it. But manufacturing systems are complex entities that can be viewed in many ways,[1] many of which are integral to sound managerial insight. A particularly important perspective, which provides an organizing framework for all others, is that of history.

A sense of history is fundamental to manufacturing managers for two main reasons. First, in manufacturing, as in all walks of life, the ultimate test of an idea is the test of time. Since short-term success may be the result of luck or exogenous circumstances, we can only identify concepts of lasting value by taking the long-term view. Second, because the requirements for success in business change over time, it is critical for managers to make decisions with the future in mind. One of the very best tools for consistently anticipating the future is a sound appreciation of the past.

The history of American manufacturing, which follows its rise from meager colonial beginnings to undisputed worldwide leadership by mid-20th century, through a period of serious decline in the 1970s and 1980s, and into a revitalization in the complex global environment of the 1990s, is a fascinating story. Sadly, we have neither the space nor the expertise to offer comprehensive coverage here. Instead, we highlight major events and trends with emphasis on themes that will be crucial later in the book. We hope the reader will be sufficiently interested in these historical issues to pursue more basic sources. The following are attractive starting points. Wren (1987) provides an excellent general overview from a management perspective. Boorstin in *The Americans* trilogy (1958, 1965, 1973) offers a number of highly readable insights into American business

[1] For example, to a mechanical engineer a manufacturing system is a set of physical processes for altering material, to an operations manager it is a logistical network of product flows, to an organization behavior specialist it is a community of people with shared concerns, to an accountant it is a collection of interrelated cash flows, and so on.

in a cultural context. Chandler (1977, 1990) gives a towering treatment of the evolution of large-scale management in America, as well as Germany and Great Britain. We have drawn heavily on these works, and their references, in what follows.

1.2 The American Experience

In many ways, America began with a clean slate. A vast, wide-open continent offered unparalleled resources and unlimited opportunities for development. Unshackled by the traditions of the Old World, Americans were free to write their own rules. Government, law, cultural practices, and social mores were all choices to be made as part of the grand American experiment.

Naturally, these choices reflected the times in which they were made. In 1776, antimonarchist sentiment, which would soon fuel the French Revolution, was on the rise in both the Old World and the New. America chose democracy. In 1776, Scotsman Adam Smith (1723–1790) proclaimed the end of the old mercantilist system and the beginnings of modern capitalism in his *Wealth of Nations,* in which he articulated the benefits of the division of labor and explained the workings of the "invisible hand" of capitalism.[2] America chose the free market system. In 1776, James Watt (1736–1819) sold his first steam engine in England and began the first industrial revolution in earnest. America embraced the new factory system, evolved a unique style of manufacturing, and eventually led the transportation and communications breakthroughs that sparked the second industrial revolution. In 1776, English common law was the standard for the civilized world. America adapted this tradition, borrowed from Roman law and the *Code Napoléon,* and rapidly became the most litigious country in the world.[3]

In almost all cases, Americans did not invent revolutionary concepts from scratch. Rather, they borrowed freely (and even stole) ideas from the Old World and adapted them to the New. Because the needs of the New World were different, because they were not bound by Old World customs and traditions, and, quite frankly, because they were naive, the social and economic institutions that resulted were uniquely American.

The very fact that America had the opportunity to create itself has done much to shape its natioual identity. Unlike the countries of the Old World, which coalesced as nations long after they had acquired a national spirit, the United States of America achieved nationhood as a composite of colonies with little sense of common identity. Hence, Americans actively sought an identity in the form of cultural symbols. The strongest and most uniquely American cultural icon was that of the rugged individualist seeking freedom on the frontier. This spawned the wild comic legends about Davy Crockett and Mike Fink and later played a large part in transforming Abraham Lincoln into a revered national icon as the "rail splitter" president. Even after the frontier was gone, the myth of the frontier lived on in popular literature and cinema about the cowboys, ranchers, gunfighters, and prospectors of the Old West.

In more recent times, the myth of the frontier evolved into the myth of the self-made person, which has roots stretching back to the aphorisms of Benjamin Franklin (1706–1790) and the essays of Ralph Waldo Emerson (1803–1882), and which found fertile ground in the Protestant work ethic. This myth made heroes out of successful industrialists of the 19th century (e.g., Carnegie, Rockefeller, Morgan) and provided

[2] It is not coincidence that Henry Ford, one of the men most visibly associated with capitalism, would write a book 150 years after Smith's and with the penultimate chapter entitled "The Wealth of Nations."

[3] Two-thirds of the world's lawyers practice in the United States where there are 1,000 lawyers to every 100 engineers. Japan, on the other hand, has 1,000 engineers to every 100 lawyers (Lamm 1988, 17).

cultural support for the unvarnished pursuit of wealth by the corporate raiders of the 1980s. The terms that referred to the players in the takeover games of that "decade of greed"—*gunslinger, white knight, masters of the universe*—were not accidental. Nor is the fact that marketing and finance have consistently been more popular in American business schools than operations management. The perception has been that in finance and marketing, one can do something "big" or "bold" by starting daring new ventures or launching exciting new products, while in operations management one can only struggle to save a few pennies on the cost side—necessary, perhaps, but not very exciting. Attention to detail may be a virtue in Europe or Japan, where resource limits have long been a fact of life; it is decidedly dull in the land of the cowboy.

A third cultural force permeating the American identity is an underlying faith in the *scientific method.* From the period of the Enlightenment, which in America took the form of the popular science of Franklin and then the pragmatic inventions of Whitney, Bell, Eastman, Edison, and others, Americans have always embraced the rational, reductionist, analytical approach of science. The first uniquely American management system became known as **scientific management.**[4] The notion of "managing by the numbers" has deep roots in our cultural propensity for things scientific.

The *reductionist* method favored by scientists analyzes systems by breaking them down into their component parts and studying each one. This was a fundamental tenet of scientific management, which worked to improve overall efficiency by decomposing work into specific tasks and then improving the efficiency of each task. Today's industrial engineers and operations researchers still use this approach almost exclusively and are very much a product of the scientific management movement.

While reductionism can be an extremely profitable paradigm for analyzing complex systems—and certainly Western science has attained many triumphs via this approach—it is not the only valid perspective. Indeed, as has become obvious from the huge gap between academic research and actual practice in industry, too much emphasis on individual components can lead to a loss of perspective for the overall system.

In contrast to the reductionism of the West, Far Eastern societies seem to maintain a more **holistic** or **systems** perspective. In this approach, individual components are viewed much more in terms of their interactions with other subsystems and in the light of the overall goals of the system. This systems perspective undoubtedly influenced the development of just-in-time (JIT) systems in Japan, as we will discuss more thoroughly in Chapter 4.

The difference between the reductionist and holistic perspectives is starkly illustrated by the differing responses taken by the Americans and the Japanese to the problem of setups in manufacturing operations. Setup time is the time required for changeover of a machine from making one product to making another. In the American industrial engineering/operations research literature, for decades, setup times were regarded as constraints, leading to the development of all sorts of complex mathematical models for determining "optimal" lot sizes that would balance setup costs against inventory carrying costs. This view made perfect sense from a reductionist perspective, in which the setups were a given for the subsystem under consideration. In contrast, the Japanese, looking at manufacturing systems in the more holistic sense, recognized that setup times were not a given—they could be reduced. Moreover, from a systems perspective, there was clear value in reducing setup times. Clever use of jigs, fixtures, off-cycle preparations, and the like, which became known as *single minute exchange of die,* or SMED (Shingo 1985), enabled some Japanese factories to realize significantly shorter setup times than those

[4]This is in spite of the fact that its developer, Frederick W. Taylor, himself preferred the terms *task management* or the *Taylor system.*

in comparable American plants. In particular, the Japanese automobile industry became among the most productive in the world. These plants became simpler to manage and more flexible than their American counterparts.

Of course, the Japanese system had its weak points as well. Its convoluted pricing and distribution systems made Japanese electronic devices cheaper in New York than in Tokyo. Competition was tightly regulated by a traditional corporate network that kept out newcomers and led to bad investments. Strong profits of the 1980s were plowed into overvalued stocks and real estate. When the bubble burst in the 1990s, Japan found itself mired in an extended recession that precipitated the "Asian crisis" throughout the Pacific Rim. But Japanese workers in many industries remain productive, their investment rate is high, and personal debt is low. These sound economic basics make it very likely that Japan will continue to be a strong source of competition well into the 21st century.

1.3 The First Industrial Revolution

Prior to the first industrial revolution, production was small-scale, for limited markets, and labor- rather than capital-intensive. Work was carried out under two systems, the **domestic system** and **craft guilds.** In the domestic system, material was "put out" by merchants to homes where people performed the necessary operations. For instance, in the textile industry, different families spun, bleached, and dyed material, with merchants paying them on a piecework basis. In the craft guilds, work was passed from one shop to another. For example, leather was tanned by a tanner, passed to curriers, then passed to shoemakers and saddlers. The result was separate markets for the material at each step of the process.

The first industrial revolution began in England during the mid-18th century in the textile industry. This revolution, which dramatically changed manufacturing practices and the very course of human existence, was stimulated by several innovations that helped mechanize many of the traditional manual operations. Among the more prominent technological advances were the *flying shuttle* developed by John Kay in 1733, the *spinning jenny* invented by James Hargreaves in 1765 (Jenny was Mrs. Hargreaves), and the *water frame* developed by Richard Arkwright in 1769. By facilitating the substitution of capital for labor, these innovations generated economies of scale that made mass production in centralized locations attractive for the first time.

The single most important innovation of the first industrial revolution, however, was the steam engine, developed by James Watt in 1765 and first installed by John Wilkinson in his iron works in 1776. In 1781 Watt developed the technology for transforming the up-and-down motion of the drive beam to rotary motion. This made steam practical as a power source for a host of applications, including factories, ships, trains, and mines. Steam opened up far greater freedom of location and industrial organization by freeing manufacturers from their reliance on water power. It also provided cheaper power, which led to lower production costs, lower prices, and greatly expanded markets.

It has been said that Adam Smith and James Watt did more to change the world around them than anyone else in their period of history. Smith told us why the modern factory system, with its division of labor and "invisible hand" of capitalism, was desirable. Watt, with his engines (and the well-organized factory in which he, his partner Matthew Boulton and their sons built them), showed us how to do it. Many features of modern life, including widespread employment in large-scale factories, mass production of inexpensive goods, the rise of big business, the existence of a professional managerial class, and others, are direct consequences of their contributions.

1.3.1 The Industrial Revolution in America

England had a decided technological edge over America throughout the 18th century, and protected her competitive advantage by prohibiting export of models, plans, or people that could reveal the technologies upon which her industrial strength was based. It was not until the 1790s that a technologically advanced textile mill appeared in America—and that was the result of an early case of industrial espionage!

Boorstin (1965, 27) reports that Americans made numerous attempts to invent machinery like that in use in England during the later years of the 18th century, going so far as to organize state lotteries to raise prize money for enticing inventors. When these efforts failed repeatedly, Americans tried to import or copy English machines. Tench Coxe, a Philadelphian, managed to get a set of brass models made of Arkwright's machinery; but British customs officers discovered them on the dock and foiled his attempt. America finally succeeded in its efforts when Samuel Slater (1768–1835)—who had been apprenticed at the age of 14 to Jedediah Strutt, the partner of Richard Arkwright (1732–1792)—disguised himself as a farmer and left England secretly, without even telling his mother, to avoid the English law prohibiting departure of anyone with technical knowledge. Using the promise of a partnership, Moses Brown (for whom Brown University was named), who owned a small textile operation in Rhode Island with his son-in-law William Almy, enticed Slater to share his illegally transported technical knowledge. With Brown and Almy's capital and Slater's phenomenal memory, they built a cotton-spinning frame and in 1793 established the first modern textile mill in America at Pawtucket, Rhode Island.

The *Rhode Island system*, as the management system used by the Almy, Brown, and Slater partnership became known, closely resembled the British system on which it was founded. Focusing only on spinning fine yarn, Slater and his associates relied little on vertical integration and much on direct personal supervision of their operations. However, by the 1820s, the American textile industry would acquire a distinctly different character from that of the English by consolidating many previously disparate operations under a single roof. This was catalyzed by two factors.

First, America, unlike England, had no strong tradition of craft guilds. In England, distinct stages of production (e.g., spinning, weaving, dying, printing, in cotton textile manufacture) were carried out by different artisans who regarded themselves as engaged in distinct occupations. Specialized traders dealt in yarn, woven goods, and dyestuffs. These groups all had vested interests in not centralizing or simplifying production. In contrast, America relied primarily on the domestic system for textile production throughout its colonial period. Americans of this time either spun and wove for themselves or purchased imported woolens and cottons. Even in the latter half of the 18th century, a large proportion of American manufacturing was carried out by village artisans without guild affiliation. As a result, there were no organized constituencies to block the move toward integration of the manufacturing process.

Second, America, unlike England, still had large untapped sources of water power in the late 18th and early 19th centuries. Thus, the steam engine did not replace water power in America on a widespread basis until the Civil War. With large sources of water power, it was desirable to centralize manufacturing operations. This is precisely what Francis Cabot Lowell (1775–1817) did. After smuggling plans for a power loom out of Britain (Chandler 1977, 58), he and his associates built the famous cotton textile factories at Waltham and Lowell, Massachusetts, in 1814 and 1821. By using a single source of water power to drive all the steps necessary to manufacture cotton cloth, they established an early example of a modern integrated factory system. Ironically, because steam facilitated power generation in smaller units, its earlier introduction in England

served to keep the production process smaller and more fragmented in England than in water-reliant America.

The result was that Americans, faced with a fundamentally different environment than that of the technologically and economically superior British firms, responded by innovating. These steps toward vertical integration in the early-19th-century textile industry were harbingers of a powerful trend that would ultimately make America the land of big business. The seeds of the enormous integrated mass production facilities that would become the norm in the 20th century were planted early in our history.

1.3.2 The American System of Manufacturing

Vertical integration was the first step in a distinctively American style of manufacturing. The second and more fundamental step was the production of interchangeable parts in the manufacture of complex multipart products. By the mid-19th century it was clear that the Americans were evolving an entirely new approach to manufacturing. The 1851 Crystal Palace Exhibition in London saw the first use of the term *American system of manufacturing* to describe the display of American products, such as the locks of Alfred Hobbs, the repeating pistol of Samuel Colt, and the mechanical reaper of Cyrus McCormick, all produced using the method of interchangeable parts.

The concept of interchangeable parts did not originate in America. The Arsenal of Venice was using some standard parts in the manufacture of warships as early as 1436. French gunsmith Honore LeBlanc had shown Thomas Jefferson musket components manufactured using interchangeable parts in 1785; but the French had abandoned his approach in favor of traditional craft methods (Mumford 1934, Singer 1958). It fell to two New Englanders, Eli Whitney (1765–1825) and Simeon North, to prove the feasibility of interchangeable parts as a sound industrial practice. At Jefferson's urging, Whitney was contracted to produce 10,000 muskets for the American government in 1801. Although it took him until 1809 to deliver the last musket, and he made only $2,500 on the job, he established beyond dispute the workability of what he called his "Uniformity System." North, a scythe manufacturer, confirmed the practicality of the concept and devised new methods for implementing it, through a series of contracts between 1799 and 1813 to produce pistols with interchangeable parts for the War Department. The inspiration of Jefferson and the ideas of Whitney and North were realized on a large scale for the first time at the Springfield Armory between 1815 and 1825, under the direction of Colonel Roswell Lee.

Prior to the innovation of interchangeable parts, the making of a complex machine was carried out in its entirety by an artisan, who fabricated and fitted each required piece. Under Whitney's uniformity system, the individual parts were mass-produced to tolerances tight enough to enable their use in any finished product. The division of labor called for by Adam Smith could now be carried out to an extent never before achievable, with individual workers producing single parts rather than completed products. The highly skilled artisan was no longer necessary.

It is difficult to overstate the importance of the idea of interchangeable parts, which Boorstein (1965) calls "the greatest skill-saving innovation in human history." Imagine producing personal computers under the skilled artisan system! The artisan would first have to fabricate a silicon wafer and then turn it into the needed chips. Then the printed-circuit boards would have to be produced, not to mention all the components that go into them. The disk drives, monitor, power supply, and so forth—all would have to be fabricated. Finally, all the components would be assembled in a handmade plastic case. Even if such a feat could be achieved, personal computers would cost millions of dollars

and would hardly be "personal." Without exaggeration, our modern way of life depends on and evolved from the innovation of interchangeable parts. Undoubtedly, the Whitney and North contracts were among the most productive uses of federal funds to stimulate technological development in all of American history.

The American system of manufacturing, emphasizing mass production through use of vertical integration and interchangeable parts, started two important trends that impacted the nature of manufacturing management in this country to the present.

First, the concept of interchangeable parts greatly reduced the need for specialized skills on the part of workers. Whitney stated his aim as to "substitute correct and effective operations of machinery for that skill of the artist which is acquired only by long practice and experience, a species of skill which is not possessed in this country to any considerable extent" (Boorstein 1965, 33). Under the American system, workers without specialized skills could make complex products. An immediate result was a difference in worker wages between England and America. In the 1820s, unskilled laborers' wages in America were one-third or one-half higher than those in England, while highly-skilled workers in America were only slightly better paid than in England. Clearly, America placed a lower premium on specialized skills than other countries from a very early point in her history. Workers, like parts, were interchangeable. This early rise of the undifferentiated worker contributed to the rocky history of labor relations in America. It also paved the way for the sharp distinction between planning (by management) and execution (by workers) under the principles of scientific management in the early 20th century.

Second, by embedding specialization in machinery instead of people, the American system placed a greater premium on general intelligence than on specialized training. In England, unskilled meant unspecialized; but the American system broke down the distinction between skilled and unskilled. Moreover, machinery, techniques, and products were constantly changing, so that open-mindedness and versatility became more important than manual dexterity or task-specific knowledge. A liberal education was useful in the New World in a way that it had never been in the Old World, where an education was primarily a mark of refinement. This trend would greatly influence the American system of education. It also very likely prepared the way for the rise of the professional manager, who is assumed able to manage any operation without detailed knowledge of its specifics.

1.4 The Second Industrial Revolution

In spite of the notable advances in the textile industry by Slater in the 1790s and the practical demonstration of the uniformity system by Whitney, North, and Lee in the early 1800s, most industry in pre-1840 America was small, family-owned, and technologically primitive. Before the 1830s, coal was not widely available, so most industry relied on water power. Seasonal variations in the power supply, due to drought or ice, plus the lack of a reliable all-weather transportation network made full-time, year-round production impractical for many manufacturers. Workers were recruited seasonally from the local farm population, and goods were sold locally or through the traditional merchant network established to sell British goods in America. The class of permanent industrial workers was small, and the class of industrial managers almost nonexistent. Prior to 1840, there were almost no manufacturing enterprises sophisticated enough to require anything more than traditional methods of direct factory management by the owners.

Before the Civil War, large factories were the exception rather than the rule. In 1832, Secretary of the Treasury Louis McLane conducted a survey of manufacturing in

10 states and found only 36 enterprises with 250 or more workers, of which 31 were textile factories. The vast majority of enterprises had assets of only a few thousand dollars, had fewer than a dozen employees, and relied on water power (Chandler 1977, 60–61). The Springfield Armory, often cited as the most modern plant of its time—it used interchangeable parts, division of labor, cost accounting techniques, uniform standards, inspection/control procedures, and advanced metalworking methods—rarely had more than 250 employees.

The spread of the factory system was limited by the dependence on water power until the opening of the anthracite coal fields in eastern Pennsylvania in the 1830s. From 1840, anthracite-fueled blast furnaces began providing an inexpensive supply of pig iron for the first time. The availability of energy and raw material prompted a variety of industries (e.g., makers of watches, clocks, safes, locks, pistols) to build large factories using the method of interchangeable parts. In the late 1840s, newly invented technologies (e.g., sewing machines and reapers) also began production using the interchangeable-parts method.

However, even with the availability of coal, large-scale production facilities did not immediately arise. The modern integrated industrial enterprise was not the consequence of the technological and energy innovations of the first industrial revolution. The mass *production* characteristic of large-scale manufacturing required coordination of a mass *distribution* system to facilitate the flow of materials and goods through the economy. Thus, the second industrial revolution was catalyzed by innovations in transportation and communication—railroad, steamship, and telegraph—that occurred between 1850 and 1880. Breakthroughs in distribution technology in turn prompted a revolution in mass production technology in the 1880s and 1890s, including the Bonsack machine for cigarettes, the "automatic-line" canning process for foods, practical implementation of the Bessemer steel process and electrolytic aluminum refining, and many others. During this time, America visibly led the way in mass production and distribution innovations and, as a result, by World War II had more large-scale business enterprises than the rest of the world combined.

1.4.1 The Role of the Railroads

Railroads were the spark that ignited the second industrial revolution for three reasons:

1. They were America's first big business, and hence the first place where large-scale management hierarchies and modern accounting practices were needed.
2. Their construction (and that of the telegraph system at the same time) created a large market for mass-produced products, such as iron rails, wheels, and spikes, as well as basic commodities such as wood, glass, upholstery, and copper wire.
3. They connected the country, providing reliable all-weather transportation for factory goods and creating mass markets for products.

Colonel John Stevens received the first railroad charter in America from the New Jersey legislature in 1815 but, because of funding problems, did not build the 23-mile-long Camden and Amboy Railroad until 1830. In 1850 there were 9,000 miles of track extending as far as Ohio (Stover 1961, 29). By 1865 there were 35,085 miles of railroad in the United States, only 3,272 of which were west of the Mississippi. By 1890, the total had reached 199,876 miles, 72,473 of which were west of the Mississippi. Unlike in the Old World and in the eastern United States, where railroads connected established population centers, western railroads were generally built in sparsely populated areas, with lines running from "Nowhere-in-Particular to Nowhere-at-All" in the anticipation of development.

The capital required to build a railroad was far greater than that required to build a textile mill or metalworking enterprise. A single individual or small group of associates was rarely able to own a railroad. Moreover, because of the complexity and distributed nature of its operations, the many stockholders or their representatives could not directly manage a railroad. For the first time, a new class of salaried employees—middle managers—emerged in American business. Out of necessity the railroads became the birthplace of the first administrative hierarchies, in which managers managed other managers.

A pioneer of methods for managing the newly emerging structures was Daniel Craig McCallum (1815–1878). Working for the New York and Erie Railroad Company in the 1850s, he developed principles of management and a formal organization chart to convey lines of authority, communication, and division of labor (Chandler 1977, 101). Henry Varnum Poor, editor of the *American Railroad Journal*, widely publicized McCallum's work in his writings and sold lithographs of his organization chart for $1 each. Although the Erie line was taken over by financiers with little concern for efficiency (i.e., the infamous Jay Gould and his associates), Poor's publicity efforts ensured that McCallum's ideas had a major impact on railroad management in America.

Because of their complexity and reliance on a hierarchy of managers, railroads required large amounts of data and new types of analysis. In response to this need, innovators like J. Edgar Thomson of the Pennsylvania Railroad and Albert Fink of the Louisville & Nashville invented many of the basic techniques of modern accounting during the 1850s and 1860s. Specific contributions included introduction of standardized ratios (e.g., the ratio between a railroad's operating revenues and its expenditures, called the *operating ratio*), capital accounting procedures (e.g., renewal accounting), and unit cost measures (e.g., cost per ton-mile). Again, Henry Varnum Poor publicized the new accounting techniques and they rapidly became standard industry practice.

In addition to being the first big businesses, the railroads, along with the telegraph, paved the way for future big businesses by creating a mass distribution network and thereby making mass markets possible. As the transportation and communication systems improved, commodity dealers, purchasing agricultural products from farmers and selling to processors and wholesalers, began to appear in the 1850s and 1860s. By the 1870s and 1880s, mass retailers, such as department stores and mail-order houses, followed suit.

1.4.2 Mass Retailers

The phenomenal growth of these mass retailers provided a need for further advances in the management of operations. For example, Sears and Roebuck's sales grew from $138,000 in 1891 to $37,789,000 in 1905 (Chandler 1977, 231). Otto Doering developed a system for handling the huge volume of orders at Sears in the early years of the 20th century, a system which used machinery to convey paperwork and transport items in the warehouse. But the key to his process was a complex and rigid scheduling system that gave departments a 15-minute window in which to deliver items for a particular order. Departments that failed to meet the schedule were fined 50 cents per item. Legend has it that Henry Ford visited and studied this state-of-the-art mail-order facility before building his first plant (Drucker 1954, 30).

The mass distribution systems of the retailers and mail-order houses also produced important contributions to the development of accounting practices. Because of their high volumes and low margins, these enterprises had to be extremely cost-conscious. Analogous to the use of operating ratios by the railroads, retailers used gross margins (sales receipts less cost of goods sold and operating expenses). But since retailers, like

the railroads, were single-activity firms, they developed specific measures of process efficiency unique to their type of business. Whereas the railroads concentrated on cost per ton-mile, the retailers focused on inventory turns or "stockturn" (the ratio of annual sales to average on-hand inventory). Marshall Field was tracking inventory turns as early as 1870 (Johnson and Kaplan 1987, 41), and maintained an average of between five and six turns during the 1870s and 1880s (Chandler 1977, 223), numbers that equal or better the performance of some retail operations today.

It is important to understand the difference between the environment in which American retailers flourished and the environment prevalent in the Old World. In Europe and Japan, goods were sold to populations in established centers with strong word-of-mouth contacts. Under such conditions, advertising was largely a luxury. Americans, on the other hand, marketed their goods to a sparse and fluctuating population scattered across a vast continent. Advertising was the life blood of firms like Sears and Roebuck. Very early on, marketing was more important in the New World than in the Old. Later on, the role of marketing in manufacturing would be further reinforced when makers of new technologies (sewing machines, typewriters, agricultural equipment) found they could not count on wholesalers or other intermediaries to provide the specialized services necessary to sell their products, and formed their own sales organizations.

1.4.3 Andrew Carnegie and Scale

Following the lead of the railroads, other industries began the trend toward big business through horizontal and vertical integration. In horizontal integration, a firm bought up competitors in the same line of business (steel, oil, etc.). In vertical integration, firms subsumed their sources of raw material and users of the product. For instance, in the steel industry, vertical integration took place when the steel mill owners purchased mining and ore production facilities on the upstream end and rolling mills and fabrication facilities on the downstream end.

In many respects, modern factory management first appeared in the metal making and working industries. Prior to the 1850s, the American iron and steel industry was fragmented into separate companies that performed the smelting, rolling, forging, and fabrication operations. In the 1850s and 1860s, in response to the tremendous growth of railroads, several large integrated rail mills appeared in which blast furnaces and shaping mills were contained in a single works. Nevertheless, in 1868, America was still a minor player in steel, producing only 8,500 tons compared with Britain's production of 110,000 tons.

In 1872, Andrew Carnegie (1835–1919) turned his hand to the steel industry. Carnegie had worked for J. Edgar Thompson on the Pennsylvania Railroad, rising from telegraph operator to division superintendent, and had a sound appreciation for the accounting and management methods of the railroad industry. He combined the new Bessemer process for making steel with the management methods of McCallum and Thompson, and he brought the industry to previously unimagined levels of integration and efficiency. Carnegie expressed his respect for his railroad mentors by naming his first integrated steel operation the Edgar Thompson Works. The goal of the E. T. Works was "a large and regular output," accomplished through the use of the largest and most technologically advanced blast furnaces in the world. More importantly, the E. T. Works took full advantage of integration by maintaining a continuous work flow—it was the first steel mill whose layout was dictated by material flow. By relentlessly exploiting his scale advantages and increasing velocity of throughput, Carnegie quickly became the most efficient steel producer in the world.

Carnegie further increased the scale of his operations by integrating vertically into iron and coal mines and other steel-related operations to improve flow even more. The effect was dramatic. By 1879, American steel production nearly equaled that of Britain. And by 1902, America produced 9,138,000 tons, compared with 1,826,000 for Britain.

Carnegie also put the cost accounting skills acquired from his railroad experience to good use. A stickler for accurate costing—one of his favorite dictums was, "Watch the costs and the profits will take care of themselves"—he instituted a strict accounting system. By doggedly focusing on unit cost, he became the low-cost producer of steel and was able to undercut competitors who had a less precise grasp of their costs. He used this information to his advantage, raising prices along with his competition during periods of prosperity and relentlessly cutting prices during recessions.

In addition to graphically illustrating the benefits from scale economies and high throughput, Carnegie's was a classic story of an entrepreneur who made use of minute data and prudent attention to operating details to gain a significant strategic advantage in the marketplace. He focused solely on steel and knew his business thoroughly, saying

> I believe the true road to preeminent success in any line is to make yourself master in that line. I have no faith in the policy of scattering one's resources, and in my experience I have rarely if ever met a man who achieved preeminence in money-making—certainly never one in manufacturing—who was interested in many concerns. The men who have succeeded are men who have chosen one line and stuck to it. (Carnegie 1920, 177)

Aside from representing one of the largest fortunes the world had known, Carnegie's success had substantial social benefit. When Carnegie started in the steel business in the 1870s, iron rails cost $100 per ton; by the late 1890s they sold for $12 per ton (Chandler 1984, 485).

1.4.4 Henry Ford and Speed

By the beginning of the 20th century, integration, vertical and horizontal, had already made America the land of big business. High-volume production was commonplace in process industries such as steel, aluminum, oil, chemicals, food, and tobacco. Mass production of mechanical products such as sewing machines, typewriters, reapers, and industrial machinery, based on new methods for fabricating and assembling interchangeable metal parts, was in full swing. However, it remained for Henry Ford (1863–1947) to make high-speed mass production of complex mechanical products possible with his famous innovation, the moving assembly line.

Like Carnegie, Ford recognized the importance of throughput velocity. In an effort to speed production, Ford abandoned the practice of skilled workers assembling substantial subassemblies and workers gathering around a static chassis to complete assembly. Instead, he sought to bring the product to the worker in a nonstop, continuous stream. Much has been made of the use of the moving assembly line, first used at Ford's Highland Park plant in 1913. However, as Ford noted, the principle was more important than the technology:

> The thing is to keep everything in motion and take the work to the man and not the man to the work. That is the real principle of our production, and conveyors are only one of many means to an end. (Ford 1926, 103)

After Ford, mass production became almost synonymous with assembly-line production.

Ford had signaled his strategy to provide cheap, reliable transportation early on with the Model N, introduced in 1906 for $600. This price made it competitive with much less sophisticated motorized buggies and far less expensive than other four-cylinder automo-

biles, all of which cost more than $1,000. In 1908, Ford followed with the legendary Model T touring car, originally priced at $850. By focusing on continual improvement of a single model and pushing his mass production techniques to new limits at his Highland Park plant, Ford reduced labor time to produce the Model T from 12.5 to 1.5 hours, and he brought prices down to $360 by 1916 and $290 by the 1920s. Ford sold 730,041 Model T's in fiscal year 1916/17, roughly one-third of the American automobile market. By the early 1920s, Ford Motor Company commanded two-thirds of the American automobile market.

Henry Ford also made his share of mistakes. He stubbornly held to the belief in a perfectible product and never appreciated the need for constant attention to the process of bringing new products to market. His famous statement that "the customer can have any color car as long as it's black" equated mass production with product uniformity. He failed to see the potential for producing a variety of end products from a common set of standardized parts. Moreover, his management style was that of a dictatorial owner. He never learned to trust his managerial hierarchy to make decisions of importance. Peter Drucker (1954) points to Henry's desire to "manage without managers" as the fundamental cause of Ford's precipitous decline in market share (from more than 60 percent down to 20 percent) between the early 1920s and World War II.

But Henry Ford's spectacular successes were not merely a result of luck or timing. The one insight he had that drove him to new and innovative manufacturing methods was his appreciation of the strategic importance of speed. Ford knew that high throughput and low inventories would enable him to keep his costs low enough to maintain an edge on his competition and to price his product so as to be available to a large segment of the public. It was his focus on speed that motivated his moving assembly line. But his concern for speed extended far beyond the production line. In 1926, he claimed, "Our finished inventory is all in transit. So is most of our raw material inventory." He boasted that his company could take ore from a mine and produce an automobile in 81 hours. Even allowing for storage of iron ore in winter and other inventory stocking, he claimed an average cycle time of not more than five days. Given this, it is little wonder that Taiichi Ohno, the originator of just-in-time systems, of whom we will have more to say in Chapter 4, was an unabashed admirer of Ford.

The insight that speed is critical, to both cost and throughput, was not in itself responsible for Ford's success. Rather, it was his attention to the details of implementing this insight that set him apart from the competition. The moving assembly line was just one technological innovation that helped him achieve his goal of unimpeded flow of materials through the entire system. He used many of the methods of the newly emerging discipline of scientific management (although Ford had evidently never heard of its founder, Frederick Taylor) to break down and refine the individual tasks in the assembly process. His 1926 book is filled with detailed stories of technical innovations—in glass making, linen manufacture, synthetic steering wheels, artificial leather, heat treating of steel, spindle screwdrivers, casting bronze bushings, automatic lathes, broaching machines, making of springs—that evidence his attention to details and appreciation of their importance. For all his shortcomings and idiosyncrasies, Henry Ford knew his business and used his intimacy with small issues to make a big imprint on the history of manufacturing in America.

1.5 Scientific Management

Although management has been practiced since ancient times (Peter Drucker credits the Egyptians who built the pyramids with being the greatest managers of all time), management *as a discipline* dates back to the late 19th century. Important as they were,

the practical experiences and rules of thumb offered by such visionaries as Machiavelli did not make management a field because they did not result from a systematized method of critical scrutiny. Only when managers began to observe their practices in the light of the rational, deductive approach of scientific inquiry could management be termed a discipline and gain some of the respectability accorded to other disciplines using the scientific method, such as medicine and engineering. Not surprisingly, the first proponents of a scientific approach to management were engineers. By seeking to introduce a management focus into the professional fabric of engineering, they sought to give it some of engineering's effectiveness and respectability.

Scientific observation of work goes back at least as far as Leonardo da Vinci, who measured the amount of earth a man could shovel more than 450 years ago (Consiglio 1969). However, as long as manufacturing was carried out in small facilities amenable to direct supervision, there was little incentive to develop systematic work management procedures. It was the rise of the large integrated business enterprise in the late 19th and early 20th centuries that caused manufacturing to become so complex as to demand more sophisticated control techniques. Since the United States led the drive toward increased manufacturing scale, it was inevitable that it would also lead the accompanying managerial revolution.

Still, before American management writers developed their ideas in response to the second industrial revolution, a few British writers had anticipated the systematizing of management in response to the first industrial revolution. One such visionary was Charles Babbage (1792–1871). A British eccentric of incredibly wide-ranging interests, he demonstrated the first mechanical calculator, which he called a "difference machine," complete with a punch card input system and external memory storage, in 1822. He turned his attention to factory management in his 1832 book *On the Economy of Machinery and Manufactures,* in which he elaborated on Adam Smith's principle of division of labor and described how various tasks in a factory could be divided among different types of workers. Using a pin factory as an example, he described the detailed tasks required in pin manufacture and measured the times and resources required for each. He suggested a profit-sharing scheme in which workers derive a share of their wages in proportion to factory profits. Novel as his ideas were, though, Babbage was a writer, not a practitioner. He measured work rates for descriptive purposes only; he never sought to improve efficiency. He never developed his computer to commercial reality, and his management ideas were never implemented.

The earliest American writings on the problem of factory management appear to be a series of letters to the editor of the *American Machinist* by James Waring See, writing under the name of "Chordal," beginning in 1877 and published in book form in 1880 (Muhs, Wrege, Murtuza 1981). See advocated high wages to attract quality workers, standardization of tools, good "housekeeping" practices in the shop, well-defined job descriptions, and clear lines of authority. But perhaps because his book *(Extracts from Chordal's Letters)* did not sound like a book on business or because he did not interact with other pioneers in the area, See was not widely recognized or cited in future work on management as a formal discipline.

The notion that management could be made into a profession began to surface during the period when engineering became recognized as a profession. The American Society of Civil Engineers was formed in 1852, the American Institute of Mining Engineers in 1871, and, most importantly for the future of management, the American Society of Mechanical Engineers (ASME) in 1880. ASME quickly became the forum for debate of issues related to factory operation and management. In 1886, Henry Towne (1844–1924), engineer, cofounder of Yale Lock Company, and president of Yale and Towne Manufacturing Company, presented a paper entitled "The Engineer as an Economist"

(Towne 1886). In it, he held that "the matter of shop management is of equal importance with that of engineering ... and the *management of works* has become a matter of such great and far-reaching importance as perhaps to justify its classification also as one of the modern arts." Towne also called for ASME to create an "Economic Section" to provide a "medium for the interchange" of experiences related to shop management. Although ASME did not form a Management Division until 1920, Towne and others kept shop management issues in prominence at society meetings.

1.5.1 Frederick W. Taylor

It is easy in hindsight to give credit to many individuals for seeking to rationalize the practice of management. But until Frederick W. Taylor (1856–1915), no one generated the sustained interest, active following, and systematic framework necessary to plausibly proclaim management as a discipline. It was Taylor who persistently and vocally called for the use of science in management. It was Taylor who presented his ideas as a coherent system in both his publications and his many oral presentations. It was Taylor who, with the help of his associates, implemented his system in many plants. And it is Taylor who lies buried under the epithet "father of scientific management."

Although he came from a well-to-do family, had attended the prestigious Exeter Academy, and had been admitted to Harvard, Taylor chose instead to apprentice as a machinist; and he rose rapidly from laborer to chief engineer at Midvale Steel Company between 1878 and 1884. An engineer to the core, he earned a degree in mechanical engineering from Stevens Institute on a correspondence basis while working full-time. He developed several inventions for which he received patents. The most important of these, high-speed steel (which enables a cutting tool to remain hard at red heat), would have been sufficient to guarantee him a place in history even without his involvement in scientific management.

But Taylor's engineering accomplishments pale in comparison to his contributions to management. Drucker (1954) wrote that Taylor's system "may well be the most powerful as well as the most lasting contribution America has made to Western thought since the Federalist Papers." Lenin, hardly a fan of American business, was an ardent admirer of Taylor. In addition to being known as the father of scientific management, he is claimed as the "father of industrial engineering" (Emerson and Naehring 1988).

But what were Taylor's ideas that accord him such a lofty position in the history of management? On the surface, Taylor was an almost fanatic champion of efficiency. Boorstein (1973, 363) calls him the "Apostle of the American Gospel of Efficiency." The core of his management system consisted of breaking down the production process into its component parts and improving the efficiency of each. In essence, Taylor was trying to do for work units what Whitney had done for material units: standardize them and make them interchangeable. Work standards, which he applied to activities ranging from shoveling coal to precision machining, represented the work rate that should be attainable by a "first-class man."

But Taylor did more than merely measure and compare the rates at which men worked. What made Taylor's work scientific was his relentless search for the best way to do tasks. Rules of thumb, tradition, standard practices were anathema to him. Manual tasks were honed to maximum efficiency by examining each component separately and eliminating all false, slow, and useless movements. Mechanical work was accelerated through the use of jigs, fixtures, and other devices, many invented by Taylor himself. The "standard" was the rate at which a "first-class" man could work *using the "best" procedure*.

With a faith in the scientific method that was singularly American, Taylor sought the same level of predictability and precision for manual tasks that he achieved with the "feed and speed" formulas he developed for metal cutting. The following formula for the time required to haul material with a wheelbarrow B is typical (Taylor 1903, 1431):

$$B = \left\{ p + [a + 0.51 + (0.0048)\text{distance hauled}]\frac{27}{L} \right\} 1.27$$

Here p represents the time loosening one cubic yard with the pick, a represents the time filling a barrow with any material, L represents the load of a barrow in cubic feet, and all times are in minutes and distances in feet.

Although Taylor was never able to extend his "science of shoveling" (as his opponents derisively termed his work) into a broader theory of work, it was not for lack of trying. He hired an associate, Sanford Thompson, to conduct extensive work measurement experiments. While he was never able to reduce broad categories of work to formulas, Taylor remained confident that this was possible:

> After a few years, say three, four or five years more, someone will be ready to publish the first book giving the laws of the movements of men in the machine shop—all the laws, not only a few of them. Let me predict, just as sure as the sun shines, that is going to come in every trade.[5]

Once the standard for a particular task had been scientifically established, it remained to motivate the workers to achieve it. Taylor advocated all three basic categories of worker motivation:

1. *The "carrot."* Taylor proposed a "differential piece rate" system, in which workers would be paid a low rate for the first increment of work and a substantially higher rate for the next increment. The idea was to give a significant reward to workers who met the standard relative to those who did not.
2. *The "stick."* Although he tried fining workers for failure to achieve the standard, Taylor ultimately rejected this approach. A worker who is unable to meet the standard should be reassigned to a task to which he is more suited and a worker who refuses to meet the standard ("a bird that can sing and won't sing") should be discharged.
3. *Factory ethos.* Taylor felt that a *mental revolution,* in which management and labor recognize their common purpose, was necessary in order for scientific management to work. For the workers this meant leaving the design of their work to management and realizing that they would share in the rewards of efficiency gains via the piece rate system. The result, he felt, would be that both productivity and wages would rise, workers would be happy, and there would be no need for labor unions. Unfortunately, when piecework systems resulted in wages that were considered too high, it was a common practice for employers to reduce the rate or increase the standard.

Beyond time studies and incentive systems, Taylor's engineering outlook led him to the conclusion that management authority should emanate from expertise rather than power. In sharp contrast to the militaristic unity-of-command character of traditional management, Taylor proposed a system of "functional foremanship" in which the traditional single foreman is replaced by eight different supervisors, each with responsibility for specific functions. These included the *inspector,* responsible for quality of work; the *gang boss,* responsible for machine setup and motion efficiency; the *speed boss,* responsible for machine speeds and tool choices; the *repair boss,* responsible for machine

[5] Abstract of an address given by Taylor before the Cleveland Advertising Club, March 3, 1915, and repeated the next day. It was his last public appearance. Reprinted in Shafritz and Ott 1990, 69–80.

maintenance and repair; the *order of work* or *route clerk*, responsible for routing and scheduling work; the *instruction card foreman*, responsible for overseeing the process of instructing bosses and workers in the details of their work; the *time and cost clerk*, responsible for sending instruction cards to the men and seeing that they record time and cost of their work; and the *shop disciplinarian*, who takes care of discipline in the case of "insubordination or impudence, repeated failure to do their duty, lateness or unexcused absence."

Finally, to complete his management system, Taylor recognized that he required an accounting system. Lacking personal expertise in financial matters, he borrowed and adapted a bookkeeping system from Manufacturing Investment Company, while working there as general manager from 1890 to 1893. This system was developed by William D. Basley, who had worked as the accountant for the New York and Northern Railroad, but was transferred to the Manufacturing Investment Company, also owned by the owners of the railroad, in 1892. Taylor, like Carnegie before him, successfully applied railroad accounting methods to manufacturing.

To Taylor, scientific management was not simply time and motion study, a wage incentive system, an organizational strategy, and an accounting system. It was a philosophy, which he distilled to four principles. Although worded in various ways in his writings, these are concisely stated as (Taylor 1911, 130)

1. The development of a true science.
2. The scientific selection of the worker.
3. His scientific education and development.
4. Intimate friendly cooperation between management and the men.

The first principle, by which Taylor meant that it was the managers' job to pursue a scientific basis for running their business, was the foundation of scientific management. The second and third principles paved the way for the activities of personnel and industrial engineering departments for years to come. However, in Taylor's time there was considerably more science in the writing about selection and education of workers than there was in practice. The fourth principle was Taylor's justification for his belief that trade unions were not necessary. Because increased efficiency would lead to greater surplus, which would be shared by management and labor (an assumption that organized labor did not accept), workers should welcome the new system and work in concert with management to achieve its potential. Taylor felt that workers would cooperate if offered higher pay for greater efficiency, and he actively opposed the rate-cutting practices by which companies would redefine work standards if the resulting pay rates were too high. But he had little sympathy for the reluctance of workers to be subjected to stopwatch studies or to give up their familiar practices in favor of new ones. As a result, Taylor never enjoyed good relations with labor.

1.5.2 Planning versus Doing

What Taylor meant in his fourth principle by "intimate friendly cooperation" was a clear separation of the jobs of management from those of the workers. Managers should do the planning—design the job, set the pace, rhythm, and motions—and workers should work. In Taylor's mind, this was simply a matter of matching each group to the work for which it was best qualified.

In concept, Taylor's views on this issue represented a fundamental observation: that planning and doing are distinct activities. Drucker describes this as one of Taylor's most

valuable insights, "a greater contribution to America's industrial rise than stopwatch or time and motion study. On it rests the entire structure of modern management" (Drucker 1954, 284). Clearly Drucker's *management by objectives* would be meaningless without the realization that management will be easier and more productive if managers plan their activities before undertaking them.

But Taylor went further than distinguishing the activities of planning and doing. He placed them in entirely separate jobs. All planning activities rested with management. Even management was separated according to planning and doing. For instance, the gang boss had charge of all work up to the time that the piece was placed in the machine (planning), and the speed boss had charge of choosing the tools and overseeing the piece in the machine (doing). The workers were expected to carry out their tasks in the manner determined by management (scientifically, of course) as best. In essence, this is the military system; officers plan and take responsibility, enlisted men do the work but are not held responsible.[6] Taylor was adamant about assigning workers to tasks for which they were suited; evidently he did not feel they were suited to planning.

But, as Drucker (1954, 284) points out, planning and doing are actually two parts of the same job. Someone who plans without even a shred of doing "dreams rather than performs," and someone who works without any planning at all cannot accomplish even the most mechanical and repetitive task. Although it is clear that workers *do* plan in practice, the tradition of scientific management has clearly discouraged American workers from thinking creatively about their work and American managers from expecting them to. Juran (1992, 365) contends that the removal of responsibility for planning by workers had a negative effect on quality and resulted in reliance by American firms on inspection for quality assurance.

In contrast, the Japanese, with their quality circles, suggestion programs, and empowerment of workers to shut down lines when problems occur, have legitimized planning on the part of the workers. On the management side, the Japanese requirement that future managers and engineers begin their careers on the shop floor has also helped remove the barrier between planning and doing. "Quality at the source" programs are much more natural in this environment, so it is not surprising that the Japanese appreciated the ideas of quality prophets, such as Deming and Juran, long before the Americans did.

Taylor's error with regard to the separation of planning and doing lay in extending a valuable conceptual insight to an inappropriate practice. He made the same error by extending his reduction of work tasks to their simplest components from the planning stage to the execution stage. The fact that it is effective to analyze work broken down into its elemental motions does not necessarily imply that it is effective to carry it out in this way. Simplified tasks could improve productivity in the short term, but the benefits are less clear in the long term. The reason is that simple repetitive tasks do not make for satisfying work, and therefore, long-term motivation is difficult. Furthermore, by encouraging workers to concentrate on motions instead of on jobs, scientific management had the unintended result of making workers inflexible. As the pace of change in technology and the marketplace accelerated, this lack of flexibility became a clear competitive burden. The Japanese, with their holistic perspective and worker empowerment practices, have consciously encouraged their workforce to be more adaptable.

By making planning the explicit duty of management and by emphasizing the need for quantification, scientific management has played a large role in spawning and shaping

[6]Taylor's functional management represented a break with the traditional management notion of a single line of authority, which the proponents of scientific management called "military" or "driver" or "Marquis of Queensberry" management (see, e.g., L. Gilbreth 1914). However, he adhered to, even strengthened, the militaristic centralization of responsibility with management.

the fields of industrial engineering, operations research, and management science. The reductionist framework established by scientific management is behind the traditional emphasis by the industrial engineers on line balancing and machine utilization. It is also at the root of the decades-long fascination by operations researchers with simplistic scheduling problems, an obsession that produced 30 years of literature and virtually no applications (Dudek, Panwalker, and Smith 1992). The flaw in these approaches is not the analytic techniques themselves, but the lack of an objective that is consistent with the overall system objective. Taylorism spawned powerful tools but not a framework in which those tools could achieve their full potential.

1.5.3 Other Pioneers of Scientific Management

Taylor's position in history is in no small part due to the legions of followers he inspired. One of his earliest collaborators was Henry Gantt (1861–1919), who worked with Taylor at Midvale Steel, Simond's Rolling Machine, and Bethlehem Steel. Gantt is best remembered for the Gantt chart used in project management. But he was also an ardent efficiency advocate and a successful scientific management consultant. Although Gantt was considered by Taylor as one of his true disciples, Gantt disagreed with Taylor on several points. Most importantly, Gantt preferred a "task work with a bonus" system, in which workers were guaranteed their day's rate but received a bonus for completing a job within the set time, to Taylor's differential piece rate system. Gantt was also less sanguine than Taylor about the prospects for setting truly fair standards, and therefore he developed explicit procedures for enabling workers to protest or revise the standards.

Others in Taylor's immediate circle of followers were Carl Barth (1860–1939), Taylor's mathematician and developer of special-purpose slide rules for setting "feeds and speeds" for metal cutting; Morris Cook (1872–1960), who applied Taylor's ideas both in industry and as Director of Public Works in Philadelphia; and Horace Hathaway (1878–1944), who personally directed the installation of scientific management at Tabor Manufacturing Company and wrote extensively on scientific management in the technical literature.

Also adding energy to the movement and luster to Taylor's reputation were less orthodox proponents of scientific management, with some of whom Taylor quarreled bitterly. Most prominent among these were Harrington Emerson (1853–1931) and Frank Gilbreth (1868–1924). Emerson, who had become a champion of efficiency independently of Taylor and had reorganized the workshops of the Santa Fe Railroad, testified during the hearings of the Interstate Commerce Commission concerning a proposed railroad rate hike in 1910–1911 that scientific management could save "a million dollars a day." Because he was the only "efficiency engineer" with firsthand experience in the railroad industry, his statement carried enormous weight and served to emblazon scientific management on the national consciousness. Later in his career, Emerson became particularly interested in the selection and training of employees. He is also credited with originating the term *dispatching* in reference to shop floor control (Emerson 1913), a phrase which undoubtedly derives from his railroad experience.

Frank Gilbreth had a somewhat similar background to that of Taylor. Although he had passed the qualifying exams for MIT, Gilbreth became an apprentice bricklayer instead. Outraged at the inefficiency of bricklaying, in which a bricklayer had to lift his own body weight each time he bent over and picked up a brick, he invented a movable scaffold to maintain bricks at the proper level. Gilbreth was consumed by the quest for efficiency. He extended Taylor's time study to what he called *motion study*, in which he made detailed analyses of the motions involved in bricklaying in the search for a more

efficient procedure. He was the first to apply the motion picture camera to the task of analyzing motions, and he categorized the elements of human motions into 18 basic components, or therbligs (Gilbreth spelled backward, sort of). That he was successful was evidenced by the fact that he rose to become one of the most prominent builders in the country. Although Taylor feuded with him concerning some of his work for nonbuilders, he gave Gilbreth's work on bricklaying extensive coverage in his 1911 book, *The Principles of Scientific Management.*

1.5.4 The Science in Scientific Management

Scientific management has been both venerated and vilified. It has generated both proponents and opponents who have made important contributions to our understanding and practice of management. One can argue that it is the root of a host of management-related fields, ranging from organization theory to operations research. But in the final analysis, it is the basic realization that management can be approached scientifically that is the primary contribution of scientific management. This is an insight we will never lose, an insight so basic that, like the concept of interchangeable parts, once it has been achieved it is difficult to picture life without it. Others intimated it; Taylor, by sheer perseverance, drove it into the consciousness of our culture. As a result, scientific management deserves to be classed as the first management *system.* It represents the starting point for all other systems. When Taylor began the search for a management system, he made it possible to envision management as a profession.

It is, however, ironic that scientific management's legacy is the application of the scientific method to management, because in retrospect we see that scientific management itself was far from scientific. Taylor's *Principles of Scientific Management* is a book of advocacy, not science. While Taylor argued for his own differential piece rate in theory, he actually used Gantt's more practical system at Bethlehem Steel. His famous story of Schmidt, a first-class man who excelled under the differential piece rate, has been accused of having so many inconsistencies that it must have been contrived (Wrege and Perroni 1947). Taylor's work measurement studies were often carelessly done, and there is no evidence that he used any scientific criteria to select workers. Despite using the word *scientific* with numbing frequency, Taylor subjected very few of his conjectures to anything like the scrutiny demanded by the scientific method.

Thus, while scientific management fostered quantification of management, it did little to place it in a real scientific framework. Still, to give Taylor his due, by sheer force of conviction, he tapped into the underlying American faith in science and changed our view of management forever. It remains for us to realize the full potential of this view.

1.6 The Rise of the Modern Manufacturing Organization

By the end of World War I, scientific management had firmly taken hold, and the main pieces of the American system of manufacturing were in place. Large-scale, vertically integrated organizations making use of mass production techniques were the norm. Although family control of large manufacturing enterprises was still common, salaried managers ran the day-to-day operations within centralized departmental hierarchies. These organizations had essentially fully exploited the potential economies of scale for producing a single product. Further organizational growth would require taking advantage of economies of *scope* (i.e., sharing production and distribution resources across

multiple products). As a result, development of institutional structures and management procedures for controlling the resulting organizations was the main theme of American manufacturing history during the interwar period.

1.6.1 Du Pont, Sloan, and Structure

The classic story of growth through diversification is that of General Motors (GM). Formed in 1908 when William C. Durant (1861–1947) consolidated his own Buick Motor Company with the Cadillac, Oldsmobile, and Oakland companies, GM rapidly became an industrial giant. The flamboyant but erratic Durant was far more interested in acquisition than in organization, and he continued to buy up units (including Chevrolet Motor Company) to the point where, by 1920, GM was the fifth largest industrial enterprise in America. But it was an empire without structure. Lacking corporate offices, demand forecasting, and coordination of production, the corporation encountered financial difficulties whenever sales slowed. Du Pont Company came to Durant's aid more than once by investing heavily in GM and finally forced him out in 1920 (Bryant and Dethloff 1990).

Pierre Du Pont (1870–1954) came out of semiretirement to succeed Durant as president with the hope of making the Du Pont Company's GM investments profitable. A more capable successor could not possibly have been found. In 1902, he and his cousins Alfred and Coleman had purchased control of E. I. du Pont de Nemours & Company, a collection of single-function explosives manufacturers, and had consolidated it into a centrally governed, multidepartmental, integrated organization (Chandler and Salsbury 1971). Well aware of scientific management principles,[7] Du Pont and his associates installed Taylor's manufacturing control techniques and accounting system, and introduced psychological testing for personnel selection. Perhaps Du Pont's most influential innovation, however, was the refined use of return on investment (ROI) to evaluate the relative performance of departments. By 1917, Du Pont Powder Company stood as the first modern American manufacturing corporation.[8]

When he moved to General Motors, Du Pont quickly identified Alfred P. Sloan (1875–1966) as his main collaborator and set out to reorganize the company. Du Pont and Sloan agreed that GM's activities were too numerous, scattered, and varied to be amenable to the centralized organization in use at Du Pont Powder Company. With Du Pont's support, Sloan crafted a plan to structure the company as a collection of autonomous operating divisions coordinated (but not run) by a strong general office. The various divisions were carefully targeted at specific markets (e.g., Cadillac at the high-priced market, Chevrolet at the low end to compete directly with Ford, and Buick and Oldsmobile in the middle; Pontiac was introduced between Chevrolet and Oldsmobile in the mid-1920s) in accordance with Sloan's goal of "a car for every purse and purpose" (Cray 1979). Under Sloan's reorganization, GM's general office borrowed ROI methods from Du Pont Powder Company for evaluating units, and also developed sophisticated new procedures for demand forecasting, inventory tracking, and market share estimation.

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[7] A. J. Moxham and Coleman du Pont had hired Frederick Taylor as a consultant at Steel Motor Company, and were instrumental in implementing Taylor's system when they later joined Du Pont as executives.

[8] The other candidate for the first modern manufacturing corporation would be General Electric, formed in 1892 by the merger of Edison General Electric and Thomson-Houston Electric, both of which were themselves products of mergers. To manage this first major consolidation of machinery-making companies, GE set up a modern structure of top and middle management patterned after that used by the railroads. However, its financial measures were not as sophisticated as those used by Du Pont and, unlike in the modern American corporation, a board of directors dominated by outside financiers held considerable veto power (Chandler 1977).

These techniques gradually became standard throughout American industry and are still used in modified form today.

Sloan's strategy was stunningly effective. In 1921, GM was a distant second with 12.3 percent of the automotive market to Ford's 55.7 percent. With its targeted product lines and regular introduction of new models, GM increased its share to 32.3 percent by 1929, while Ford, which waited until 1927 to replace the Model T with the Model A, fell to 31.3 percent. By 1940, Ford, which was still run by Henry, his son Edsel, and a tiny group of executives, was in serious trouble, having fallen to 18.9 percent and third place behind Chrysler's 23.7 percent share and far behind GM's 47.5 percent (Chandler 1990). Only a massive reorganization by Henry Ford II, beginning in 1945 and following the GM model, saved Ford from extinction.

In addition to forging hugely successful firms, Pierre Du Pont and Alfred Sloan shaped the American manufacturing corporation of the 20th century. While exhibiting many variations, all large industrial enterprises in the 20th century have used one of two basic structures. The centralized, functional department organization developed at Du Pont is used predominantly by firms with a single line of products in a single market. The multidivisional, decentralized structure developed at GM is the rule for firms with several product lines or markets. The environment in which we practice manufacturing today owes its existence to the efforts of these two innovators and their many associates.

1.6.2 Hawthorne and the Human Element

As industrial organizations grew larger and more technologically complex, the role of the worker took on increased importance. Indeed, the primary goals of scientific management—motivating workers and matching workers to tasks—were essentially behavioral. However, Taylor, being the true engineer, seemed to believe that human beings could be optimized in the same sense as a metal-cutting machine. For example, he observed that because a worker "strains every nerve to secure victory for his side" in a baseball game (Taylor 1911, 13), he or she should be capable of similar exertion at work. Despite the fact that he was an accomplished athlete, Taylor did not show the slightest appreciation for the psychological difference between work and play. Similarly, while he could spend countless hours studying and educating workers in the science of shoveling, he had no patience for a worker's sentimental attachment to the shovel he had handled for years. Although his writings certainly indicate a concern for the workers, Taylor never managed to understand their points of view.

In spite of Taylor's personal blind spots, scientific management served to catalyze the behavioral approach to management by systematically raising questions on authority, motivation, and training. The earliest writers in the field of industrial psychology acknowledged their debt to scientific management and framed their discussions in terms consistent with Taylor's system.

The acknowledged father of industrial psychology was Hugo Munsterberg (1863–1916). Born and educated in Germany, Munsterberg came to America and established a famous psychology laboratory at Harvard, where he studied a wide range of psychological questions in education, crime, and philosophy as well as industry. In his 1913 book *Psychology and Industrial Efficiency*, he paid tribute to scientific management and directly addressed it in three parts entitled "The Best Possible Man" (i.e., worker selection), "The Best Possible Work" (i.e., training and working conditions), and "The Best Possible Effect" (i.e., achieving management goals). Munsterberg's groundbreaking work paved the way for a steady stream of industrial psychology textbooks and a psychological testing fad shortly after World War I.

Among the Americans who led the way in the application of psychology to industry was Walter Dill Scott (1869–1955), who studied worker selection and rating for promotion (Scott 1913). A series of articles he wrote in 1910 to 1911 for *System* magazine (now *Business Week*) under the title "The Psychology of Business" were highly influential in raising awareness of the field of psychology among managers. He later turned to psychological research in advertising, defined the proper role of the newly arising personnel management function, and served as president of Northwestern University.

Lillian Gilbreth (1878–1972) was an early and visible proponent of industrial psychology from inside the ranks of scientific management. Wife of scientific management pioneer Frank Gilbreth and matriarch of the brood made famous by the book *Cheaper by the Dozen* (Gilbreth and Carey 1949), Gilbreth was one of the pioneers of the scientific management movement. In addition to collaborating with her husband on his motion studies work and carrying on this work after his death, she became one of the first advocates of psychology in management with her book *The Psychology of Management* (1914), based on her doctoral thesis in psychology at Brown University. In this book she contrasted scientific management with traditional management along various dimensions, including individuality. Her premise was that because of its emphasis on scientific selection, training, and functional foremanship, scientific management offered ample opportunity for individual development, while traditional management stifled such development by concentrating power in a central figure. Although the details of her work in psychology read today like an apology for scientific management and have largely been forgotten, Lillian Gilbreth deserves a place in management history for her early call for the humanization of the management process.

Mary Parker Follett (1868–1933) belonged chronologically to the scientific management era, but her thinking on the sociology and psychology of work was far ahead of its time. Like Lillian Gilbreth, she found in Taylor's functional foremanship a sound basis for allocating authority:

> One person should not give orders to another person, but both should agree to take their orders from the situation ...We have here, I think, one of the largest contributions of scientific management; it tends to depersonalize orders. (Follett 1942, 59)

However, Follett was repelled by the relegation of the worker to simply carrying out tasks given and designated by management. She held that "not consent but participation is the right basis for all social relations" (Follett 1942, 211). By "participation," Follett meant to include the workers' ideas as well as their labor. Her rationale was that the ideas are valuable in themselves, but more importantly, the very process of participation is essential to establishing a functional work environment. Although at times her ideas sound idealistic, the depth and range of her work are astonishing and many of her insights still apply today.

A major episode in the quest to understand the human side of manufacturing was the series of studies conducted at the Western Electric Hawthorne plant in Chicago between 1924 and 1932. The studies originally began with a simple question: How does workplace illumination affect worker productivity? Under sponsorship of the National Academy of Science, a team of researchers from Massachusetts Institute of Technology observed groups of coil-winding operators under different lighting levels. They observed that productivity relative to a control group went up as illumination was increased, as had been expected. Then, in another experiment, they observed that productivity also went up when illumination was *decreased,* even to the level of moonlight (Roethlisberger and Dickson 1939).

Unable to explain the results, the original team abandoned the illumination studies and began other tests—of the effects on productivity of rest periods, length of work week, incentive plans, free lunches, and supervisory styles. In most cases, the trend was for higher-than-normal output by the groups under study.

Various experts were brought in to study the puzzling Hawthorne data, most notably George Elton Mayo (1880–1949) from Harvard. Approaching the problem from the perspective of the "psychology of the total situation," he came to the conclusion that the results were primarily due to "a remarkable change of mental attitude in the group." In the legend that subsequently grew up around the Hawthorne studies, Mayo's interpretation was reduced to the simple explanation that productivity increased as a result of the attention received by the workers under study, and this was dubbed the *Hawthorne effect*. However, in his writings, Mayo (1933, 1945) was not satisfied with this simple explanation and modified his view beyond this initial insight, arguing that work is essentially a group activity and that workers strive for a sense of belonging, not simply financial gain, in their jobs. By emphasizing the need for listening and counseling by managers in order to improve worker collaboration, the industrial psychology movement shifted the emphasis of management from technical efficiency, the focus of Taylorism, to a richer, more complex, human relations orientation.

1.6.3 Management Education

In addition to fostering the human relations perspective, the rise of the modern integrated business enterprise solidified the position of the professional managerial class. Prior to 1920, the majority of large-scale businesses were run by owner-entrepreneurs such as Carnegie, Ford, and Du Pont. Growth and integration after World War I resulted in systems too large to be run by owners (although Henry Ford tried, with disastrous results). Consequently, more and more decision-making responsibility was given to managers, middle and upper, who were without significant holdings in the firm.

In the 19th and early 20th centuries, it was not uncommon for these professional managers to be drawn from the ranks of the skilled workers (e.g., machinists). But as the modern business enterprises matured, formal university training became increasingly necessary. Many managers of this era were educated in traditional engineering disciplines (e.g., mechanical, electrical, civil, chemical). Some, however, began to seek education directly related to management, in either business schools or industrial engineering programs, both of which were emerging in the wake of the scientific management movement at the turn of the century.

The first American undergraduate business program was established in 1881 at the University of Pennsylvania's Wharton School. This was followed by schools at Chicago and Berkeley in 1898, and at Dartmouth (with the first master's level program), New York University, and Wisconsin in 1900. By 1910 there were more than a dozen separately organized schools of business at American universities, although the programs were generally small and had curricula restricted to background (e.g., economics, law, foreign languages) with anecdotes about the best industrial practices. The leading program of the time, Harvard, was organized in large part by Arch Shaw who had previously lectured at Northwestern and, as head of a Chicago publishing house, had published *Library of Factory Management*. Shaw relied heavily on outside lecturers from the scientific management movement (e.g., Frederick Taylor, Harrington Emerson, Carl Barth, Morris Cooke) and was instrumental in introducing the case method, which became Harvard's trademark and would heavily influence business education across America (Chandler 1977).

Between 1914 and 1940, American business schools grew and diversified their curricula. During this period most of the state universities introduced business programs; among them were Ohio State (1916); Alabama, Minnesota, North Carolina (1919); Virginia (1920); Indiana (1921); Kansas and Michigan (1924) (Pierson 1959). As the number of programs grew, so did the number of degrees granted: from 1,576 BAs and 110 MBAs in 1920, to 18,549 BAs and 1,139 MBAs in 1940 (Gordon and Howell 1959). At the same time, the functional areas of a business education were being standardized; by the mid-1920s, more than half of the 34 schools belonging to the American Association of Collegiate Schools of Business required students to take courses in accounting, business law, finance, statistics, and marketing. Textbooks supporting this functional orientation also began to appear (e.g., Hodge and McKinsey 1921 in accounting, Lough 1920 and Bonneville 1925 in finance, and Cherington 1920 in marketing).

American engineering schools also responded to the need for management education by introducing industrial engineering (IE) programs. Like the early business schools, the first IE departments were heavily influenced by the scientific management movement. Hugo Diemer taught the first shop management course in the mechanical engineering department of the University of Kansas in 1901 to 1902 and later went on to found the first IE curriculum at Penn State in 1908. Other engineering schools followed, and by the end of World War II there were more than 25 IE curricula in American universities. After the war, growth of the IE field tracked that of the economy: by the 1980s the number of IE programs had reached about 100 (Emerson and Naehring 1988).

The tools of industrial engineering evolved as the field grew during the interwar period. In addition to the methods of time and motion study (Gilbreth 1911; Barnes 1937), techniques of cost engineering (Fish 1915; Grant 1930), quality control (Shewhart 1931; Grant and Leavenworth 1946), and production/inventory management (Spriegel and Lansburgh 1923; Mitchell 1931; Raymond 1931; Whitin 1953) were presented in textbook form and widely introduced into industrial engineering curricula. By the end of World War II, all the major components of the IE discipline were in place, with the exception of the quantitative tools of operations research, which did not appear in a major way until after the war.

1.7 Peak, Decline, and Resurgence of American Manufacturing

Although the modern American manufacturing enterprise had largely been formed by the 1920s, the depression of the 1930s and the war of the 1940s prevented the country from reaping the full benefits of its powerful manufacturing sector. Thus, it was not until the post–World War II period, in the 1950s and 1960s, that America enjoyed a golden era of manufacturing. This era shaped the attitudes of a generation of managers, heavily influenced business and engineering schools, and set the stage for the not-so-golden era of manufacturing in the 1980s and 1990s.

1.7.1 The Golden Era

American manufacturing went into World War II in an extremely strong position, having mastered the techniques of mass production and distribution and management of large-scale enterprises. It emerged from the war in a position of undisputed global dominance. In 1945 the American industrial plant was easily the strongest in the world. The American market was eight times the size of the next-largest market in the world, giving American firms a huge scale advantage. American per capita income was eight times that of Japan

in the 1950s, providing a vast source of capital, despite the fact that savings rates were lower than those in other countries. The American primary and secondary education system was the finest in the world. And with the GI Bill added to the land grant college system, America outpaced the rest of the world in higher education as well. Labor productivity (measured as gross domestic product per worker-hour) was nearly double that of any European country, and fully three times that of Germany and seven times that of Japan (Maddison 1984). With its huge domestic market, ready capital, and well-trained, productive workforce, America could produce and distribute goods at a pace and scale unthinkable to anyone else.

In contrast, the rest of the world lay virtually in ruins. The industrial plant in Europe and Japan had been physically devastated by the war. The scientific establishments of many countries were in disarray as America inherited some of their best brains. Furthermore, at the war's end, because transportation was expensive and trade policies protectionist, economies were far less global than they are today. Because the primary market for almost everything was in America, other countries would have been at a huge disadvantage even without their inferior physical plants and disrupted R&D base.

The resulting postwar boom in American manufacturing was undoubtedly exhilarating and was certainly profitable. Americans saw per capita income (in constant 1958 dollars) rise from $1 in 1950 to $3 in 1970 (U.S. Department of Commerce 1972). In 1947, the 200 largest industrial firms in America were responsible for 30 percent of the world's value added in manufacturing and 47.2 percent of total corporate manufacturing assets. By 1963, they accounted for 41 percent of value added and 56.3 percent of assets. By 1969 the top 200 American industrials accounted for 60.9 percent of the world's manufacturing assets (Chandler 1977, 482). For a while the living was easy. But as many of the baby boom generation enjoyed "Leave It to Beaver" lives in suburbia, the competitive world that would be their inheritance was being shaped as America's former enemies and allies recovered from the war.

1.7.2 Accountants Count and Salesmen Sell

During the golden era following World War II, the principal opportunities for American manufacturing firms were plainly in the areas of marketing, to develop the huge potential markets for new products, and finance, to fuel growth. As we mentioned earlier, America already had a stronger history in advertising than the Old World. Moreover, as indicated by the reliance of Du Pont and GM on financial measures to coordinate their large-scale enterprises, American manufacturers were well acquainted with the tools of finance. The manufacturing function itself became of secondary importance. American dominance in manufacturing was so formidable that eminent economist John Kenneth Galbraith proclaimed the problem of production "solved" (Galbraith 1958).

But as the manufacturing boom of the 1950s and 1960s turned into the manufacturing bust of the 1970s and 1980s, it became plain that something was wrong. The simplest explanation is that since the details of manufacturing didn't matter during the golden era, American firms became lax. Because American goods were the envy of the world, firms could largely dictate the quality specifications of their products, and managers learned to take quality for granted. Because of the American technological advantage and the lack of competition, continual improvement was unnecessary to maintain market share, and managers learned to take the status quo for granted. When foreign firms, which could not afford to take anything for granted, recovered sufficiently to present a legitimate challenge, many American firms lacked the vigor to meet it.

While this simple explanation may be accurate for some firms or industries, it does not give the whole story. The influences of the golden era on the current condition of American manufacturing are subtle and complex. Besides promoting a deemphasis on manufacturing details, the emphasis on marketing and finance in the 1950s and 1960s profoundly influenced today's American manufacturing firms. Recognizing these areas as having the greatest career potential, more and more of the "best and brightest" chose careers in marketing and finance. These became the glamour functions, while manufacturing and operations were increasingly viewed as dead-end "career breakers." This led to the simultaneous rise of the marketing and finance outlooks as dominant perspectives in American manufacturing firms. We trace some of the consequences below.

The Marketing Outlook. With top executives and rising stars increasingly preoccupied with selling, the organizations themselves took on more of the marketing outlook. While there is nothing intrinsically wrong with the marketing outlook for the marketing department, for the firm itself it can be an overly conservative perspective. The principal task of marketing is to analyze the introduction of new products. But the products that are most amenable to analysis tend to be imitative, rather than innovative.

A good case history that illustrates the pitfalls of the marketing outlook is that of IBM and the xerography process. In the late 1950s, Haloid Company (which had introduced the first commercial xerographic copier in 1949 and later changed its name to Xerox) offered IBM the opportunity to jointly develop the first practical office copier. IBM enlisted Arthur D. Little, a Boston management consulting firm, to conduct a market study on the potential for such a product. A. D. Little, basing its conclusions on consumption of carbon paper and assessments of which offices needed to make paper copies, estimated maximum demand to be no more than 5,000 machines, far less than necessary to justify the development costs (Kearns and Nadler 1992). IBM declined the offer, and Xerox went on to make so much money that royalties to Battelle Memorial Institute, the research laboratory where the process was developed, threatened its not-for-profit status.

The conclusion is that the marketing outlook will often not justify the high-risk, high-payoff ventures associated with truly innovative new products. The Xerox machine *created* a demand for paper copies that did not previously exist. While hard to analyze, revolutionary products such as this can be enormously profitable. An overreliance on marketing may have caused large American manufacturing firms to take on fewer of these ventures than they should have. As evidence of this, consider that the last major automotive innovation to appear first on an American car was the automatic transmission in the 1940s. Four-wheel drive, four-wheel steering, turbocharging, and antilock brakes were all introduced first by foreign automakers (Dertouzos, Lester, Solow 1989, 19).

The Finance Outlook. As noted earlier, Du Pont pioneered the use of ROI as a measure of the effectiveness of capital in a large-scale enterprise shortly after the turn of the century. However, in the 1910s, Du Pont Powder Company was primarily owned and managed by the Du Pont family; so there was no question that it was to be managed for the long-term benefit of its owners. Pierre Du Pont would never have used short-term ROI to evaluate the performance of individual managers. By the 1950s and 1960s, high-level managers were no longer owners, and the pervasiveness of the finance outlook had extended short-term ROI in the form of quarterly reports to a measure of individual performance.

An overreliance on short-term ROI discouraged managers from pursuing high-risk or long-term ventures and thus further aggravated the tendency toward the conservatism

promoted by the marketing outlook. Short-term ROI can be artificially inflated for a while, possibly many years, through reduction in the investment base by forgoing process upgrades, equipment maintenance, and replacement, and by purchasing less than state-of-the-art facilities. However, in the long run, such practices can put a firm at a distinct competitive disadvantage. For instance, Dertouzos, Lester, and Solow (1989, 57) cite statistics showing that the rate of business-sector capital investment as a percentage of net output in Japan and West Germany has significantly outpaced that of America since 1965, precisely the period over which these countries significantly narrowed the productivity gap between themselves and America.

Moreover, the finance outlook, which views manufacturing management as essentially analogous to portfolio management, implies that the way to minimize risk is to diversify. The portfolio manager diversifies investments by purchasing various types of securities. The manufacturing executive diversifies by acquiring businesses outside the firm's core activities. As the rest of the world recovered from the war and began to give American firms serious competition in the 1960s, manufacturing firms increasingly turned to the financial response of diversification, almost to the point of mania in the late 1960s. In 1965 there were 2,000 mergers and acquisitions in America; by 1969 the number had risen to more than 6,000. Moreover, of the assets acquired during the 1963–1972 merger wave, nearly three-fourths were for product diversification, and one-half of these were in unrelated products (Chandler 1977). The effect was a dramatic change in the character of America's large manufacturing firms. In 1949, 70 percent of the 500 largest American firms earned 95 percent of revenues from a single business. By 1969, 70 percent of the largest firms no longer had a dominant business (Davidson 1990).

Like the marketing outlook, the finance outlook is too restrictive a perspective for the entire firm. While managers of purely financial portfolios are certainly rational in their use of diversification to achieve stable returns, manufacturing firms that use the same strategy are neglecting an important difference between portfolio and manufacturing management: Manufacturing firms influence their destinies in a far more direct way than do investors. The profitability of a manufacturing business is a function of many things, including product design, product quality, process efficiency, customer service, and so forth. When a firm moves away from its core business, there is a danger that it will fail to perform on these key measures. This can more than offset any potential advantage from diversification and can even threaten the existence of the company.

Indeed, the preponderance of statistical evidence paints a negative picture of the effectiveness of the merger-and-acquisition strategy. A detailed survey by Ravenscraft and Scherer (1987) of mergers during the 1960s and early 1970s showed that, on average, profitability and efficiency of firms decline after they are acquired. Hayes and Wheelwright (1984, 13) cite further statistics from Fruhan (1979) and *Forbes* magazine showing that highly diversified conglomerates tend to underperform relative to firms with highly focused product markets. In the realm of popular culture, books like *Barbarians at the Gate* (Burrough 1990) and *Merchants of Debt* (Anders 1992) graphically illustrate how far pure unbridled greed can take the merger-and-acquisition process from any consideration of manufacturing effectiveness. Scherer and Ross (1990, 173), in a comprehensive survey of firm structure and economic performance, sum up the effectiveness of the merger-and-acquisition approach with this statement: "The picture that emerges is a pessimistic one: widespread failure, considerable mediocrity, and occasional successes."

1.7.3 The Professional Manager

The rapid growth following World War II profoundly shaped the manufacturing manager in two additional ways. First, strong demand for managers prompted an acceleration of

the promotion process, under the "fast-track manager" system. Second, unable to nurture enough managers internally, industry increasingly looked to the universities to provide professional management training. Before the war, MBA-trained managers were still a rarity; only 1,139 master's degrees in business were granted in 1940 (Gordon and Howell 1959, 21). After the war, this tripled to 3,357 in 1948 and continued growing steadily, so that by the 1980s the MBA had become the standard credential for the business executive in America. This intensified emphasis led to changes in the character of both corporations and business schools.

The Fast-Track Manager. As Hayes and Wheelwright (1984) point out, before the war, it was traditional for managers to spend considerable time—a decade or more—in a job before being moved up the managerial ladder. After the war, however, there were simply not enough qualified people to fill the expanding need for managers. To fill the gap, business organizations identified rising stars and put them on fast tracks to executive levels. These individuals did shorter rotations through lower-level assignments—two or three years—on their way to upper-level positions. As a result, top manufacturing managers who came of age in the 1960s and 1970s were likely to have substantially less depth of experience at the operating levels than their predecessors.

Worse yet, the concept of a fast-track manager, first introduced to fill a genuine postwar need, gradually became institutionalized. Once some "stars" had moved up the promotion ladder quickly, it became impossible to convince those who followed to return to the slower, traditional pace. A bright young manager who was not promoted quickly enough would look for opportunities elsewhere. Lifelong loyalty to a firm became a thing of the past in America, and it became commonplace for top managers in one industry to have come up from the ranks of an entirely different one.[9] American business schools preached the concept of the professional manager who could manage any firm regardless of the technological or customer details, and American industry practiced it.[10] The days of Carnegie and Ford, owner-entrepreneur-managers who knew the details of their businesses from the bottom up, were gone.

Academization of Business Schools. As business schools expanded after the war to meet the demand for professional managers, their pedagogical approaches came under increasing scrutiny. In 1959, two influential studies of American business schools, commissioned by Ford Foundation (Gordon and Howell 1959) and Carnegie Corporation (Pierson 1959), were released. These studies criticized American universities for taking an overly vocational approach to business education and called for an increase in academic standards and a broadening of emphasis to promote general knowledge, based on the "fundamental disciplines" of the behavioral sciences, economics, and mathematics and statistics. The studies advocated an interesting mix of specialization (i.e., emphasis on more sophisticated analytical techniques[11]) and generalization (i.e., development of professional managers who are prepared to deal with virtually any management problem).

Having been on the fringe of academic respectability from their inception, the business schools took the studies' recommendations seriously. They hired faculty specialists in psychology, sociology, economics, mathematics, and statistics—many without any

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[9]For example, John Scully came from Pepsi to head Apple Computer, and Archie McCardle came from Xerox to head International Harvester.

[10]For that matter, American government practiced it. When Secretary of the Treasury Donald Regan and White House Chief of Staff James Baker exchanged jobs during the Reagan administration, there was little mention of it in the press—except to note the different management styles of the two men.

[11]Presumably this had something to do with the fact that the studies were done in the era of Sputnik—a time of widespread faith in science.

business background whatever. They revised curricula to include more courses in these basic "theoretical" subjects and reduced courses aimed at training students for specific jobs. Operations research, which had burst onto the scene with some well-publicized military successes during World War II and was developing rapidly in the 1960s with the evolution of the digital computer, was quickly absorbed into operations management. The concept of the professional manager became the ruling paradigm in American business education.

This "modernizing" of the business schools did more than produce a generation of managers long on general theories and short on specific practical skills. It eroded the business schools' traditional, albeit small, role as repositories of the best of industry practice. With specialists in psychology and mathematics pursuing narrowly focused research in arcane academic journals, it is hardly surprising that when productivity growth declined in the late 1970s and early 1980s, industry did not look to the universities for help. Instead, it turned to Japanese examples (e.g., Schonberger 1982) and anecdotal surveys of industry practice by consultants (e.g., Peters and Waterman 1982). Thus, after being educated in the "scientific" tools of management, the MBA-trained professional managers of the 1980s and 1990s were wooed by an endless stream of quick fixes for their management woes. Fads based on buzzwords, such as theory Z, management by objectives, zero-based budgeting, decentralization, quality circles, restructuring, "excellence," management by walking around, matrix management, entrepreneuring, value chain analysis, one-minute managing, just-in-time, total quality management, time-based competition, business process reengineering, and many others, came and went with numbing regularity. While many of these "theories" contain valuable insights, the sheer number of them is evidence that the fix is not quick.

The ultimate irony occurred in the 1980s when, in a desperate attempt to win back the trust of students alienated by the almost total disconnect between classroom and boardroom, many operations management courses began to teach the buzzword fads themselves. In doing so, business schools gave up their role as arbiter of what works and what does not. Instead of being trendsetters, they became trend followers.

It is apparent that business schools and corporations have swung far apart since the Ford and Carnegie studies of 1959, with industry naively relying on glib buzzword approaches and academia leaning too far toward specialized research and imitative teaching. It is time for a reappraisal of both. Business schools need to recover their foundation in practice, in order to focus their tools on problems of real industry interest instead of on abstract intellectual challenge. Industry needs to recover its appreciation of the importance of the technical details of manufacturing and develop the capacity to systematically evaluate which management practices work, instead of lurching from one bandwagon to the next. By adjusting the attitudes of both academics and practitioners, we have the potential to apply the tools and technology developed in the decades since World War II to sustain manufacturing as a solid base of the American economy well into the 21st century.

1.7.4 Recovery and Globalization of Manufacturing

In many respects, the 1990s represented a resurgence of American manufacturing after the decline of the 1970s and 1980s. In 1997, manufacturing profits were at a 40-year high, and unemployment was at its lowest level in more than two decades. Annual productivity increases in manufacturing had returned to a healthy rate above three percent. Seven years of economic growth had spurred investment in physical plant, so that nonresidential equipment owned by business nearly doubled between 1987 and 1996 (*Business Week*, June 9, 1970, 70).

Good times for American manufacturers also extended beyond the domestic market. The Institute for Management Development in Lausanne, Switzerland, ranked America as the most globally competitive nation in the world every year during the period 1993 to 1997. A 1993 survey by the Center for the Study of American Business (CSAB) at Washington University in St. Louis of 48 manufacturing executives found that 90 percent considered their firms more competitive than they had been five years earlier (Chilton 1995). Large majorities of these executives also reported that quality and product development time had improved substantially over this same period.

While encouraging, the situation in the mid-1990s was far from a return to that of the mid-1960s. The American economy was strong but hardly dominant. World-class firms in struggling economies retained their potential to offer intense competition; for example, despite improved profitability of America's "big three," Toyota is still widely regarded as the premier automaker in the world (Taylor 1997). American manufacturers remained keenly aware of competition from around the globe. The CSAB survey reported that 75 percent of manufacturing executives strongly agreed (and an additional 10 percent somewhat agreed) that the competition they faced in 1993 was much stiffer than that 10 years earlier, and large majorities agreed that even more improvements in quality and product development times would be needed in the next five years in order to keep pace.

Furthermore, some statistics gave troublesome or ambiguous signs. For instance, trade deficits in the 1990s remained at or near record levels, although the deficit as a percentage of exports fell significantly. Also, labor productivity increases that were ascribed to downsizing of the manufacturing workforce in the 1980s and 1990s may have been partially a by-product of a workforce reclassification, from permanent employees to temporary employees and consultants. Finally, the productivity increases and economic recovery did not translate to a surge in real wages. From 1970 to 1985 productivity grew at a pace of 1.9 percent per year while real wages grew 0.87 percent per year. However, from 1985 to 1996 the growth in productivity was 2.5 percent while wage growth was only 0.26 percent per year. This may have been partly due to the increasing influence of Wall Street. The bull market of the 1990s (driven at least in part by baby-boomers seeking retirement investments) encouraged analysts to look more closely than ever at anticipated earnings. This in turn motivated management to continue making sharp cuts in the workforce (both labor and middle management), using temporary help, and instituting many other productivity improvements, all while keeping wages nearly constant. Clearly, the world of manufacturing has become a very different, and much more intensely competitive, place than it was during America's golden era.

1.8 The Future

America's manufacturing future cannot help but be influenced by its past. The practices and institutions used today have evolved over the past 200 years. The influences range from the ramifications of the myth of the frontier to our love affair with finance and marketing, and they will not evaporate overnight. An appreciation of what has gone before can at least make us conscious of what we are dealing with (a brief summary of manufacturing milestones is given in Table 1.1). But history shapes only the possibilities for the future, not the future itself. It is up to the next generation of manufacturing managers to evolve the American system of manufacturing to its next level.

What will this level be? Although no one can say for sure, it is our belief that the concept of the professional manager is bankrupt. In a world of intense global competition, simply setting appropriate general guidelines is not enough. Managers need detailed knowledge about their business, knowledge that must include *technical* details.

TABLE 1.1 Milestones in the History of Manufacturing

Date	Event
4000 B.C.	Egyptians coordinate large-scale projects to build pyramids.
1500	Leonardo da Vinci systematically studies shoveling.
1733	John Kay invents flying shuttle.
1765	James Hargreaves invents spinning jenny.
1765	James Watt invents steam engine.
1776	Adam Smith publishes *Wealth of Nations,* introducing the notions of division of labor and the invisible hand of capitalism.
1776	James Watt sells first steam engine.
1781	James Watt invents system for producing rotary motion from up-and-down stroke of steam engine.
1785	Honore LeBlanc shows Thomas Jefferson interchangeable musket parts.
1793	First modern textile mill in America established in Pawtucket, RI.
1801	Eli Whitney contracted by U.S. government to produce muskets, using system of interchangeable parts.
1814	Integrated textile facility established in Waltham, MA.
1832	Charles Babbage publishes *On the Economy of Machinery and Manufactures,* dealing with organization and costing procedures for factories.
1840	Opening of anthracite coal fields in eastern Pennsylvania provides first American source of inexpensive nonwater power.
1851	Crystal Palace Exhibition in London displays "American system of manufacturing."
1854	Daniel C. McCallum develops and implements earliest large-scale organization management system at New York and Erie Railroad.
1855	Henry Bessemer patents a process for refining iron into steel that was far better suited to mass production than earlier "puddling" processes.
1869	The first transcontinental railroad, the Union Pacific–Central Pacific, is completed.
1870	Marshall Field makes use of inventory turns as a measure of retail operation performance.
1875	Andrew Carnegie opens the Edgar Thompson Steel Works in Pittsburgh, the first integrated Bessemer rail mill built from scratch and for decades the largest steel works in the world.
1877	Arthur Wellington publishes *The Economic Theory of the Location of Railways,* the first book to present methods of capital budgeting.
1880	American Society of Mechanical Engineers (ASME) founded.
1886	Charles Hall of the United States and Paul Heroult in Europe simultaneously invent electrolytic method for reducing bauxite into aluminum.
1886	Henry Towne presents paper at ASME calling for an "Economic Section" devoted to shop management.
1910	Hugo Diemer publishes *Factory Organization and Administration,* the first industrial engineering textbook.
1911	F. W. Taylor publishes *The Principles of Scientific Management.*
1913	Henry Ford introduces first moving automotive assembly line in Highland Park, MI.
1913	Ford W. Harris publishes *How Many Parts to Make at Once.*
1914	Lillian Gilbreth publishes *The Psychology of Management.*
1915	John C. L. Fish publishes *Engineering Economics: First Principles,* the first text to present discounted cash flow methods.
1916	Henri Fayol publishes first overall theory of management as *Administration industrielle et générale* (not translated into English until 1929).
1920	Alfred P. Sloan reorganizes General Motors to consist of a general office and several autonomous divisions.
1924	Hawthorne studies begin at Western Electric plant in Chicago; they continue to 1932.
1931	Walter Shewhart publishes *Economic Control of Quality of Manufactured Product,* introducing the concept of the control chart.
1945	ENIAC (Electronic Numerical Integrator and Calculator), the first fully electronic digital computer, is built at the Univeristy of Pennsylvania.
1947	Herbert Simon publishes *Administrative Behavior,* marking a change in focus of organization theory from the structure of organizations to the process of decision making.
1953	Thomson Whitin publishes *The Theory of Inventory Management,* the first book to develop a theory to underlie the practice of inventory control.
1954	Peter Drucker publishes *The Practice of Management,* introducing the concept of Management by Objectives (MBO) on a wide scale.
1964	The IBM 360 becomes the first computer based on silicon chips.
1975	Joseph Orlicky publishes *Material Requirements Planning.*
1977	Introduction of the Apple II starts the personal computer revolution in earnest.
1978	Taichi Ohno publishes *Toyota seisan hoshiki* on the Toyota production system.

Unfortunately, the rise of such monolithic software packages as Enterprise Requirements Planning (the subject of Chapter 3) which purport to encapsulate "best practices" may prove to be a giant step backward in terms of managers better understanding their practices.

In the future, survival itself is likely to depend on understanding these details. The manufacturing function is no longer a necessary evil that can be taken for granted; it is a vital strategic function. In an era when products move from cutting-edge technology to commodities in the blink of an eye, inefficient manufacturing is likely to be fatal. The economic recovery of the 1990s and the fact that several universities have initiated programs in manufacturing management that stress the technical aspects and operating details of manufacturing are encouraging signs that we are adjusting to the new era.

But change will not come uniformly to all of American manufacturing. Some firms will adapt—indeed, have already adapted—to the new globally competitive world of manufacturing; others will resist change or will continue to seek some kind of technological quick fix. American firms will not rise or fall as a group. Firms that master the intricacies of manufacturing under the new world order will thrive. Those that cling to the methods evolved under the unique, and long-gone, conditions following World War II will not. Those that continue to increase profits by squeezing their employees to increase productivity without allowing real wages to rise will also fail (it appears that the General Motors strike in the summer of 1998 was a crack in the veneer of new American juggernaut).

To make the transition to the new era of manufacturing, it is crucial to remember the lessons of history. Consistently, the key to effective manufacturing has been not technology alone, but also the organization in which the technology was used. The only way for a manufacturing firm of the future to gain a significant strategic advantage over the long term will be to focus and coordinate its manufacturing operation, in conjunction with product and market development, with customer needs. The goal of this book is to provide the manufacturing manager with the intuition and tools needed to do just this.

Discussion Points

1. Before 1900, despite its weaknesses in effective management of workers, manufacturing leadership was well provided by top management. They were technological entrepreneurs, architects of productive systems, veritable lions of industry. But when they delegated their production responsibilities to a second-level department, the factory institution never recovered its vitality. The lion was tamed. Its management systems became protective and generally were neither entrepreneurial nor strategic. Production managers since then have typically had little to do with initiating substantially new process technology—in contrast to their predecessors before 1900. Wickham Skinner (1985)
   a. Do you agree with Skinner?
   b. What structural differences between manufacturing enterprises before 1890 and after 1920 contributed to this difference in managerial orientation?
   c. Why have manufacturing managers become increasingly seen as "custodians of financial assets"? (What were the impacts on the role of manufacturing as part of a business strategy?)
   d. How is Japan (or Germany) different from (or the same as) America with regard to this trend in manufacturing leadership?
   e. Taking the structural characteristics of manufacturing enterprises (e.g., scale, complexity, pace of technological change) as given, what can be done to revitalize manufacturing leadership?

2. America's industrial rise took place following a war with its principal rival (England); Japan's rise also took place following a war with its primary rival (America). America's success could be attributed to its system (i.e., interchangeable parts and vertical integration), while Japan's success could be attributed to its system (i.e., just-in-time).
   a. What other parallels can be drawn between the manufacturing stories of America and Japan?
   b. What are key differences?
   c. What relevance do these similarities and differences have to the manufacturing manager and policy maker of today?

Study Questions

1. What events characterized the first and second industrial revolutions? What effects did these changes have on the nature of manufacturing management?
2. List three key impacts of Frederick W. Taylor's scientific management on the practice of manufacturing management in America.
3. Proponents of a service economy for America sometimes compare the recent decline in manufacturing jobs to the earlier decline in agriculture jobs. In what way are these two declines different? How might this affect the argument that a shift to a service economy will not reduce our standard of living?
4. What are some signs of the decline of American manufacturing? How long has this been going on?
5. Give a counterargument for each of the following "usual answers" as to why American manufacturing is in decline:
   a. Growth of government regulation, taxes, etc.
   b. Deterioration in the American work ethic combined with adversary relationship between labor and management.
   c. Interruptions in supply and price increases in energy since first OPEC oil shock.
   d. Massive influx of new people into workforce—teenagers, women, and minority groups—who had to be conditioned and trained.
   e. Advent of unusually high capital costs caused by high inflation.
   If the real answer is none of the above, what else is left?
6. Name two post–World War II trends in management that have contributed to the decline of American manufacturing.
7. Why was it unimportant for a manager to be terribly concerned with production details in the 1950s and early 1960s? How did this affect the nature of American business schools during this period and their impact on management practices today?
8. Give some pros and cons of the portfolio management approach to managing a complex manufacturing enterprise.
9. What caused the need for the fast-track manager in the 1950s and 1960s? What potential impacts on the perspective of management might this practice have?
10. Compare a professional manager (i.e., a manager who is allegedly capable of managing any business) to a manager of a purely financial portfolio. List some strengths and weaknesses that such a person might bring to the manufacturing environment.
11. What attitudes does a modern professional manager in America share with the early settlers of this country? What negative consequences might this have?
12. Even in circumstances where it can be documented that innovative designs have had markedly better long-term performance, why do many managers pursue *imitative* designs?
13. It has been widely claimed that many of the troubles of American manufacturing can be traced to an overreliance on short-term financial measures. Name some policies, at both the government and firm levels, that might be used to discourage this type of mind-set.

14. What essential skill does a manufacturing manager need to be able to appreciate the big picture and still pay attention to important details without becoming completely overwhelmed?
15. In very rough terms, one could attribute the success of American manufacturing to effective competition on the cost dimension (i.e., via economies of scale due to mass production), the success of German manufacturing to effective competition on the quality dimension (i.e., via a reputation for superior product design and conformance with performance specifications), and the success of Japanese manufacturing to effective competition on the time dimension (i.e., via short manufacturing cycle times and rapid introduction of new products). Of course, each newly ascendant manufacturing power had to compete on the dimensions of its predecessors as well, so Germany had to be cost-competitive and Japan used cost and quality in addition to time. Thinking in terms of this simple model, that represents global competition as a succession of new competitive dimensions, give some suggestions for what might be the next important dimension of competition.

CHAPTER

2 INVENTORY CONTROL: FROM EOQ TO ROP

When your pills get down to four
Order more.

Anonymous, from Hadley and Whitin (1963)

2.1 Introduction

Scientific management (SM) made the modern discipline of operations management (OM) possible. Not only did SM establish management as a discipline worthy of study, but also it placed a premium on quantitative precision that made mathematics a management tool for the first time. Taylor's primitive work formulas were the precursors to a host of mathematical models designed to assist decision making at all levels of plant design and control. These models became standard subjects in business and engineering curricula, and entire academic research disciplines sprang up around various OM problem areas, including inventory control, scheduling, capacity planning, forecasting, quality control, and equipment maintenance. The models, and the SM focus that motivated them, are now part of the standard language of business.

Of the operations management subdisciplines that spawned mathematical models, none was more central to factory management, nor more typical of the American approach to OM, than that of inventory control. In this chapter, we trace the history of the mathematical modeling approach to inventory control in America. Our reasons for doing this are as follows:

1. The inventory models we discuss are among the oldest results of the OM field and are still widely used and cited. As such, they are essential components of the language of manufacturing management.
2. Inventory plays a key role in the logistical behavior of virtually all manufacturing systems. The concepts introduced in these historical models will come back in our factory physics development in Part II and our discussion of inventory management in Chapter 17.
3. These classical inventory results are central to more modern techniques of manufacturing management, such as material requirements planning (MRP), just-in-time (JIT), and time-based competition (TBC), and are therefore important as a foundation for the remainder of Part I.

We begin with the oldest, and simplest, model—the economic order quantity (EOQ), and we work our way up to the more sophisticated reorder point (ROP) models. For each model we give a motivating example, a presentation of its development, and a discussion of its underlying insight.

2.2 The Economic Order Quantity Model

One of the earliest applications of mathematics to factory management was the work of Ford W. Harris (1913) on the problem of setting manufacturing lot sizes. Although the original paper was evidently incorrectly cited for many years (see Erlenkotter 1989, 1990), Harris's EOQ model has been widely studied and is a staple of virtually every introductory production and operations management textbook.

2.2.1 Motivation

Consider the situation of MedEquip, a small manufacturer of operating-room monitoring and diagnostic equipment, which produces a variety of final products by mounting electronic components in standard metal racks. The racks are purchased from a local metalworking shop, which must set up its equipment (presses, machining stations, and welding stations) each time it produces a "run" of racks. Because of the time wasted setting up the shop, the metalworking shop can produce (and sell) the racks more cheaply if MedEquip purchases them in quantities greater than one. However, because MedEquip does not want to tie up too much of its precious cash in stores of racks, it does not want to buy in excessive quantities.

This dilemma is precisely the one studied by Harris in his paper "How Many Parts to Make at Once." He puts it thus:

> Interest on capital tied up in wages, material and overhead sets a maximum limit to the quantity of parts which can be profitably manufactured at one time; "set-up" costs on the job fix the minimum. Experience has shown one manager a way to determine the economical size of lots. (Harris 1913)

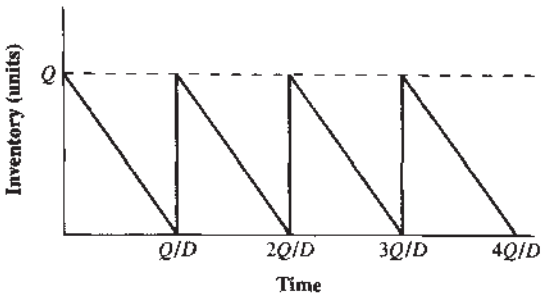
The problem Harris had in mind was that of a factory producing various products and switching between products entails a costly setup. As an example, he described a metalworking shop that produced copper connectors. Each time the shop changed from one type of connector to another, machines had to be adjusted, clerical work had to be done, and material might be wasted (e.g., copper used up as test parts in the adjustment process). Harris defined the sum of the labor and material costs to ready the shop to produce a product to be the **setup cost.** (Notice that if the connectors had been purchased, instead of manufactured, then the problem would remain similar, but setup cost would correspond to the cost of placing a purchase order.)

The basic tradeoff is the same in the MedEquip example and Harris's copper connector case. Large lots reduce setup costs by requiring less frequent changeovers. But small lots reduce inventory by bringing in product closer to the time it is used. The EOQ model was Harris's systematic approach to striking a balance between these two concerns.

2.2.2 The Model

Despite his claim in the above quote that the EOQ is based on experience, Harris was consistent with the scientific management emphasis of his day on precise mathematical

FIGURE 2.1
Inventory versus time in the EOQ model



approaches to factory management. To derive a lot size formula, he made the following assumptions about the manufacturing system:[1]

1. *Production is instantaneous.* There is no capacity constraint, and the entire lot is produced simultaneously.
2. *Delivery is immediate.* There is no time lag between production and availability to satisfy demand.
3. *Demand is deterministic.* There is no uncertainty about the quantity or timing of demand.
4. *Demand is constant over time.* In fact, it can be represented as a straight line, so that if annual demand is 365 units, this translates to a daily demand of one unit.
5. *A production run incurs a fixed setup cost.* Regardless of the size of the lot or the status of the factory, the setup cost is the same.
6. *Products can be analyzed individually.* Either there is only a single product or there are no interactions (e.g., shared equipment) between products.

With these assumptions, we can use Harris's notation, with slight modifications for ease of presentation, to develop the EOQ model for computing optimal production lot sizes. The notation we will require is as follows:

D = demand rate (in units per year)
c = unit production cost, not counting setup or inventory costs (in dollars per unit)
A = fixed setup (ordering) cost to produce (purchase) a lot (in dollars)
h = holding cost (in dollars per unit per year); if the holding cost consists entirely of interest on money tied up in inventory, then $h = ic$, where i is the annual interest rate
Q = lot size (in units); this is the decision variable

For modeling purposes, Harris represented both time and product as continuous quantities. Since he assumed constant, deterministic demand, ordering Q units each time the inventory reaches zero results in an average inventory level of $Q/2$ (see Figure 2.1). The holding cost associated with this inventory is therefore $hQ/2$ per year. The setup cost is A per order, or AD/Q per year, since we must place D/Q orders per year to satisfy demand. The production cost is c per unit, or cD per year. Thus, the total (inventory,

[1]The reader should keep in mind that *all* models are based on simplifying assumptions of some sort. The real world is too complex to analyze directly. Good modeling assumptions are those that facilitate analysis while capturing the essence of the real problem. We will be explicit about the underlying assumptions of the models we discuss in order to allow the reader to personally gauge their reasonableness.

setup, and production) cost per year can be expressed as

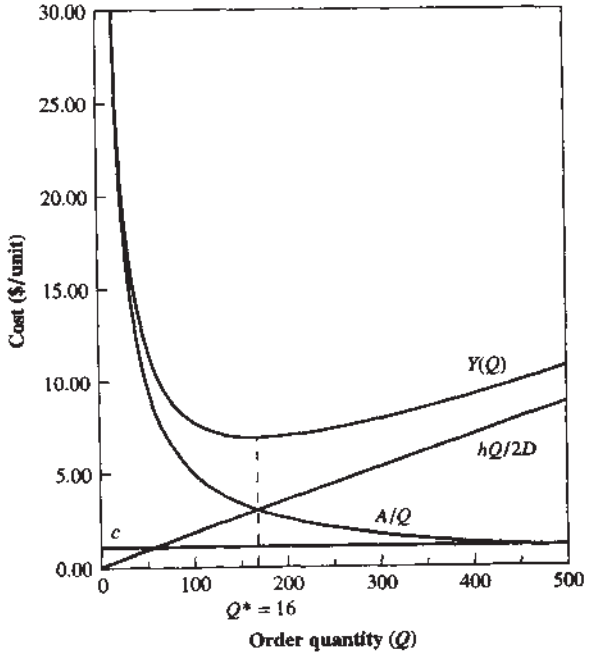
$$Y(Q) = \frac{hQ}{2} + \frac{AD}{Q} + cD \tag{2.1}$$

Example:
To illustrate the nature of $Y(Q)$, let us return to the MedEquip example. Suppose that its demand for metal racks is fairly steady and predictable at $D = 1{,}000$ units per year. The unit cost of the racks is $c = \$250$, but the metalworking shop also charges a fixed cost of $A = \$500$ per order, to cover the cost of shutting down the shop to set up for a MedEquip run. MedEquip estimates its opportunity cost or hurdle rate for money at 10 percent per year. It also estimates that the floorspace required to store a rack costs roughly \$10 per year in annualized costs. Hence, the annual holding cost per rack is $h = (0.1)(250) + 10 = \$35$. Substituting these values into expression (2.1) yields the plots in Figure 2.2.

We can make the following observations about the cost function $Y(Q)$ from Figure 2.2:

1. The holding cost term hQ/D increases linearly in the lot size Q and eventually becomes the dominant component of total annual cost for large Q.
2. The setup cost term AD/Q diminishes quickly in Q, indicating that while increasing lot size initially generates substantial savings in setup cost, the returns from increased lot sizes decrease rapidly.
3. The unit-cost term cD does not affect the relative cost for different lot sizes, since it does not include a Q term.
4. The total annual cost $Y(Q)$ is minimized by some lot size Q. Interestingly, this minimum turns out to occur precisely at the value of Q for which the holding cost and setup cost are exactly balanced (i.e., the hQ/D and AD/Q cost curves cross).

FIGURE 2.2
Costs in the EOQ model



Harris wrote that finding the value of Q that minimizes $Y(Q)$ "involves higher mathematics" and simply gives the solution without further derivation. The mathematics he is referring to (calculus) does not seem quite as high today, so we will fill in some of the details he omitted in the following technical note. Those not interested in such details can skip this and subsequent technical notes without loss of continuity.

Technical Note

The standard approach for finding the minimum of an unconstrained function, such as $Y(Q)$, is to take its derivative with respect to Q, set it equal to zero, and solve the resulting equation for Q^*. This will find a point where the slope is zero (i.e., the function is flat). If the function is convex (as we will verify below), then the zero-slope point will be unique and will correspond to the minimum of $Y(Q)$.

Taking the derivative of $Y(Q)$ and setting the result equal to zero yields

$$\frac{dY(Q)}{dQ} = \frac{h}{2} - \frac{AD}{Q^2} = 0 \tag{2.2}$$

This equation represents the *first-order condition* for Q to be a minimum. The *second-order condition* makes sure that this zero-slope point corresponds to a minimum (i.e., as opposed to a maximum or a saddle point) by checking the second derivative of $Y(Q)$:

$$\frac{d^2Y(Q)}{dQ^2} = 2\frac{AD}{Q^3} \tag{2.3}$$

Since this second derivative is positive for any positive Q (that is, $Y(Q)$ is convex), it follows that solving (2.2) for Q^* (as we do in (2.4) below) does indeed minimize $Y(Q)$.

The lot size that minimizes $Y(Q)$ in cost function (2.1) is

$$Q^* = \sqrt{\frac{2AD}{h}} \tag{2.4}$$

This square root formula is the well-known **economic order quantity (EOQ),** also referred to as the **economic lot size.** Applying this formula to the example in Figure 2.2, we get

$$Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2(500)(1{,}000)}{35}} = 169$$

The intuition behind this result is that the large fixed cost ($500) associated with placing an order makes it attractive for MedEquip to order racks in fairly large batches (169).

2.2.3 The Key Insight of EOQ

The obvious implication of the above result is that the optimal order quantity increases with the square root of the setup cost or the demand rate and decreases with the square root of the holding cost. However, a more fundamental insight from Harris's work is the one he observed in his abstract, namely, the realization that

> There is a tradeoff between lot size and inventory.

Increasing the lot size increases the average amount of inventory on hand, but reduces the frequency of ordering. By using a setup cost to penalize frequent replenishments, Harris articulated this tradeoff in clear economic terms.

The basic insight on the previous page is incontrovertible. However, the specific mathematical result (i.e., the EOQ square root formula) depends on the modeling assumptions, some of which we could certainly question (e.g., how realistic is instantaneous production?). Moreover, the usefulness of the EOQ formula for computational purposes depends on the realism of the input data. Although Harris claimed that "The set-up cost proper is generally understood" and "may, in a large factory, exceed *one dollar* per order," estimating setup costs may actually be a difficult task. As we will discuss in detail later in Parts II and III, setups in a manufacturing system have a variety of other impacts (e.g., on capacity, variability, and quality) and are therefore not easily reduced to a single invariant cost. In purchasing systems, however, where some of these other effects are not an issue and the setup cost can be cleanly interpreted as the cost of placing a purchase order, the EOQ model can be very useful.

It is worth noting that we can use the insight that there is a tradeoff between lot size and inventory without even resorting to Harris's square root formula. Since the average number of lots per year F is

$$F = \frac{D}{Q} \tag{2.5}$$

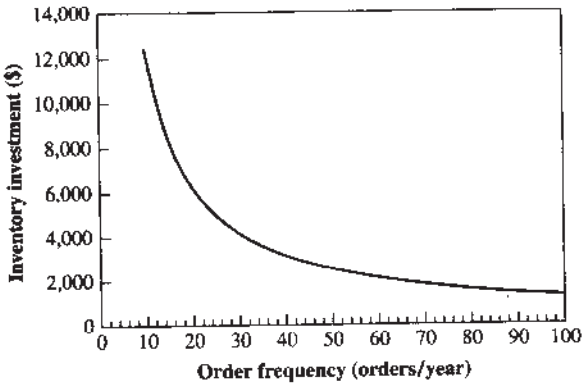
and the total inventory investment is

$$I = \frac{cQ}{2} = \frac{cD}{2F} \tag{2.6}$$

we can simply plot inventory investment I as a function of replenishment frequency F in lots per year. We do this for the MedEquip example with $D = 1{,}000$ and $c = \$250$ in Figure 2.3. Notice that this graph shows us that the inventory is cut in half (from \$12,500 to \$6,250) when we produce or order 20 times per year rather than 10 times per year (i.e., change the lot size from 100 to 50). However, if we replenish 30 times per year instead of 20 times per year (i.e., decrease the lot size from 50 to 33), inventory only falls from \$6,250 to \$4,125, a 34 percent decrease.

This analysis shows that there are decreasing returns to additional replenishments. If we can attach a value to these production runs or purchase orders (i.e., the setup cost A), then we can compute the optimal lot size using the EOQ formula as we did in Figure 2.2. However, if this cost is unknown, as it may well be, then the curve in Figure 2.3 at least gives us an idea of the impact on total inventory of an additional annual replenishment. Armed with this tradeoff information, a manager can select a reasonable number of changeovers or purchase orders per year and thereby specify a lot size.

FIGURE 2.3
Inventory investment versus lots per year



2.2.4 Sensitivity

A second insight that follows from the EOQ model is that

Holding and setup costs are fairly insensitive to lot size.

We can see this in Figure 2.2, where the total cost only varies between seven and eight for values of Q between 96 and 306. This implies that if, for any reason, we use a lot size that is slightly different than Q^*, the increase in the holding plus setup costs will not be large. This feature was qualitatively observed by Harris in his original paper. The earliest quantitative treatment of it of which we are aware is by Brown (1967, 16).

To examine the sensitivity of the cost to lot size, we begin by substituting Q^* for Q into expression (2.1) for Y (but omitting the c term, since this is not affected by lot size), and we find that the minimum holding plus setup cost per unit is given by

$$\begin{aligned} Y^* = Y(Q^*) &= \frac{hQ^*}{2} + \frac{AD}{Q^*} \\ &= \frac{h\sqrt{2AD/h}}{2} + \frac{AD}{\sqrt{2AD/h}} \\ &= \sqrt{2ADh} \end{aligned} \tag{2.7}$$

Now, suppose that instead of using Q^*, we use some other arbitrary lot size Q', which might be larger or smaller than Q^*. From expression (2.1) for $Y(Q)$, we see that the annual holding plus setup cost under Q' can be written

$$Y(Q') = \frac{hQ'}{2} + \frac{AD}{Q'}$$

Hence, the ratio of the annual cost using lot size Q' to the optimal annual cost (using Q^*) is given by

$$\begin{aligned} \frac{Y(Q')}{Y^*} &= \frac{hQ'/2 + AD/Q'}{\sqrt{2ADh}} \\ &= \frac{Q'}{2}\sqrt{\frac{h^2}{2ADh}} + \frac{1}{Q'}\sqrt{\frac{A^2D^2}{2ADh}} \\ &= \frac{Q'}{2}\sqrt{\frac{h}{2AD}} + \frac{1}{2Q'}\sqrt{\frac{2AD}{h}} \\ &= \frac{Q'}{2Q^*} + \frac{Q^*}{2Q'} \\ &= \frac{1}{2}\left(\frac{Q'}{Q^*} + \frac{Q^*}{Q'}\right) \end{aligned} \tag{2.8}$$

To appreciate (2.8), suppose that $Q' = 2Q^*$, which implies that we use a lot size twice as large as optimal. Then the ratio of the resulting holding plus setup cost to the optimum is $\frac{1}{2}(2 + \frac{1}{2}) = 1.25$. That is, a 100 percent error in lot size results in a 25 percent error in cost. Notice that if $Q' = Q^*/2$, we also get an error of 25 percent in the cost function.

We can get further sensitivity insight from the EOQ model by noting that because demand is deterministic, the order interval is completely determined by the order quantity. We can express the time between orders T as

$$T = \frac{Q}{D} \tag{2.9}$$

Hence, dividing (2.4) by D, we get the following expression for the optimal order interval

$$T^* = \sqrt{\frac{2A}{hD}} \tag{2.10}$$

and by substituting (2.9) into (2.8), we get the following expression for the ratio of the cost resulting from an arbitrary order interval T' and the optimum cost:

$$\frac{\text{Annual cost under } T'}{\text{Annual cost under } T^*} = \frac{1}{2}\left(\frac{T'}{T^*} + \frac{T^*}{T'}\right) \tag{2.11}$$

Expression (2.11) is useful in multiproduct settings, where it is desirable to order such that different products are frequently replenished at the same time (e.g., to facilitate sharing of delivery trucks). A method for facilitating this that has been widely proposed in the operations research literature is to order items at intervals given by *powers of* 2. That is, make the order interval one week, two weeks, four weeks, eight weeks, etc.[2] The result is that items ordered at 2^n week intervals will be placed at the same time as orders for items with 2^k intervals for all k smaller than n (see Figure 2.4). This will facilitate sharing of trucks, consolidation of ordering effort, simplification of shipping schedules, etc.

Moreover, the sensitivity results we derived above for the EOQ model imply that the error introduced by restricting order intervals to powers of 2 will not be excessive. To see this, suppose that the optimal order interval for an item T^* lies between 2^m and 2^{m+1} for some m (see Figure 2.5). Then T^* lies either in the interval $[2^m, 2^m\sqrt{2}]$ or in the interval $[2^m\sqrt{2}, 2^{m+1}]$. All points in $[2^m, 2^m\sqrt{2}]$ are no more than $\sqrt{2}$ times as large as 2^m. Likewise, all points in the interval $[2^m\sqrt{2}, 2^{m+1}]$ are no less than 2^{m+1} divided by $\sqrt{2}$. For instance, in Figure 2.5, 2^m is within a multiplicative factor of $\sqrt{2}$ of T_1^*, and 2^{m+1} is within a multiplicative factor of $1/\sqrt{2}$ of T_2^*. Hence, the power-of-2 order interval T' must lie in the interval $[T^*/\sqrt{2}, \sqrt{2}T^*]$ around the optimal order interval T^*. Thus, the maximum error in cost will occur when $T' = \sqrt{2}T^*$, or $T' = T^*/\sqrt{2}$. From (2.11), the error from using $T' = \sqrt{2}T^*$ is

$$\frac{1}{2}\left(\sqrt{2} + \frac{1}{\sqrt{2}}\right) = 1.06$$

and is the same when $T' = T^*/\sqrt{2}$. Hence, the error in the holding plus setup cost resulting from using the optimal power-of-2 order interval instead of the optimal order interval is guaranteed to be no more than six percent. Jackson, Maxwell, and Muckstadt (1985); Roundy (1985, 1986); and Federgruen and Zheng (1992) give algorithms for

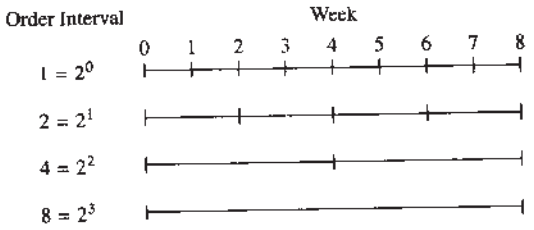


FIGURE 2.4
Powers-of-2 order intervals

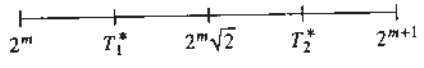


FIGURE 2.5
The "root-2" interval

[2] To be complete, we must also consider negative powers of 2 or one-half week, one-fourth week, one-eighth week, etc. However, if we use a sufficiently small unit of time as our baseline (e.g., days instead of weeks), this will not be necessary in practice.

computing the optimal power-of-2 policy and extend the above results to more general multipart settings.

As a concrete illustration of these concepts, consider once again the MedEquip problem. We computed the optimal order quantity for racks to be $Q^* = 169$. Hence, the optimal order interval is $T^* = Q^*/D = 169/1,000 = 0.169$ year, or $0.169 \times 52 = 8.78$ weeks. Suppose further that MedEquip orders a variety of other parts from the same supplier. The unit price of \$250 for racks is a delivered price, assuming an average shipping cost. However, if MedEquip combines orders for different parts, total shipping costs can be reduced. If the minimum order interval for any of the products under consideration is one week, then the order interval for racks can be rounded to the nearest power of 2 of $T = 8$ weeks or $8/52 = 0.154$ year. This implies an order quantity of $Q = TD = 0.154(1,000) = 154$. The holding plus order cost of this modified order quantity is

$$Y(Q) = \frac{hQ}{2} + \frac{AD}{Q} = \frac{35(154)}{2} + \frac{500(1,000)}{154} = \$5,942$$

The optimal annual cost (i.e., from using $Q^* = 169$) is given by

$$Y^* = \sqrt{2ADh} = \sqrt{2(500)(1,000)(35)} = \$5,916$$

So the modified order quantity results in less than a one percent increase in cost. The other parts ordered from the same supplier will have similar increases in holding plus order cost—but none of more than six percent. If these increases are offset by the reduced transportation cost, then the power-of-2 order schedule is worthwhile.

2.2.5 EOQ Extensions

Harris's original formula has been extended in a variety of ways over the years. One of the earliest extensions (Taft 1918) was to the case in which replenishment is not instantaneous; instead, there is a finite, but constant and deterministic, production rate. This model is sometimes called the **economic production lot (EPL)** model and results in a similar square root formula to the regular EOQ. Other variations of the basic EOQ include backorders (i.e., orders that are not filled immediately, but have to wait until stock is available), major and minor setups, and quantity discounts among others (see Johnson and Montgomery 1974; McClain and Thomas 1985; Plossl 1985; Silver, Pyke, and Peterson 1998).

2.3 Dynamic Lot Sizing

As we noted above, the EOQ formulation is predicated on a number of assumptions, specifically,

1. Instantaneous production.
2. Immediate delivery.
3. Deterministic demand.
4. Constant demand.
5. Known constant setup costs.
6. Single product or separable products.

We have already noted that Taft relaxed the assumption of instantaneous production. Introducing delivery delays is straightforward if delivery times are known and fixed (i.e.,

compute order quantities according to the EOQ formula and place the orders at times equal to desired delivery minus delivery time). If delivery times are uncertain, then a different approach is required. However, a more prevalent and important source of randomness than delivery times is in demand. The topic of relaxing the assumption of deterministic demand will be taken up in the next section on statistical inventory models. We have already discussed an approach for getting around the specification of a constant setup cost (i.e., by examining the inventory versus order frequency tradeoff). In Chapter 17 we will discuss approaches for handling multiproduct cases where parts cannot be analyzed separately. This leaves the assumption of constant demand.

2.3.1 Motivation

Consider the situation of RoadHog, Inc., which is a small manufacturer of motorcycle accessories. It makes a muffler with fins (that does little to suppress engine noise) on a line that is also used to make a variety of other products. Because it is costly to set up the line to produce the gauges, RoadHog has an incentive to produce them in batches. However, while customer demand is known over a 10-week planning horizon (because it is entered into a master production schedule and "frozen"), it is not necessarily constant from week to week. Since this violates a key assumption of the EOQ model, we need a fundamentally different model to balance the setup and holding costs.

The main historical approach to relaxing the constant-demand assumption is the Wagner–Whitin model (Wagner and Whitin 1958). This model considers the problem of determining production lot sizes when demand is deterministic but time-varying and all the other assumptions for the EOQ model are valid. The importance of this **dynamic lot-sizing** approach is that it has had a substantial impact on the literature in production control, and later influenced the development of materials requirements planning (MRP), as we will discuss in Chapter 3. For these reasons, we now present an overview of the Wagner–Whitin dynamic lot-sizing procedure.

2.3.2 Problem Formulation

When demand varies over time, a continuous time model, like the EOQ model, is awkward to specify. So, instead, we will clump demand into discrete periods, which could correspond to days, weeks, or months, depending on the system. A daily production schedule might make sense for a high-volume system with rapidly changing demand, while a monthly schedule may be adequate for a low-volume system with demand that changes more slowly.

To specify the problem and model, we will make use of the following notation, which represents the dynamic counterpart to the static notation used for the EOQ model:

t = a time period (e.g., day, week, month); we will consider $t = 1, \ldots, T$, where T represents the **planning horizon**

D_t = demand in period t (in units)

c_t = unit production cost (in dollars per unit), not counting setup or inventory costs in period t

A_t = setup (order) cost to produce (purchase) a lot in period t (in dollars)

h_t = holding cost to carry a unit of inventory from period t to period $t + 1$ (in dollars per unit per period); for example, if holding cost consists entirely of interest on money tied up in inventory, where i is the annual interest rate and periods correspond to weeks, then $h_t = ic_t/52$

I_t = inventory (in units) left over at the end of period t

Q_t = lot size (in units) in period t; there are T such decision variables, one for each period

Example:

With this notation, we can specify the RoadHog problem precisely. We suppose that the data for the next 10 weeks are as given in Table 2.1. Note that for simplicity we have assumed that the setup costs A_t, the production cost c_t, and the holding cost h_t are all constant over time, although this is not necessary for the Wagner–Whitin model. The basic problem is to satisfy all demands at minimal cost (i.e., production plus setup plus holding cost). The only controls are the production quantities Q_t. However, since all demands must be filled, only the timing of production is open to choice, not the total production quantity. Hence if the unit production cost is constant (that is, c_t does not vary with t), then production cost will be the same regardless of timing and therefore can be omitted altogether.

The simplest lot-sizing procedure one might think of is to produce exactly what is required in each period. This is called the **lot-for-lot rule,** and as we will see in Chapter 3, it can make sense in some situations. However, in this problem, the lot-for-lot rule implies that we will have to produce, and hence pay a setup cost, in every period. Table 2.2 shows the production schedule and resulting costs for this policy. Since we never carry inventory, the total cost is just that of the 10 setups, or $1,000.

Another plausible policy is to produce a fixed amount each time we perform a setup. This is known as the **fixed order quantity** lot-sizing rule. Since there are 300 units to produce, one possible fixed order quantity would be 100 units. This would require us to produce exactly three times, resulting in three setups, and would not leave any product left over at the end of period 10. Table 2.3 illustrates the production schedule and resulting costs for this policy. Notice that under this policy we frequently produce more

TABLE 2.1 Data for the RoadHog Dynamic Lot-Sizing Example

t	1	2	3	4	5	6	7	8	9	10
D_t	20	50	10	50	50	10	20	40	20	30
c_t	10	10	10	10	10	10	10	10	10	10
A_t	100	100	100	100	100	100	100	100	100	100
h_t	1	1	1	1	1	1	1	1	1	1

TABLE 2.2 Lot-for-Lot Solution to the RoadHog Example

t	1	2	3	4	5	6	7	8	9	10	Total
D_t	20	50	10	50	50	10	20	40	20	30	300
Q_t	20	50	10	50	50	10	20	40	20	30	300
I_t	0	0	0	0	0	0	0	0	0	0	0
Setup cost	100	100	100	100	100	100	100	100	100	100	1,000
Holding cost	0	0	0	0	0	0	0	0	0	0	0
Total cost	100	100	100	100	100	100	100	100	100	100	1,000

TABLE 2.3 Fixed Order Quantity Solution to the RoadHog Example

t	1	2	3	4	5	6	7	8	9	10	Total
D_t	20	50	10	50	50	10	20	40	20	30	300
Q_t	100	0	0	100	0	0	100	0	0	0	300
I_t	80	30	20	70	20	10	90	50	30	0	0
Setup cost	100	0	0	100	0	0	100	0	0	0	300
Holding cost	80	30	20	70	20	10	90	50	30	0	400
Total cost	180	30	20	170	20	10	190	50	30	0	700

than is required in a given period and therefore pay inventory carrying costs. However, the total inventory carrying cost is only \$400, which, when added to the \$300 setup cost, results in a total cost of \$700. This is lower than the cost from the lot-for-lot policy. But can we do better? We will find out below by developing a procedure that is guaranteed to find the minimum setup plus inventory cost.

2.3.3 The Wagner–Whitin Procedure

A key observation for solving the dynamic lot-sizing problem is that if we produce items in period t (and incur a setup cost) for use to satisfy demand in period $t + 1$, then it cannot possibly be economical to produce in period $t + 1$ (and incur another setup cost). Either it is cheaper to produce *all* of period $t + 1$'s demand in period t, or all of it in $t + 1$; it is never cheaper to produce some in each. (Notice that we violated this property in the fixed order quantity solution given in Table 2.3.) In more general terms, we can state this result as follows:

> **Wagner–Whitin Property**
> Under an optimal lot-sizing policy either the inventory carried to period $t + 1$ from a previous period will be zero or the production quantity in period $t + 1$ will be zero.

This result greatly facilitates computation of optimal production quantities, as we will see.[3]

The Wagner–Whitin property implies that either $Q_t = 0$ or Q_t will be one of the following: D_t, $D_t + D_{t+1}$, $D_t + d_{t+1} + D_{t+2}$, ..., $D_t + D_{t+1} + \cdots + D_T$. That is, we will produce either nothing or exactly enough to satisfy demand in the current period plus some integer number of future periods. We could compute the minimum-cost production schedule by enumerating all possible combinations of periods in which production occurs. However, since we can either produce or not produce in each period, the number of such combinations is 2^{N-1}, which can be quite large if many periods are considered. To be more efficient, Wagner and Whitin suggested an algorithm that is well suited to computer implementation. We will describe this algorithm by means of the RoadHog example.

[3]Some pundits have noted that, while useful mathematically, in real systems the Wagner–Whitin property is either obvious or ridiculous. In essence, it states we should not produce until inventory falls to zero. If one really accepts all the modeling assumptions, particularly those of known, deterministic demand and well-defined fixed setup costs, then the property is nearly tautological. However, in real systems where uncertainty complicates things, one almost always starts production before inventory is exhausted (i.e., to provide protection against stockouts caused by random disruptions).

The Wagner–Whitin algorithm proceeds forward in time, starting with period 1 and finishing with period N. By the Wagner–Whitin property, we know that we will only produce in a period if the inventory carried to that period is zero. If this is the case, then our decision can be thought of in terms of how many periods of demand to produce. For instance, in a six-period problem, there are six possibilities for the amount we can produce in period 1, namely, D_1, $D_1 + D_2$, $D_1 + D_2 + D_3$, ..., $D_1 + D_2 + D_3 + D_4 + D_5 + D_6$. If we choose to produce $D_1 + D_2$, then inventory will run out in period 3 and so we will have to produce again in that period. In period 3, then, we will have the option of producing for period 3 only; periods 3 and 4; periods 3, 4, and 5; or periods 3, 4, 5, and 6.

Step 1

We begin the algorithm by looking at the one-period problem. That is, we act as though the world ends after one period. The optimal policy for this problem is trivial; we produce 20 units to satisfy demand in period 1, and we are done. Since there is no inventory carried from one period to another, and we are neglecting production cost, the minimum cost in the one-period problem, which we denote by Z_1^*, is

$$Z_1^* = A_1 = 100$$

As we will see as the algorithm unfolds, it is also useful to keep track of the last period in which production occurs in each problem we consider. Here, obviously, production takes place only in period 1, so the last period of production in the one-period problem, which we denote by j_1^*, is

$$j_1^* = 1$$

Step 2

In the next step of the algorithm we increase the time horizon and consider the two-period problem. Now we have two options for the production in period 2; we can cover demand in period 2 with production either in period 1 or in period 2. If we produce in period 1, we will incur a holding cost associated with carrying inventory from period 1 to period 2. If we produce in period 2, we will incur an extra setup cost in period 2. Notice also that if we produce in period 2, then the cost of satisfying previous demand (i.e., demand in period 1) is given by Z_1^*. Since we are trying to minimize cost, the optimal policy is to choose the period with the lower total cost, that is,

$$\begin{aligned} Z_2^* &= \min \begin{Bmatrix} A_1 + h_1 D_2 & \text{produce in period 1} \\ Z_1^* + A_2 & \text{produce in period 2} \end{Bmatrix} \\ &= \min \begin{Bmatrix} 100 + 1(50) = 150 \\ 100 + 100 = 200 \end{Bmatrix} \\ &= 150 \end{aligned}$$

The optimal decision is to produce for both periods 1 and 2 in period 1. Therefore, the last period in which production takes place in an optimal two-period policy is

$$j_2^* = 1$$

Step 3

Now, we proceed to the three-period problem. Ordinarily four possible production schedules would need to be considered: produce in period 1 only, produce in periods 1 and 2, produce in periods 1 and 3, or produce in periods 1, 2, and 3. However, we

need to consider only three of these: one only, one and two, and one and three. This is because we only need to consider when we are going to produce the demand for period 3. We have already solved the two- and one-period problems. Note that the gain in speed grows sharply as the number of periods grows. For instance, for the 10-period problem we reduce the number of schedules we must check from 512 to 10. We will reduce these even more with the "planning horizon" result discussed later.[4]

If we decide to produce in period 3, then we know from our solution to the two-period problem that it will be optimal to produce for periods 1 and 2 in period 1.

$$
\begin{aligned}
Z_3^* &= \min \begin{Bmatrix} A_1 + h_1 D_2 + (h_1 + h_2) D_3 & \text{produce in period 1} \\ Z_1^* + A_2 + h_2 D_3 & \text{produce in period 2} \\ Z_2^* + A_3 & \text{produce in period 3} \end{Bmatrix} \\
&= \min \begin{Bmatrix} 100 + 1(50) + (1+1)(10) & = 170 \\ 100 + 100 + 1(10) & = 210 \\ 150 + 100 & = 250 \end{Bmatrix} \\
&= 170
\end{aligned}
$$

Again, it is optimal to produce everything in period 1, so

$$j_3^* = 1$$

Step 4

The situation changes when we move to the next step, the four-period problem. Now there are four options for the timing of production for period 4, namely, periods 1 to 4:

$$
\begin{aligned}
Z_4^* &= \min \begin{Bmatrix} A_1 + h_1 D_2 + (h_1 + h_2) D_3 + (h_1 + h_2 + h_3) D_4 & \text{produce in period 1} \\ Z_1^* + A_2 + h_2 D_3 + (h_2 + h_3) D_4 & \text{produce in period 2} \\ Z_2^* + A_3 + h_3 D_4 & \text{produce in period 3} \\ Z_3^* + A_4 & \text{produce in period 4} \end{Bmatrix} \\
&= \min \begin{Bmatrix} 100 + 1(50) + (1+1)(10) + (1+1+1)(50) & = 320 \\ 100 + 100 + 1(10) + (1+1)(50) & = 310 \\ 150 + 100 + 1(50) & = 300 \\ 170 + 100 & = 270 \end{Bmatrix} \\
&= 270
\end{aligned}
$$

This time, it turned out to be optimal not to produce in period 1, but rather to meet period 4's demand with production in period 4. Hence,

$$j_4^* = 4$$

If our planning horizon were only 4 periods, we would be done at this point. We would translate our results to a lot-sizing policy by reading the j_t^* values backward in time. The fact that $j_4^* = 4$ means that we would produce $D_4 = 50$ units in period 4. This would leave us with a three-period problem. Since $j_3^* = 1$, it would be optimal to produce $D_1 + D_2 + D_3 = 80$ units in period 1.

[4]This technique of solving successively longer horizon problems and using the solutions from previous steps to reduce the amount of computation in each step is known as *dynamic programming*. Dynamic programming is a form of *implicit enumeration*, which allows us to consider all possible solutions without explicitly computing the cost of each one.

Step 5 and Beyond

But our planning horizon is not 4 periods; it is 10 periods. Hence, we must continue the algorithm. However, before doing this, we will make an observation that will further reduce the computations we must make. Notice that up to this point, each step in the algorithm has increased the number of periods we must consider for the last period's production. So, by step 4, we had to consider producing for period 4 in all periods 1 through 4. It turns out that this is not always necessary.

Notice that in the four-period problem it is optimal to produce in period 4 for period 4. What this means is that the cost of setting up in period 4 is less than the cost setting up in period 1, 2, or 3 and carrying the inventory to period 4. If it weren't, then we would have chosen to produce in one of these periods. Now consider what this means for period 5. For instance, could it be cheaper to produce for period 5 in period 3 than in period 4? Production in periods 3 and 4 must be held in inventory from period 4 to period 5 and therefore incur the same carrying cost for that period. Therefore the only question is whether it is cheaper to set up in period 3 and carry inventory from period 3 to period 4 than it is to set up in period 4. But we already know the answer to this question. The fact that $j_4^* = 4$ tells us that it is cheaper to set up in period 4. Therefore, it is unnecessary to consider producing in periods 1, 2, and 3 for the demand in period 5. We need to consider only periods 4 and 5.

This reasoning can more generally be stated as follows:

Planning Horizon Property

If $j_t^* = \bar{t}$, then the last period in which production occurs in an optimal $t + 1$ period policy must be in the set $\bar{t}, \bar{t} + 1, \ldots, t + 1$.

Using this property, the calculation required to compute the minimum cost for the five-period problem is

$$
\begin{aligned}
Z_5^* &= \min \begin{Bmatrix} Z_3^* + A_4 + h_4 D_5 & \text{produce in period 4} \\ Z_4^* + A_5 & \text{produce in period 5} \end{Bmatrix} \\
&= \min \begin{Bmatrix} 170 + 100 + 1(50) & = 320 \\ 270 + 100 & = 370 \end{Bmatrix} \\
&= 320
\end{aligned}
$$

Given that we are going to set up in period 4 anyway, it is cheaper to carry inventory from period 4 to period 5 than to set up again in period 5. Hence,

$$j_5^* = 4$$

We solve the remaining five periods, using the same approach, and summarize the results of these calculations in Table 2.4. Notice the blank spaces in the upper right-hand corner of this table. These are the result of our use of the planning horizon property. Without this property, we would have had to calculate values for each of these spaces.

2.3.4 Interpreting the Solution

The minimum total setup plus inventory carrying cost is given by $Z_{10} = \$580$, which we note is indeed lower than the cost achieved by either the lot-for-lot or fixed order quantity solutions we offered earlier. The optimal lot sizes are determined from the j_t^* values. Since $j_{10}^* = 8$, it is optimal to produce for periods 8, 9, and 10 in period 8. Hence, $Q_8^* = D_8 + D_9 + D_{10} = 90$. With periods 8, 9, and 10 taken care of, we are

TABLE 2.4 Solution to Wagner–Whitin Example

Last Period with Production	Planning Horizon t									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	100	150	170	320						
2		200	210	310						
3			250	300						
4				270	320	340	400	560		
5					370	380	420	540		
6						420	440	520		
7							440	480	520	610
8								500	520	580
9									580	610
10										620
Z_t^*	100	150	170	270	320	340	400	480	520	580
j_t^*	1	1	1	4	4	4	4	7	7 or 8	8

left with a seven-period problem. Since $j_7^* = 4$, it is optimal to produce for periods 4, 5, 6, and 7 in period 4. Hence, $Q_4^* = D_4 + D_5 + D_6 + D_7 = 130$. This leaves us with a three-period problem. Since $j_3^* = 1$, we should produce for periods 1, 2, and 3 in period 1, so $Q_1^* = D_1 + D_2 + D_3 = 80$.

2.3.5 Caveats

Although the calculations underlying Table 2.4 are certainly tedious to do by hand, they are not difficult for a computer. Given this, it is rather surprising that many production and operations management textbooks have omitted the Wagner–Whitin algorithm in favor of simpler heuristics that do not always give the optimal solution. Presumably, "simpler" meant both less computationally burdensome and easier to explain. Given that the algorithm is only used where production planning is computerized, the computational-burden argument is not compelling. Furthermore, the concepts underlying the algorithm are not difficult—certainly not so difficult as to prevent practitioners from using commercial software incorporating it!

However, there are more important concerns about the entire concept of "optimal" lot sizing whether one is using the Wagner–Whitin algorithm or any of the heuristic approaches that approximate it.

1. Like the EOQ model, the Wagner–Whitin model assumes setup costs known in advance of the lot-sizing procedure. But, as we noted earlier, setup costs can be very difficult to estimate in manufacturing systems. Moreover, the true cost of a setup is influenced by capacity. For instance, shutting down to change a die is very costly in terms of lost production when operating close to capacity, but not nearly as costly when there is a great deal of excess capacity. This issue cannot be addressed by any model that assumes independent setup costs. Thus, it would appear that the Wagner–Whitin model, like EOQ, is better suited to purchasing than production systems.

2. Also like the EOQ model, the Waguer–Whitin model assumes deterministic demand and deterministic production. Uncertainties, such as order cancellations, yield loss, and delivery schedule deviations are not considered. The result is that the "optimal"

production schedule given by the Wagner–Whitin algorithm will have to be adjusted to meet real conditions (e.g., reduced to accommodate leftover inventory from order cancellations or inflated for expected yield loss). The fact that these adjustments will be made on an ad hoc basis, coupled with the speculative nature of the setup costs, could make this theoretically optimal schedule perform poorly in practice.

3. Another key assumption is that of *independent products,* that is, that production for different products does not make use of common resources. This assumption is clearly violated in many instances. This can be important if some resources are highly utilized.

4. The Wagner–Whitin property leads us to the conclusion that we should produce either nothing in a period or the demand for an integer number of future periods. This property follows from (1) the fact that a fixed setup cost is incurred each time production takes place and (2) the assumption of infinite capacity. In the real world, where setups have more subtle consequences and capacity is finite, a sensible production plan may be quite different. For instance, it may be reasonable to produce according to a level production plan (i.e., produce approximately the same amount in each period), in order to achieve a degree of pacing or rhythm in the line. Wagner–Whitin, by focusing exclusively on the tradeoff between fixed and holding costs, may actually serve to steer our intuition away from realistic concerns.

2.4 Statistical Inventory Models

All the models discussed up to this point have assumed that demand is fixed and known. Although there are cases in which this assumption may approximate reality (e.g., when the schedule is literally frozen over the horizon of interest), often it does not. If demand is random, then there are two basic approaches to take:

1. Model demand as if it were deterministic for modeling purposes and then modify the solution to account for randomness.
2. Explicitly represent randomness in the model.

Neither approach is correct or incorrect in any absolute sense. The real question is, Which is more *useful?* In general, the answer depends on the circumstances. When planning is over a sufficiently long horizon to ensure that random deviations "average out," a deterministic model may work well. Also, a deterministic model with appropriate "fudge factors" to anticipate randomness, coupled with a suitably frequent regeneration cycle to get back on track, can be effective. However, to determine these fudge factors or to help design policies for dealing with time frames in which randomness is critical, a model that explicitly incorporates randomness may be more appropriate.

Historically, the operations management literature has pursued both approaches. The most prevalent deterministic model for production scheduling is materials requirements planning (MRP), the subject of Chapter 3. The most prevalent probabilistic models are the **statistical reorder point** approaches, which we examine in this section.

Statistical modeling of production and inventory control problems is not new, dating back at least to Wilson (1934). In this classic paper, Wilson breaks the inventory control problem into two distinct parts:

1. Determining the **order quantity,** or the amount of inventory that will be purchased or produced with each replenishment.
2. Determining the **reorder point,** or the inventory level at which a replenishment (purchase or production) will be triggered.

In this section, we will address this two-part problem in three stages.

First, we will consider the situation in which we are only interested in a single replenishment, so that the only issue is to determine the appropriate order quantity in the face of uncertain demand. This has traditionally been called the **news vendor model** because it could apply to a person who purchases newspapers at the beginning of the day, sells a random amount, and then must discard any leftovers.

Second, we will consider the situation in which inventory is replenished one unit at a time as random demands occur, so that the only issue is to determine the reorder point. The target inventory level we set for the system is known as a *base stock level*, and hence the resulting model is termed the **base stock model.**

Third, we will consider the situation where inventory is monitored continuously and demands occur randomly, possibly in batches. When the inventory level reaches (or goes below) r, an order of size Q is placed. After a lead time of ℓ, during which a stockout might occur, the order is received. The problem is to determine appropriate values of Q and r. The model we use to address this problem is known as the **(Q, r) model.**

These models will make use of the concepts and notation found in the field of *probability.* If it has been awhile since the reader has reviewed these, now might be a good time to peruse Appendix 2A.

2.4.1 The News Vendor Model

Consider the situation that a manufacturer of Christmas lights faces each year. Demand is somewhat unpredictable and occurs in such a short burst just prior to Christmas that if inventory is not on the shelves, sales are lost. Therefore, the decision of how many sets of lights to produce must be made prior to the holiday season. Additionally, the cost of collecting unsold inventory and holding it until next year is too high to make year-to-year storage an attractive option. Instead, any unsold sets of lights are sold after Christmas at a steep discount.

To choose an appropriate production quantity, the important pieces of information to consider are (1) anticipated demand and (2) the costs of producing too much or too little. To develop a formal model, we make the following assumptions:

1. *Products are separable.* We can consider products one at a time since there are no interactions (e.g., shared resources).
2. *Planning is done for a single period.* We can neglect future periods since the effect of the current decision on them is negligible (e.g., because inventory cannot be carried across periods).
3. *Demand is random.* We can characterize demand with a known probability distribution.
4. *Deliveries are made in advance of demand.* All stock ordered or produced is available to meet demand.
5. *Costs of overage or underage are linear.* The charges for having too much or too little inventory is proportional to the amount of the overage or underage.

We make use of these assumptions to develop a model using the following notation:

X = demand (in units), a random variable

$G(x) = P(X \leq x)$ = cumulative distribution function of demand; for this model we will assume that G is a continuous distribution because it is analytically convenient, but the results are essentially the same if G is discrete (i.e., restricted to integer values), as we will note

$g(x) = \dfrac{d}{dx}G(x)$ = density function of demand

μ = mean demand (in units)
σ = standard deviation of demand (in units)
c_o = cost (in dollars) per unit left over after demand is realized
c_s = cost (in dollars) per unit of shortage
Q = production or order quantity (in units); this is the decision variable

Example:
Now consider the Christmas lights example with some numbers. Suppose that a set of lights costs \$1 to make and distribute and sells for \$2. Any sets not sold by Christmas will be discounted to \$0.50. In terms of the above modeling notation, this means that the unit overage cost is the amount lost per excess set, or $c_o = \$(1 - 0.50) = \$0.50$. The unit shortage cost is the lost profit from a sale, or $c_s = \$(2 - 1) = \$1$. Suppose further that demand has been forecast to be 10,000 units with a standard deviation of 1,000 units and that the normal distribution is a reasonable representation of demand.

The firm could choose to produce 10,000 sets of lights. But recall that the symmetry (i.e., bell shape) of the normal distribution implies that it is equally likely for demand to be greater or less than 10,000 units. If demand is less than 10,000 units, the firm will lose $c_o = \$0.50$ per unit of overproduction. If demand is greater than 10,000 units, the firm will lose $c_s = \$1$ per unit of underproduction. Clearly, shortages are worse than overages. This suggests that perhaps the firm should produce more than 10,000 units. But how much more? The model we develop below is aimed at answering exactly this question.

To develop a model, observe that if we produce Q units and demand is X units, then the number of units of overage is given by

$$\text{Units over} = \max\{Q - X, 0\}$$

That is, if $Q \geq X$, then the overage is simply $Q - X$; but if $Q < X$, then there is a shortage and so the overage is zero. We can calculate the expected overage as

$$\begin{aligned} E[\text{units over}] &= \int_0^\infty \max\{Q - x, 0\} g(x)\, dx \\ &= \int_0^Q (Q - x) g(x)\, dx \end{aligned} \tag{2.12}$$

Similarly, the number of units of shortage is given by

$$\text{Units short} = \max\{X - Q, 0\}$$

That is, if $X \geq Q$, then the shortage is simply $X - Q$; but if $X < Q$, then there is an overage and so the shortage is zero. We can calculate the expected shortage as

$$\begin{aligned} E[\text{units short}] &= \int_0^\infty \max\{x - Q, 0\} g(x)\, dx \\ &= \int_Q^\infty (x - Q) g(x)\, dx \end{aligned} \tag{2.13}$$

Using (2.12) and (2.13), we can express the expected cost as a function of the production quantity as

$$Y(Q) = c_o \int_0^Q (Q - x) g(x)\, dx + c_s \int_Q^\infty (x - Q) g(x)\, dx \tag{2.14}$$

We will find the value of Q that minimizes this expected cost in the following technical note.

Technical Note

As we did for the EOQ model, we will find the minimum of $Y(Q)$ by taking its derivative and setting it equal to zero. To do this, however, we need to take the derivative of integrals with limits that are functions of Q. The tool we require for this is *Leibnitz's rule*, which can be written as

$$\frac{d}{dQ}\int_{a_1(Q)}^{a_2(Q)} f(x, Q)\,dx = \int_{a_1(Q)}^{a_2(Q)} \frac{\partial}{\partial Q}[f(x, Q)]\,dx + f(a_2(Q), Q)\frac{da_2(Q)}{dQ} - f(a_1(Q), Q)\frac{da_1(Q)}{dQ}$$

Applying this to take the derivative of $Y(Q)$ and setting the result equal to zero yields

$$\begin{aligned}\frac{dY(Q)}{dQ} &= c_o\int_0^Q 1g(x)\,dx + c_s\int_Q^\infty (-1)g(x)\,dx \\ &= c_oG(Q) - c_s[1 - G(Q)] = 0 \end{aligned} \tag{2.15}$$

Solving (2.15) (which we simplify below in (2.16)) for Q^* yields the production (order) quantity that minimizes $Y(Q)$.

To minimize expected overage plus shortage cost, we should choose a production or order quantity Q^* that satisfies

$$G(Q^*) = \frac{c_s}{c_o + c_s} \tag{2.16}$$

First, note that since $G(Q^*)$ represents the probability that demand is less than or equal to Q^*, this result implies that Q^* should be chosen such that the probability of having enough stock to meet demand is $c_s/(c_o + c_s)$. Second, notice that since $G(x)$ increases in x (cumulative distribution functions are always monotonically increasing), so that anything that makes the right-hand side of (2.16) larger will result in a larger Q^*. This implies that increasing c_s will increase Q^*, while increasing c_o will decrease Q^*, as we would intuitively expect.

We can further simplify expression (2.16) if we assume that G is normal. For this case we can write

$$G(Q^*) = \Phi\left(\frac{Q^* - \mu}{\sigma}\right) = \frac{c_s}{c_o + c_s}$$

where Φ is the cumulative distribution function (cdf) of the standard normal distribution.[5] This means that

$$\frac{Q^* - \mu}{\sigma} = z$$

where z is the value in the standard normal table (see Table 1 at the end of the book) for which $\Phi(z) = c_s/(c_o + c_s)$, and hence

$$Q^* = \mu + z\sigma \tag{2.17}$$

[5] We are making use of the well-known result that if X is normally distributed with mean μ and standard deviation σ, then $(X - \mu)/\sigma$ is normally distributed with mean zero and standard deviation one (i.e., the standard normal distribution).

Expression (2.17) implies that for the normal case, Q^* is an increasing function of the mean demand μ. It is also increasing in the standard deviation of demand σ, provided that z is positive. This will be the case whenever $c_s/(c_o + c_s)$ is greater than one-half, since $\Phi(0) = 0.5$ and $\Phi(z)$ is increasing in z. However, if costs are such that $c_s/(c_o + c_s)$ is less than one-half, then the optimal order size Q^* will decrease as σ increases.

Example:
Now we return to the Christmas lights example. Because demand is normally distributed, we can compute Q^* from (2.17). To do this, we must find z by computing

$$\frac{c_s}{c_o + c_s} = \frac{1}{1 + 0.5} = 0.67$$

and by looking up in a standard normal table to find that $\Phi(0.44) = 0.67$. Hence $z = 0.67$ and

$$Q^* = \mu + z\sigma = 10,000 + (0.44)1,000 = 10,440$$

Notice that this answer can be interpreted as telling us to produce 0.44 standard deviation above mean demand. Therefore, if the standard deviation of demand had been 2,000 units, instead of 1,000, the answer would have been to produce $0.44 \times 2,000 = 880$ units above mean demand, or 10,880 units.

The news vendor problem, and its intuitive critical ratio solution given in (2.16), can be extended to a variety of applications that, unlike the Christmas lights example, have more than one period. One common situation is the problem in which

1. A firm faces periodic (e.g, monthly) demands that are independent and have the same distribution $G(x)$.
2. All orders are backordered (i.e., met eventually).
3. There is no setup cost associated with producing an order.

It can be shown that an "order up to Q" policy (i.e., after each demand, produce enough to bring the inventory level up to Q) is optimal under these conditions. Moreover, the problem of finding the optimal order-up-to level Q^* can be formulated as a news vendor model (see Nahmias 1993, 291–294). The solution Q^* therefore satisfies Equation (2.16), where c_o represents the cost to hold one unit of inventory in stock for one period and c_s represents the cost of carrying a unit of backorder (i.e., an unfilled order) for one period. Similarly, under the same conditions, except that sales are lost instead of backordered, the optimal order-up-to level is found by solving (2.16) for Q^* with c_o equal to the one-period holding cost and c_s equal to the unit profit (i.e., selling price minus production cost).

We conclude this section by summarizing the basic insights from the news vendor model:

1. In an environment of uncertain demand, the appropriate production or order quantity depends on both the distribution of demand *and* the relative costs of overproducing versus underproducing.
2. If demand is normally distributed, then increasing the variability (i.e., standard deviation) of demand will increase the production or order quantity if $c_s/(c_s + c_o) > 0.5$ and decrease it if $c_s/(c_s + c_o) < 0.5$.

2.4.2 The Base Stock Model

Consider the situation facing Superior Appliance, a store that sells a particular model of refrigerator. Because space is limited and because the manufacturer makes frequent deliveries of other appliances, Superior finds it practical to order replacement refrigerators each time one is sold. In fact, it has a system that places purchase orders automatically whenever a sale is made. But because the manufacturer is slow to fill replenishment orders, the store must carry some stock in order to meet customer demands promptly. Under these conditions, the key question concerns how much stock to carry.

To answer this question, we need a model. To develop one, we make use of a continuous-time framework (e.g., like the EOQ model) and the following modeling assumptions:

1. *Products can be analyzed individually.* There are no product interactions (e.g., shared resources).
2. *Demands occur one at a time.* There are no batch orders.
3. *Unfilled demand is backordered.* There are no lost sales.
4. *Replenishment lead times are fixed and known.* There is no randomness in delivery lead times. (We will show how to relax this assumption to consider variable lead times later in this chapter.)
5. *Replenishments are ordered one at a time.* There is no setup cost or constraint on the number of orders that can be placed per year, which would motivate batch replenishment.

We will relax the last assumption in the next section on the (Q, r) model, where ordering in bulk will become a potentially attractive option.

We also make use of the following notation:

ℓ = replenishment lead time (in days), assumed constant throughout this section

X = demand during replenishment lead time (in units), a random variable

$p(x) = P(X = x)$ = probability demand during replenishment lead time equals x (probability mass function). We are assuming demand is discrete (i.e., countable), but sometimes it is convenient to approximate demand with a continuous distribution. When we do this, we assume a density function $g(x)$ in place of the probability mass function

$G(x) = P(X \leq x) = \sum_{i=0}^{x} p(i)$ = probability demand during replenishment lead time is less than or equal to x (cumulative distribution function)

$\theta = E[X]$, mean demand (in units) during lead time ℓ

σ = standard deviation of demand (in units) during lead time ℓ

h = cost to carry one unit of inventory for one year (in dollars per unit per year)

b = cost to carry one unit of backorder for one year (in dollars per unit per year)

r = reorder point (in units), which represents inventory level that triggers a replenishment order; this is the decision variable

$R = r + 1$, base stock level (in units)

$s = r - \theta$, safety stock level (in units)

$S(R)$ = fill rate (fraction of orders filled from stock) as a function of R

$B(R)$ = average number of outstanding backorders as a function of R

$I(R)$ = average on-hand inventory level (in units) as a function of R

Since we place an order when there are r units in stock and expect to incur demand for θ units while we wait for the replenishment order to arrive, $r - \theta$ is the amount of inventory we expect to have on hand when the order arrives. If $s = r - \theta > 0$, then we call this the **safety stock** for this system, since it represents inventory that protects it against stockouts due to fluctuations in either demand or deliveries. Since finding $r - \theta$ is equivalent to finding r (because θ is a constant), we can view the problem as finding the optimal base stock level ($R = r + 1$), reorder point r, or safety stock level ($s = r - \theta$).

We can approach the problem of finding an optimal base stock level in one of two ways. We can follow the procedure we have used up to now (in the EOQ, Wagner–Whitin, and news vendor models) and formulate a cost function and find the reorder point that minimizes this cost. Or we can simply specify the desired customer service level and find the smallest reorder point that attains it. We will develop both approaches below. But first we need to develop expressions for the performance measures $S(R)$, $B(R)$, and $I(R)$.

We begin by analyzing the relationship between inventory, replenishment orders, and backorders under a base stock policy. To do this, we distinguish between **on-hand inventory,** which represents physical inventory in stock (and hence can never be negative), and **inventory position,** which represents the balance of on-hand inventory, backorders, and replenishment orders and is given by

$$\text{Inventory position} = \text{on-hand inventory} - \text{backorders} + \text{orders} \tag{2.18}$$

Under a base stock policy we place a replenishment order every time a demand occurs. Hence, at all times the following holds:

$$\text{Inventory position} = R \tag{2.19}$$

Using (2.18) and (2.19), we can derive expressions for the performance measures.

Service Level. Consider a specific replenishment order. Because lead times are constant, we know that all the other $R - 1$ items either in inventory or on order will be available to fill new demand before the order under consideration arrives. Therefore, the only way the order can arrive after the demand for it has occurred is if demand during the replenishment lead time is greater than or equal to R (that is, $X \geq R$). Hence, the probability that the order arrives *before* its demand (i.e., does not result in a backorder) is given by $P(X < R) = P(X \leq R - 1) = G(R - 1) = G(r)$. Since all orders are alike with regard to this calculation, the fraction of demands that are filled from stock is equal to the probability that an order arrives before the demand for it has occurred, or

$$S(R) = G(R - 1) = G(r) \tag{2.20}$$

Hence, $G(R - 1)$ represents the fraction of demands that will be filled from stock. This is normally called the **fill rate** and represents a reasonable definition of customer service for many inventory control systems.

Backorder Level. At any time, the number of orders is exactly equal to the number of demands that have occurred during the last ℓ time units. If we let X represent this (random) number of demands, then from (2.18) and (2.19)

$$\text{On-hand inventory} - \text{backorders} = R - X \tag{2.21}$$

Notice that on-hand inventory and backorders can never be positive at the same time (i.e., because if we had both inventory and backorders, we would fill backorders until either stock ran out or the backorders were all filled). So, at a point where the number of outstanding orders is $X = x$, the backorder level is given by

$$\text{Backorders} = \begin{cases} 0 & \text{if } x < R \\ x - R & \text{if } x \geq R \end{cases}$$

The expected backorder level can be computed by averaging over possible values of x:

$$B(R) = \sum_{x=R}^{\infty} (x - R)p(x) \tag{2.22}$$

Expression (2.22) is a very important and useful function in the theory of inventory control. Because it measures the amount of unmet demand (backorder level), it is referred to as a *loss function*. While it can be computed in the form given in (2.22), it is frequently more convenient to write it in terms of the cumulative distribution function as follows:

$$B(R) = \theta - \sum_{x=0}^{R} [1 - G(x)] \tag{2.23}$$

This loss function will come up again in the (Q, r) model. Even simpler spreadsheet-implementable formulas for computing $B(R)$ are given in Appendix 2B for the cases where demand is Poisson-distributed and also for the case where demand is approximated by the (continuous) normal distribution.

Inventory Level. Taking the expectation of both sides of Equation (2.21) and noting that $I(R)$ represents expected on-hand inventory, $B(R)$ represents expected backorder level, and $E[X] = \theta$ is the expected lead time demand, we get

$$I(R) = R - \theta + B(R) \tag{2.24}$$

Example:
We can now analyze the Superior Appliance example. Suppose from past experience we know that mean demand for the refrigerator under consideration is 10 units per month and replenishment lead time is one month. Therefore, mean demand during lead time is $\theta = 10$ units. Further suppose that we model demand using the Poisson distribution.[6] Specifically, for any integer values of k and x, we set

$$p(R) = \text{Prob}\{\text{demand during lead time} = R\} = \frac{\theta^R e^{-\theta}}{R!} = \frac{10^R e^{-10}}{R!}$$

and

$$G(R) = \sum_{k=0}^{R} p(k) = \sum_{k=0}^{R} \frac{10^k e^{-10}}{k!}$$

With these we can also compute the $B(r)$ function by using the formulas from Appendix 2B. We summarize the results in Table 2.5. If we want to achieve a fill rate of at least

[6] The Poisson distribution is a good modeling choice for demand processes where demands occur one by one and do not exhibit cyclic fluctuations. It is completely specified by only one parameter, the mean, and is therefore convenient when one lacks information concerning the variability of demand. The standard deviation of the Poisson is equal to the square root of the mean.

TABLE 2.5 Fill Rates for Various Values of R

R	$p(R)$	$G(R)$	$B(R)$	R	$p(R)$	$G(R)$	$B(R)$
0	0.000	0.000	10.000	12	0.095	0.792	0.531
1	0.000	0.000	9.000	13	0.073	0.864	0.322
2	0.002	0.003	8.001	14	0.052	0.917	0.187
3	0.008	0.010	7.003	15	0.035	0.951	0.103
4	0.019	0.029	6.014	16	0.022	0.973	0.055
5	0.038	0.067	5.043	17	0.013	0.986	0.028
6	0.063	0.130	4.110	18	0.007	0.993	0.013
7	0.090	0.220	3.240	19	0.004	0.997	0.006
8	0.113	0.333	2.460	20	0.002	0.998	0.003
9	0.125	0.458	1.793	21	0.001	0.999	0.001
10	0.125	0.583	1.251	22	0.000	0.999	0.000
11	0.114	0.697	0.834	23	0.000	1.000	0.000

90 percent, we must choose R such that $G(R-1) \geq 0.9$. From Table 2.5 we see this requires $R - 1 = 14$, or $R = 15$, which results in a 91.7 percent fill rate. Since average demand during a replenishment lead time is 10 units, this is equivalent to setting a safety stock level of $r - \theta = 14 - 10 = 4$ units. The average backorder level resulting from $R = 15$ is given by $B(15) = 0.103$. The average inventory level is given by

$$I(R) = R - \theta + B(R) = 15 - 10 + 0.103 = 5.103$$

If we were to increase the base stock level from 15 to 16, the fill rate would increase to 95.1 percent, the backorder level would fall to 0.055, and the average inventory level would increase to 6.055. Whether or not the improved customer service (as measured by fill rate and backorder level) is worth the additional inventory investment is a value judgment for Superior Appliance. One way to balance these competing issues is to use a cost optimization model, as we show below.

In general, the higher the mean demand during replenishment lead time, the higher the base stock level required to achieve a particular fill rate. This is hardly surprising, since the reorder point r must contain enough inventory to cover demand while orders are coming. If the distribution of demand during lead time is symmetric (e.g., bell-shaped), then the probability of demand exceeding θ during the lead time is one-half. Hence, any fill rate greater than one-half will require r to be greater than θ.

In addition to mean demand, the variability of the demand process affects the choice of base stock level. The higher the standard deviation of demand during a replenishment lead time, the larger r will have to be for a given fill rate. If, in the previous example, we had approximated $G(x)$ by the normal distribution with mean θ and standard deviation σ, the choice of σ would have influenced the results in Table 2.5. Choosing $\sigma = \sqrt{\theta}$ would give results similar to those generated by using the Poisson distribution for $G(x)$ (since the standard deviation is always the square root of the mean in the Poisson). Higher values of σ would have given lower fill rates for the various values of r, while lower values of σ would have resulted in higher fill rates.

The base stock model has been widely studied in the operations management literature. This is partly because it is comparatively simple to analyze, but also because it is easily extended to a range of situations. For instance, base stocks can be used to

control work releases in a multistage production line. In such a system, a base stock level is established for each inventory buffer in the line (e.g., in front of the workstations). Whenever an item is removed from the buffer, a replenishment order is triggered. As we will discuss in Chapter 4, this is essentially what the Japanese kanban system does.

Finally, we consider an optimization approach to setting the base stock level. To do this, we approximate demand with a continuous distribution $G(x)$ with density $g(x)$. Then we can write the cost function consisting of the sum of inventory holding costs plus backorder costs as

$$\begin{aligned} Y(R) &= \text{holding cost} + \text{backorder cost} \qquad (2.25)\\ &= hI(R) + bB(R)\\ &= h(R - \theta + B(R)) + bB(R)\\ &= h(R - \theta) + (b + h)B(R) \qquad (2.26) \end{aligned}$$

We compute the base stock level R that minimizes $Y(R)$ in the following technical note.

Technical Note

Treating R as a continuous variable, we can take the derivative of $Y(R)$ as follows:

$$\frac{dY(R)}{dR} = h + (b + h)\frac{dB(R)}{dR}$$

The continuous-version expression of (2.22), the backorder function, $B(R)$, is given by

$$B(R) = \int_R^\infty (x - R)g(x)\,dx \qquad (2.27)$$

so $dB(R)/dR$ can be computed as

$$\begin{aligned} \frac{dB(R)}{dR} &= \frac{d}{dR}\int_R^\infty (x - R)g(x)\,dx\\ &= -\int_R^\infty g(x)\,dx\\ &= -[1 - G(R)] \end{aligned}$$

Setting $dY(R)/dR$ equal to zero yields

$$\frac{dY(R)}{dR} = h - (b + h)[1 - G(R)] = 0 \qquad (2.28)$$

Solving (2.28) yields the optimal value of R.

The base stock level R that minimizes holding plus backorder cost is given by

$$G(R^*) = \frac{b}{b + h} \qquad (2.29)$$

Notice that this formula has the same critical ratio structure that we saw in the news vendor solution given in (2.16). This implies that the optimal base stock level is the one for which the fill rate is given by $b/(b + h)$. This result makes intuitive sense, since increasing the holding cost h causes R^* to decrease, while increasing the backorder cost b causes R^* to increase. Note that when backorder and holding costs are equal, the

resulting fill rate is one-half so that $R^* = \theta$, the average demand during the replenishment time, and thus there is no safety stock.

As we did for the news vendor problem, we can simplify (2.29) for the case where G is normal. Using the same arguments we used to derive expression (2.17), we can show that

$$R^* = \theta + z\sigma \tag{2.30}$$

where z is the value from the standard normal table for which $\Phi(z) = b/(b + h)$ and μ and σ are the mean and standard deviation, respectively, of lead-time demand. Note that R^* increases in θ and also increases in σ provided that $z > 0$. This will be the case as long as $b/(b + h) > 0.5$, or equivalently $b > h$. Since carrying a unit of backorder is typically more costly than carrying a unit of inventory, it is generally the case that the optimal base stock level is an increasing function of demand variability.

Example:
Let us return to the Superior Appliance example. To approximate demand with a continuous distribution, we assume lead-time demand is normally distributed with mean $\theta = 10$ units per month and standard deviation $\sigma = \sqrt{\theta} = 3.16$ units per month. (Choosing $\sigma = \sqrt{\theta}$ makes the standard deviation the same as that for the Poisson distribution used in the earlier example.) Suppose that the wholesale cost of the refrigerators is \$750 and Superior uses an interest rate of two percent per month to charge inventory costs, so that $h = 0.02(750) = \$15$ per unit per month. Further suppose that the backorder cost is estimated to be \$25 per unit per month, because Superior typically has to offer discounts to get sales on out-of-stock items.

Then the optimal base stock level can be found from (2.30) by first computing z by calculating

$$\frac{b}{b+h} = \frac{25}{25+15} = 0.625$$

and looking up in a standard normal table to find $\Phi(0.32) = 0.625$. Hence, $z = 0.32$ and

$$R^* = \theta + z\sigma = 10 + 0.32(3.16) = 11.01 \approx 11$$

Using Table 2.5, we can compute the fill rate for this base stock level as $S(R) = G(R - 1) = G(10) = 0.583$. (Notice that even though we used a continuous model to find R^*, we used the discrete formula in Table 2.5 to compute the actual fill rate because in real life, demand for refrigerators is discrete.) This is a pretty low fill rate, which may indicate that our choice for the backorder cost b was too low.

If we were to increase the backorder cost to $b = \$200$, the critical ratio would increase to 0.93, which (because $z_{0.93} = 1.48$) would increase the optimal base stock level to $R^* = 10 + 1.48(3.16) = 14.67 \approx 15$. This is the base stock level we got in our previous analysis where we set it to achieve a fill rate of 90 percent, and we recall that the actual fill rate it achieves is 91.7 percent. We can make two observations from this. First, the actual fill rate computed from Table 2.5 using the Poisson distribution—91.7 percent even after rounding R up to 15—is generally lower than the critical ratio in (2.29), 93 percent, because a continuous demand distribution tends to make inventory look more efficient than it really is. Second, the backorder cost necessary to get a base stock level of 15, and hence a fill rate greater than 90 percent, is very large

($200 per unit per month!), which suggests that such a high fill rate is not a economical.[7]

We conclude by noting that the primary insights from the simple base stock model are as follows:

1. Reorder points control the probability of stockouts by establishing **safety stock.**
2. The required base stock level (and hence safety stock) that achieves a given fill rate is an increasing function of the mean and (provided that unit backorder cost exceeds unit holding cost) standard deviation of the demand during replenishment lead time.
3. The "optimal" fill rate is an increasing function of the backorder cost and a decreasing function of the holding cost. Hence, if we fix the holding cost, we can use either a service constraint or a backorder cost to determine the appropriate base stock level.
4. Base stock levels in multistage production systems are very similar to kanban systems, and therefore the above insights apply to those systems as well.

2.4.3 The (Q, r) Model

Consider the situation of Jack, a maintenance manager, who must stock spare parts to facilitate equipment repairs. Demand for parts is a function of machine breakdowns and is therefore inherently unpredictable (i.e., random). But, unlike in the base stock model, suppose that the costs incurred in placing a purchase order (for parts obtained from an outside supplier) or the costs associated with setting up the production facility (for parts produced internally) are significant enough to make one-at-a-time replenishment impractical. Thus, the maintenance manager must determine not only how much stock to carry (as in the base stock model), but also how many to produce or order at a time (as in the EOQ and news vendor models). Addressing both of these issues simultaneously is the focus of the (Q, r) model.

From a modeling perspective, the assumptions underlying the (Q, r) model are identical to those of the base stock model, except that we will assume that either

1. There is a fixed cost associated with a replenishment order. or
2. There is a constraint on the number of replenishment orders per year.

and therefore replenishment quantities greater than 1 may make sense.

The basic mechanics of the (Q, r) model are illustrated in Figure 2.6, which shows the net inventory level (on-hand inventory minus backorder level) and inventory position (net inventory plus replenishment orders) for a single product being continuously monitored. Demands occur randomly, but we assume that they arrive one at a time, which is why net inventory always drops in unit steps in Figure 2.6. When the inventory position reaches the reorder point r, a replenishment order for quantity Q is placed. (Notice that because the order is placed exactly when inventory position reaches r, inventory position

[7] Part of the reason that b must be so large to achieve $R = 15$ is that we are rounding to the nearest integer. If instead we always round up, which would be reasonable if we want service to be at least $b/(b + h)$, then a (still high) value of $b = \$135$ makes $b/(b + h) = 0.9$ and results in $R = 14.05$ which rounds up to 15. Since the continuous distribution is an approximation for demand anyway, it does not really matter whether a large b or an aggressive rounding procedure is used to obtain the final result. What does matter is that the user perform sensitivity analysis to understand the solution and its impacts.

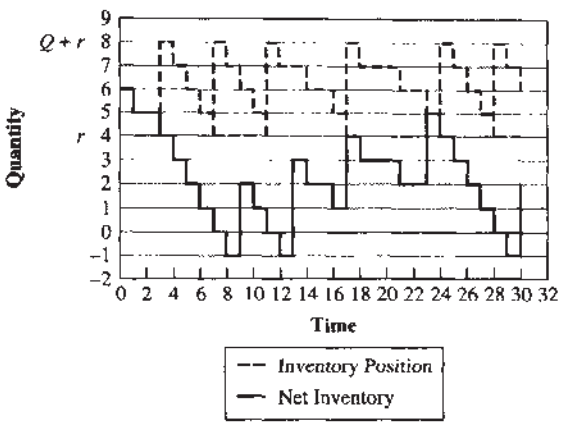


FIGURE 2.6
Net inventory and inventory position versus time in the (Q, r) model with $Q = 4$, $r = 4$

immediately jumps to $r + Q$ and hence never spends time at level r.) After a (constant) lead time of ℓ, during which stockouts might occur, the order is received. The problem is to determine appropriate values of Q and r.

As Wilson (1934) pointed out in the first formal publication on the (Q, r) model, the two controls Q and r have essentially separate purposes. As in the EOQ model, the replenishment quantity Q affects the tradeoff between production or order frequency and inventory. Larger values of Q will result in few replenishments per year but high average inventory levels. Smaller values will produce low average inventory but many replenishments per year. In contrast, the reorder point r affects the likelihood of a stockout. A high reorder point will result in high inventory but a low probability of a stockout. A low reorder point will reduce inventory at the expense of a greater likelihood of stockouts.

Depending on how costs and customer service are represented, we will see that Q and r can interact in terms of their effects on inventory, production or order frequency, and customer service. However, it is important to recognize that the two parameters generate two fundamentally different kinds of inventory. The replenishment quantity Q affects **cycle stock** (i.e., inventory that is held to avoid excessive replenishment costs). The reorder point r affects **safety stock** (i.e., inventory held to avoid stockouts). Note that under these definitions, all the inventory held in the EOQ model is cycle stock, while all the inventory held in the base stock model is safety stock. In some sense, the (Q, r) model represents the integration of these two models.

To formulate the basic (Q, r) model, we combine the costs from the EOQ and base stock models. That is, we seek values of Q and r to solve either

$$\min_{Q,r} \{\text{fixed setup cost} + \text{backorder cost} + \text{holding cost}\} \tag{2.31}$$

or

$$\min_{Q,r} \{\text{fixed setup cost} + \text{stockout cost} + \text{holding cost}\} \tag{2.32}$$

The difference between formulations (2.31) and (2.32) lies in how customer service is represented. Backorder cost assumes a charge per unit time a customer order is unfilled, while stockout cost assumes a fixed charge for each demand that is not filled from stock (regardless of the duration of the backorder). We will make use of both approaches in the analysis that follows.

Notation. To develop expressions for each of these costs, we will make use of the following notation:

D = expected demand per year (in units)

ℓ = replenishment lead time (in days); initially we assume this is constant, although we will show how to incorporate variable lead times at the end of this section

X = demand during replenishment lead time (in units), a random variable

$\theta = E[X] = D\ell/365$ = expected demand during replenishment lead time (in units)

σ = standard deviation of demand during replenishment lead time (in units)

$p(x) = P(X = x)$ = probability demand during replenishment lead time equals x (probability mass function). As in the base stock model, we assume demand is discrete. But when it is convenient to approximate it with a continuous distribution, we assume the existence of a density function $g(x)$ in place of the probability mass function

$G(x) = P(X \le x) = \sum_{i=0}^{x} p(i)$ = probability demand during replenishment lead time is less than or equal to x (cumulative distribution function)

A = setup or purchase order cost per replenishment (in dollars)

c = unit production cost (in dollars per unit)

h = annual unit holding cost (in dollars per unit per year)

k = cost per stockout (in dollars)

b = annual unit backorder cost (in dollars per unit of backorder per year); note that failure to have inventory available to fill a demand is penalized by using either k or b but not both

Q = replenishment quantity (in units); this is a decision variable

r = reorder point (in units); this is the other decision variable

$s = r - \theta$ = safety stock implied by r (in units)

$F(Q, r)$ = order frequency (replenishment orders per year) as a function of Q and r

$S(Q, r)$ = fill rate (fraction of orders filled from stock) as a function of Q and r

$B(Q, r)$ = average number of outstanding backorders as a function of Q and r

$I(Q, r)$ = average on-hand inventory level (in units) as a function of Q and r

Costs

Fixed Setup Cost. There are two basic ways to address the desirability of having an order quantity Q greater than one. First, we could simply put a constraint on the number of replenishment orders per year. Since the number of orders per year can be computed as

$$F(Q, r) = \frac{D}{Q} \tag{2.33}$$

we can compute Q for a given order frequency F as $Q = D/F$. Alternatively, we could charge a fixed order cost A for each replenishment order that is placed. Then the annual fixed order cost becomes $F(Q, r)A = (D/Q)A$.

Stockout Cost. As we noted earlier, there are two basic ways to penalize poor customer service. One is to charge a cost each time a demand cannot be filled from stock (i.e., a stockout occurs). The other is to charge a penalty that is proportional to the length of time a customer order waits to be filled (i.e., is backordered).

The annual stockout cost is proportional to the average number of stockouts per year, given by $D[1 - S(Q, r)]$. We can compute $S(Q, r)$ by observing from Figure 2.6 that inventory position can only take on values $r + 1, r + 2, \ldots, r + Q$ (note it cannot be equal to r since whenever it reaches r, another order of Q is placed immediately). In fact, it turns out that over the long term, inventory position is equally likely to take on any value in this range. We can exploit this fact to use our results from the base stock model in the following analysis (see Zipkin 1999 for a rigorous version of this development).

Suppose we look at the system[8] after it has been running a long time and we observe that the current inventory position is x. This means that we have inventory on hand and on order sufficient to cover the next x units of demand. So we ask the question, What is the probability that the $(x + 1)$st demand will be filled from stock? The answer to this question is precisely the same as it was for the base stock model. That is, since all outstanding orders will have arrived within the replenishment lead time, the only way the $(x + 1)$st demand can stock out is if demand during the replenishment lead time is greater than or equal to x. From our analysis of the base stock model, we know that the probability of a stockout is

$$\begin{aligned} P\{X \geq x\} &= 1 - P\{X < x\} \\ &= 1 - P\{X \leq x - 1\} \\ &= 1 - G(x - 1) \end{aligned}$$

Hence, the fill rate given an inventory position of x is one minus the probability of a stockout, or $G(x - 1)$. Since the Q possible inventory positions are equally likely, the fill rate for the entire system is computed by simply averaging the fill rates over all possible inventory positions:

$$S(Q, r) = \frac{1}{Q} \sum_{x=r+1}^{r+Q} G(x - 1) = \frac{1}{Q}[G(r) + \cdots + G(r + Q - 1)] \qquad (2.34)$$

We can use (2.34) directly to compute the fill rate for a given (Q, r) pair. However, it is often more convenient to convert this to another form. By using the fact that the base stock backorder level function $B(R)$ can be written in terms of the cumulative distribution function as in (2.23), it is straightforward to show that the following is an equivalent expression for the fill rate in the (Q, r) model:

$$S(Q, r) = 1 - \frac{1}{Q}[B(r) - B(r + Q)] \qquad (2.35)$$

This exact expression for $S(Q, r)$ is simple to compute in a spreadsheet, especially using the formulas given in Appendix 2B. However, it is sometimes difficult to use in analytic expressions. For this reason, various approximations have been offered. One approximation, known as the **base stock** or **type I service** approximation, is simply the (continuous demand) base stock formula for fill rate, which is given by

$$S(Q, r) \approx G(r) \qquad (2.36)$$

From Equation (2.34) it is apparent that $G(r)$ underestimates the true fill rate. This is because the cdf $G(x)$ is an increasing function of x. Hence, we are taking the smallest

[8] This technique is called *conditioning* on a random event (i.e., the value of the inventory position) and is a very powerful analysis tool in the field of probability.

term in the average. However, while it can seriously underestimate the true fill rate, it is very simple to work with because it involves only r and not Q. It can be the basis of a very useful heuristic for computing good (Q, r) policies, as we will show below.

A second approximation of fill rate, known as **type II service,** is found by ignoring the second term in expression (2.35) (Nahmias 1993). This yields

$$S(Q, r) \approx 1 - \frac{B(r)}{Q} \tag{2.37}$$

Again, this approximation tends to underestimate the true fill rate, since the $B(r+Q)$ term in (2.35) is positive. However, since this approximation still involves both Q and r, it is not generally simpler to use than the exact formula. But as we will see below, it does turn out to be a useful intermediate approximation for deriving a reorder point formula.

Backorder Cost. If, instead of penalizing stockouts with a fixed cost per stockout k, we penalize the time a backorder remains unfilled, then the annual backorder cost will be proportional to the average backorder level $B(Q, r)$. The quantity $B(Q, r)$ can be computed in a similar manner to the fill rate, by averaging the backorder level for the base stock model over all inventory positions between $r + 1$ and $r + Q$:

$$B(Q, r) = \frac{1}{Q} \sum_{x=r+1}^{r+Q} B(x) = \frac{1}{Q}[B(r+1) + \cdots + B(r+Q)] \tag{2.38}$$

Again, this formula can be used directly or converted to simpler form for computation in a spreadsheet, as shown in Appendix 2B. As with the expression for $S(Q, r)$, it is sometimes convenient to approximate this with a simpler expression that does not involve Q. One way to do this is to use the analogous formula to the type I service formula and simply use the base stock backorder formula

$$B(Q, r) \approx B(r) \tag{2.39}$$

Notice that to make an exact analogy with the type I approximation for fill rate, we should have taken the minimum term in expression (2.38), which is $B(r + 1)$. While this would work just fine, it is a bit simpler to use $B(r)$ instead. The reason is that we typically use such an approximation when we are also approximating demand with a continuous function; under this assumption the backorder expression for the base stock model really does become $B(r)$ [instead of $B(R)$].

Holding Cost. The last cost in problems (2.31) and (2.32) is the inventory holding cost, which can be expressed as $hI(Q, r)$. We can approximate $I(Q, r)$ by looking at the *average* net inventory and acting as though demand were deterministic, as in Figure 2.7, which depicts a system with $Q = 4$, $r = 4$, $\ell = 2$, and $\theta = 2$. Demands are perfectly regular, so that every time inventory reaches the reorder point ($r = 4$), an order is placed, which arrives two time units later. Since the order arrives just as the last demand in the replenishment cycle occurs, the lowest inventory level ever reached is $r - \theta + 1 = s + 1 = 3$. In general, under these deterministic conditions, inventory will decline from $Q + s$ to $s + 1$ over the course of each replenishment cycle. Hence, the average inventory is given by

$$I(Q, r) \approx \frac{(Q+s) + (s+1)}{2} = \frac{Q+1}{2} + s = \frac{Q+1}{2} + r - \theta \tag{2.40}$$

In reality, however, demand is variable and sometimes causes backorders to occur. Since on-hand inventory cannot go below zero, the above deterministic approximation underestimates the true average inventory by the average backorder level. Hence, the exact

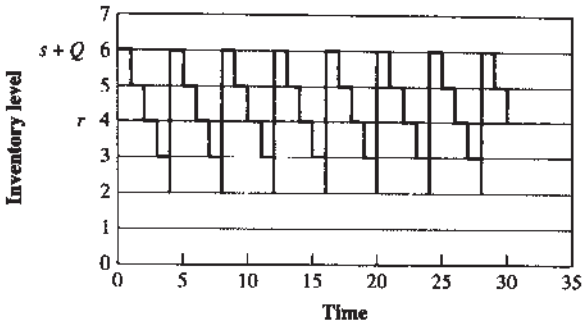


FIGURE 2.7
Expected inventory versus time in the (Q, r) model with $Q = 4$, $r = 4$, $\theta = 2$

expression is

$$I(Q,r) = \frac{Q+1}{2} + r - \theta + B(Q,r) \tag{2.41}$$

Backorder Cost Approach. We can now make verbal formulation (2.31) into a mathematical model. The sum of setup and purchase order cost, backorder cost, and inventory carrying cost can be written as

$$Y(Q,r) = \frac{D}{Q}A + bB(Q,r) + hI(Q,r) \tag{2.42}$$

Unfortunately, there are two difficulties with the cost function $Y(Q, r)$. The first is that the cost parameters A and b are difficult to estimate in practice. In particular, the backorder cost is nearly impossible to specify, since it involves such intangibles as loss of customer goodwill and company reputation. Fortunately, however, the objective is not really to minimize this cost; it is to strike a reasonable balance between setups, service, and inventory. Using a cost function allows us to conveniently use optimization tools to derive expressions for Q and r in terms of problem parameters. But the quality of the policy must be evaluated directly in terms of the performance measures, as we will illustrate in the next example. The expressions for $B(Q, r)$ and $I(Q, r)$ involve both Q and r in complicated ways. So using exact expressions for these quantities does not lead us to simple expressions for Q and r. Therefore, to achieve tractable formulas, we approximate $B(Q, r)$ by expression (2.39) and use this in place of the true expression for $B(Q, r)$ in the formula for $I(Q, r)$ as well. With this approximation our objective function becomes

$$Y(Q,r) \approx \tilde{Y}(Q,r) = \frac{D}{Q}A + bB(r) + h\left[\frac{Q+1}{2} + r - \theta + B(r)\right] \tag{2.43}$$

We compute the Q and r values that minimize $\tilde{Y}(Q, r)$ in the following technical note.

Technical Note

Treating Q as a continuous variable, differentiating $\tilde{Y}(Q, r)$ with respect to Q, and setting the result equal to zero yield

$$\frac{\partial \tilde{Y}(Q,r)}{\partial Q} = \frac{-DA}{Q^2} + \frac{h}{2} = 0 \tag{2.44}$$

Approximating lead-time demand with a continuous distribution with density $g(x)$, differentiating $\tilde{Y}(Q, r)$ with respect to r, and setting the result equal to zero yield

$$\frac{\partial \tilde{Y}(Q, r)}{\partial r} = (b + h)\frac{dB(r)}{dr} + h = 0 \tag{2.45}$$

Since, as in the base stock case, the continuous analog for the $B(r)$ function is

$$B(r) = \int_r^\infty (x - r)g(x)\,dx$$

we can compute the derivative of $B(r)$ as

$$\begin{aligned}\frac{dB(r)}{dr} &= \frac{d}{dr}\int_r^\infty (x - r)g(x)\,dx \\ &= -\int_r^\infty g(x)\,dx \\ &= -[1 - G(r)]\end{aligned}$$

and rewrite (2.45) as

$$-(b + h)[1 - G(r)] + h = 0 \tag{2.46}$$

Hence, we must solve (2.44) and (2.46) to minimize $\tilde{Y}(Q, r)$, which we do in (2.47) and (2.48).

The optimal reorder quantity Q^* and reorder point r^* are given by

$$Q^* = \sqrt{\frac{2AD}{h}} \tag{2.47}$$

$$G(r^*) = \frac{b}{b + h} \tag{2.48}$$

Notice that Q^* is given by the EOQ formula and the expression for r^* is given by the critical ratio formula for the base stock model. (The latter is not surprising, since we used a base stock approximation for the backorder level.) If we further assume that lead-time demand is normally distributed with mean θ and standard deviation σ, then we can simplify (2.48) as we did for the base stock model in (2.30) to get

$$r^* = \theta + z\sigma \tag{2.49}$$

where z is the value in the standard normal table such that $\Phi(z) = b/(b + h)$.

It is important to remember that these values for Q^* and r^* are only approximate. So we should check their performance (e.g., in terms of average inventory, fill rate, order frequency, and backorder level) by using exact formulas. If performance is not adequate, then the cost parameters can be adjusted. Typically, it makes sense to leave holding cost h alone and adjust the fixed order cost A and the backorder cost b, since these are more difficult to estimate in advance. Note that increasing A increases Q^* and hence reduces average order frequency, while increasing b increases r^* and hence reduces stockout rate and average backorder level. We illustrate this in the example on page 82.

Stockout Cost Approach. As an alternative to the backorder cost approach, we can make verbal formulation (2.32) into a mathematical model by writing the sum of the annual setup or purchase order cost, stockout cost, and inventory carrying cost as

$$Y(Q, r) = \frac{D}{Q}A + kD[1 - S(Q, r)] + hI(Q, r) \tag{2.50}$$

As was the case for the backorder model, this cost function involves parameters that are difficult to specify. In particular, the stockout cost k is dependent on the same intangibles (lost customer goodwill and company reputation) as is the backorder cost b. Hence, again, this cost function is merely a means for deriving expressions for Q and r that reasonably balance setups, service, and inventory. It is not a performance measure in itself.

Also like the backorder model, the stockout model cost function contains expressions $S(Q, r)$ and $I(Q, r)$ that involve both Q and r and therefore do not lead to simple expressions. So we will make two levels of approximation to generate closed-form expressions for Q and r.

First, analogous to what we did in the backorder cost model above, we will assume that the effect of Q on the fill rate $S(Q, r)$ and the backorder correction factor $B(Q, r)$ in the inventory term $I(Q, r)$ can be ignored. This leads to the familiar EOQ formula for the order quantity

$$Q^* = \sqrt{\frac{2AD}{h}}$$

Second, to compute an expression for the reorder point, we make two approximations in (2.50). We replace the service $S(Q, r)$ by type II approximation (2.37) and the backorder correction term $B(Q, r)$ in the inventory term by base stock approximation (2.39). This yields the following approximate cost function

$$Y(Q, r) \approx \tilde{Y}(Q, r) = \frac{D}{Q}A + kD\frac{B(r)}{Q} + h\left[\frac{Q+1}{2} + r - \theta + B(r)\right] \quad (2.51)$$

Going through the usual optimization procedure (taking the derivative with respect to r, setting the result equal to zero, and solving for r) yields the following expression for the optimal reorder point:

$$G(r^*) = \frac{kD}{kD + hQ} \quad (2.52)$$

If we further assume that lead-time demand is normally distributed with mean θ and standard deviation σ, then we can simplify the expression for the reorder point to

$$r^* = \theta + z\sigma \quad (2.53)$$

where $\Phi(z) = kD/(kD + hQ)$.

Notice that unlike formula (2.49), expression (2.53) is sensitive to Q. Specifically, making Q larger makes the ratio $kD/(kD + hQ)$ smaller and hence reduces r^*. The reason is that larger Q values serve to increase the fill rate (because the reorder point is crossed less frequently) and hence require a smaller reorder point to achieve a given level of service.

Example:
Jack, the maintenance manager, has collected historical data that indicate one of the replacement parts he stocks has annual demand (D) of 14 units per year. The unit cost c of the part is \$150, and since the firm uses an interest rate of 20 percent, the annual holding cost h has been set at $0.2(\$150) = \$30$ per year. It takes 45 days to receive a replenishment order, so average demand during a replenishment lead time is

$$\theta = \frac{14}{365} \times 45 = 1.726$$

The part is purchased from an outside supplier, and Jack estimates that the cost of time and materials required to place a purchase order A is about \$15. The one remaining

cost required by our model is the cost of either a backorder or stockout. Although he is very uncomfortable trying to estimate these, when pressed, Jack made a guess that the annualized cost of a backorder is about $b = \$100$ per year, and the cost per stockout event can be approximated by $k = \$40$.[9] Finally, Jack has chosen to model demands using the Poisson distribution.[10]

Regardless of whether we use the backorder cost model or the stockout cost model, the order quantity is computed by using (2.47), which yields

$$Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2(15)(14)}{30}} = 3.7 \approx 4$$

To compute the reorder point, we can use either the backorder cost or the stockout cost model. To use expression (2.49) from the normal demand version of the backorder model, we approximate the Poisson by the normal, with mean $\theta = 1.726$ and standard deviation $\sigma = \sqrt{1.726} = 1.314$. The critical ratio is given by

$$\frac{b}{b+h} = \frac{100}{100+30} = 0.769$$

and from a standard normal table, $\Phi(0.736) = 0.769$. Hence, $z = 0.736$ and

$$r^* = \theta + z\sigma = 1.726 + 0.736(1.314) = 2.693 \approx 3$$

As we noted earlier, the performance of this policy should not be judged in terms of the cost function or the approximate performance measures. Instead, it should be evaluated in terms of the exact expressions for order frequency, fill rate, backorder level, and inventory level. To compute these, we need to compute $p(r)$, $G(r)$, and $B(r)$. We do this by using the formulas from Appendix 2B for the Poisson demand case, and we summarize the results in Table 2.6. With these we can compute the order frequency, fill rate, backorder level, and average inventory level for the policy $(Q = 4, r = 3)$ as follows:

$$F(Q,r) = \frac{D}{Q} = \frac{14}{4} = 3.5$$

$$\begin{aligned} S(Q,r) &= 1 - \frac{1}{Q}[B(r) - B(r+Q)] \\ &= 1 - \tfrac{1}{4}[B(3) - B(3+4)] \\ &= 1 - \tfrac{1}{4}(0.140 - 0.001) \\ &= 0.965 \end{aligned}$$

$$\begin{aligned} B(Q,r) &= \frac{1}{Q}\sum_{x=r+1}^{r+Q} B(x) \\ &= \tfrac{1}{4}[B(4) + B(5) + B(6) + B(7)] \\ &= \tfrac{1}{4}(0.042 + 0.011 + 0.003 + 0.001) \\ &= 0.014 \end{aligned}$$

[9]Notice that either approach for penalizing backorders or stockouts assumes that the cost is independent of which machine it affects. Of course, in reality, stockouts for heavily used critical machines are far more costly than stockouts affecting lightly used machines with excess capacity.

[10]The Poisson is a good assumption when demands are generated by many independent sources, such as failures of different machines. However, if demands were generated by a more regular process, such as scheduled preventive maintenance procedures, the Poisson distribution will tend to overestimate variability and lead to conservative, possibly excessive, safety stock levels.

TABLE 2.6 $p(r)$, $G(r)$, $B(r)$ for $\theta = 1.726$

r	$p(r)$	$G(r)$	$B(r)$
0	0.178	0.178	1.726
1	0.307	0.485	0.904
2	0.265	0.750	0.389
3	0.153	0.903	0.140
4	0.066	0.969	0.042
5	0.023	0.991	0.011
6	0.007	0.998	0.003
7	0.002	1.000	0.001
8	0.000	1.000	0.000
9	0.000	1.000	0.000
10	0.000	1.000	0.000

$$\begin{aligned} I(Q,r) &= \frac{Q+1}{2} + r - \theta + B(Q,r) \\ &= \frac{4+1}{2} + 3 - 1.726 + 0.014 \\ &= 3.79 \end{aligned}$$

As an alternative to using the backorder cost model, we could have computed the reorder point by using expression (2.53) from the stockout cost model. The critical ratio in this formula is

$$\frac{kD}{kD + hQ} = \frac{40(14)}{40(14) + 30(4)} = 0.824$$

and from a standard normal table $\Phi(0.929) = 0.824$ so $z = 0.929$ and

$$r^* = \theta + z\sigma = 1.726 + 0.929(1.314) = 2.946 \approx 3$$

Since this policy ($Q = 4, r = 3$) is the same as that resulting from the backorder cost model, the performance measures will also be the same. So, in a practical sense, the backorder and stockout costs chosen by Jack are equivalent. In the single-product case, either model could be used—increasing either b or k will serve to increase service and decrease backorder level (at the expense of a higher inventory level). So either model can be used to generate a set of efficient solutions by varying these cost parameters. But we will see in Chapter 17 that the two models can behave differently in multiproduct systems.

The policy generated by the current cost coefficients will require placing replenishment orders three and one-half times per year, the fill rate is fairly high (96.5 percent), there will be few backorders (only 0.014 on average), and on-hand inventory will average a bit under four units (3.79). The decision maker might look at these values and feel that the policy is just fine. If not, then sensitivity analysis should be used to find variants of the solution.

For instance, suppose that the decision maker felt that three and one-half replenishment orders per year were too few and that, given the capacity of the purchasing department, $F = 7$ orders per year would be manageable. Then we could use

$Q = D/F = 14/7 = 2$. But if we stick with a reorder point of $r = 3$, then the fill rate becomes

$$S(Q, r) = 1 - \frac{1}{Q}[B(r) - B(r + Q)] = 1 - \tfrac{1}{2}(0.140 - 0.011) = 0.936$$

which may be too low for a repair part. If we increase the reorder point to $r = 4$, then the fill rate becomes

$$S(Q, r) = 1 - \frac{1}{Q}[B(r) - B(r + Q)] = 1 - \tfrac{1}{2}(0.042 - 0.003) = 0.980$$

For this new policy $(Q = 2, r = 4)$ we can easily compute the backorder level and average inventory level to be

$$\begin{aligned} B(Q, r) &= \frac{1}{Q}[B(5) + B(6)] \\ &= \tfrac{1}{2}(0.011 + 0.003) \\ &= 0.007 \end{aligned}$$

$$\begin{aligned} I(Q, r) &= \frac{Q + 1}{2} + r - \theta + B(Q, r) \\ &= \frac{2 + 1}{2} + 3 - 1.726 + 0.007 \\ &= 3.78 \end{aligned}$$

The increased reorder point has lowered the backorder rate, and the increased order frequency has reduced the average inventory level relative to the original policy of $(Q = 4, r = 3)$. Of course, the cost of doing this is an additional three and one-half replenishment orders per year.

An alternate method for doing sensitivity analysis would be to modify the fixed order cost A until the order frequency $F(Q, r)$ is satisfactory and then modify the backorder cost b or the stockout cost k (depending on which model is being used) until the fill rate $S(Q, r)$ and/or the backorder level $B(Q, r)$ is acceptable. In a single-product problem like this, there is no great advantage to this approach, since we are still searching over two variables (that is, A and b or k instead of Q and r). But as we will see in Chapter 17, this approach is *much* more efficient in multiproduct problems, where one can search over a single (A, b) or (A, k) pair instead of (Q, r) values for each product. Furthermore, since expressions (2.47), (2.49), and (2.53) are simple closed-form equations involving the problem data, they are extremely simple to compute in a spreadsheet.

Modeling Lead-Time Variability. Throughout our discussion of the base stock and (Q, r) models we have assumed that the replenishment lead time ℓ is fixed. All the variability in the system was assumed to be due to demand variability. However, in many practical situations, the lead time may also be subject to variability. For instance, a supplier of a part may sometimes be late (or early) on a delivery. The primary effect of this additional variability is to inflate the standard deviation of the demand during the replenishment lead time σ. By computing a formula for σ that considers lead-time variability, we can easily incorporate this additional source of variability into the base stock and (Q, r) models.

To develop the appropriate formula, we must introduce a bit of additional notation:

L = replenishment lead time (in number of periods), a random variable

$\ell = E[L]$ = expected replenishment lead time (in number of periods)

σ_L = standard deviation of replenishment lead time (in days)

D_t = demand on day t (in units), a random variable. We assume that demand is stationary over time, so that D_t has same distribution for each day t; we also assume daily demands are independent of one another

$d = E[D_t]$ = expected daily demand (in units)

σ_D = standard deviation of daily demand (in units)

As before, we let X represent the (random) demand during the replenishment lead time. With the above notation, this can be written as

$$X = \sum_{t=1}^{L} D_t \tag{2.54}$$

Because daily demands are independent and identically distributed, we can compute the expected demand during the replenishment lead time as

$$E[X] = E[L]E[D_t] = \ell d = \theta \tag{2.55}$$

which is what we have been using all along. However, variable lead times change the variance of demand during replenishment lead time. Using the standard formula for sums of independent, identically distributed random variables, we can compute

$$\text{Var}(X) = E[L]\,\text{Var}(D_t) + E[D_t]^2\,\text{Var}(L) = \ell\sigma_D^2 + d^2\sigma_L^2 \tag{2.56}$$

Although the "units" of (2.56) look wrong (the first term appears to have units of time while the second has units of time-squared), both terms are actually dimensionless. The reason is that L is defined as a random variable representing the *number* of periods and not the periods themselves. While the mean and variance of L do not have to be an integer, realizations of the random variable itself, do. For instance, by counting the number of days, we might observe lead times of five, six, and three days yielding a mean of 4.667. However, it is not valid to observe 5/7, 6/7, and 3/7 weeks and then compute a mean. Hence, the standard deviation of lead-time demand is

$$\sigma = \sqrt{\text{Var}(X)} = \sqrt{\ell\sigma_D^2 + d^2\sigma_L^2} \tag{2.57}$$

To get a better feel for how formula (2.57) behaves, consider the case where demand is Poisson. This implies that $\sigma_D = \sqrt{d}$, since the standard deviation is always the square root of the mean for Poisson random variables. Substituting this into (2.57) yields

$$\sigma = \sqrt{\ell d + d^2\sigma_L^2} = \sqrt{\theta + d^2\sigma_L^2} \tag{2.58}$$

Notice that if $\sigma_L = 0$, which represents the case where the replenishment lead time is constant, then this reduces to $\sigma = \sqrt{\theta}$, which is exactly what we have been using for the Poisson demand case. If $\sigma_L > 0$, then formula (2.58) serves to inflate σ above what it would be for the constant-lead-time case.

To illustrate the use of the above formula in an inventory model, let us return to the Superior Appliance example from Section 2.4.2. There we assumed that demand for refrigerators was Poisson-distributed with a mean of 10 per month and that lead time was one month (30 days). So mean daily demand is $d = \frac{10}{30} = \frac{1}{3}$. Because we are assuming Poisson demand, we can use (2.58) to calculate σ. Using the same holding and backorder cost as in Section 2.4.2, $h = 15$ and $b = 25$, the critical ratio is $b/(h+b) = 25/(15+25) = 0.625$, so $z = 0.32$ since $\Phi(0.32) = 0.625$. The optimal base stock level (assuming the normal approximation of demand) is therefore

$$R^* = \theta + z\sigma = \theta + z\sqrt{\theta + d^2\sigma_L^2}$$

If $\sigma_L = 0$, then we get $R^* = 11.01$, which is what we got previously. If $\sigma_L = 30$ (i.e., the variability in replenishment lead time is so large that the standard deviation is equal to the mean), then we get $R^* = 13.34$. The additional 3.33 units of inventory are required to achieve the same service level in the face of the more variable demand.

Formula (2.57) or (2.58) can be used in this same fashion to inflate the reorder point in the (Q, r) model in either equation (2.49) or (2.53) to account for variable replenishment lead times.

Basic (Q, r) Insights. Apart from all the mathematical and modeling complexity, the basic insights behind the (Q, r) model are essentially those of the EOQ and base stock models, namely that

> Cycle stock increases as replenishment frequency decreases.

and

> Safety stock provides a buffer against stockouts.

The (Q, r) model places these insights into a unified framework.

Historically, the (Q, r) model (including the special case of the base stock model, which is just a (Q, r) model with $Q = 1$) was one of the earliest attempts to explicitly model variability in the demand process and provide quantitative understanding of just how safety stock affects service level. In terms of rough intuition, this model suggests that safety stock, service level, and backorder level are primarily affected by the reorder point r, while cycle stock and order frequency are essentially functions of replenishment quantity Q. However, the mathematics of the model show that the situation is somewhat more subtle. As we saw above, the expressions for service and backorder level depend on Q as well as r. The reason is that if Q is large, so that the part is replenished infrequently in large batches, then stock level seldom reaches the reorder point and therefore has few opportunities for stockouts. If, on the other hand, Q is small, then stock level frequently falls to the reorder point and therefore has a greater chance of stocking out.

Beyond these qualitative observations, the (Q, r) model offers some quantitative insight into the factors that affect the optimal stocking policy. From approximate formulas (2.47), (2.49), and (2.53) we can draw the following conclusions.

1. Increasing the average annual demand D tends to increase the optimal order quantity Q.

2. Increasing the average demand during a replenishment lead time θ will tend to increase the optimal reorder point r. Note that increasing either the annual demand D or the replenishment lead time ℓ will serve to increase θ. The implication is that either high demand or long replenishment lead times will tend to require more inventory in stock.

3. Increasing the variability of the demand process σ will tend to increase the optimal reorder point r.[11] The key insight here is that a highly variable demand process typically requires more safety stock as protection against stockouts than does a very stable demand process.

4. Increasing the holding cost h will tend to decrease both the optimal replenishment quantity Q and reorder point r. Note that the holding cost can be increased by increasing

[11] Note that this is only true if the critical ratio in (2.49) or (2.53) is at least one-half. If this ratio is less than one-half, then z will be negative and the optimal order point will actually decrease in the standard deviation of lead-time demand. But this only occurs when the costs are such that it is optimal to set a relatively low fill rate for the product. So, the case where z is positive is very common in practice.

the cost of the item, the interest rate associated with inventory, or the noninterest holding costs (e.g., handling and spoilage). The point is that the more expensive it is to hold inventory, the less we should hold.

The (Q, r) model is a happy example of an approach that provides both powerful general insights and useful practical tools. As such it is a basic component of any manufacturing manager's skill set.

2.5 Conclusions

Although this chapter has covered a wide range of inventory modeling approaches, we have barely scratched the surface of this vast branch of the OM literature. The complexity and variety of inventory systems have spawned a wide array of models. Table 2.7 summarizes some of the dimensions along which these models differ and classifies the five models we have treated in this chapter (i.e., EOQ, Wagner–Whitin (WW), news vendor (NV), base stock (BS), and (Q, r)), plus the economic production lot (EPL) model that we mentioned as an EOQ extension. (Notice that some of the entries in Table 2.7 contain dashes, which indicates that the particular modeling decision has been rendered meaningless by other modeling assumptions and therefore does not apply.) The OM literature contains models representing all reasonable combinations of these dimensions, as well as models with features that go beyond them (e.g., substitution between products, explicit links between spare-parts inventory and utilization of maintenance personnel, and perishable inventories). In this book, we will return to the important subject of inventory management in Chapter 17, where we will extend some of the models of this chapter into the important practical environments of multiple products and multilevel inventory systems. The reader interested in a more comprehensive summary than we can provide in two chapters is encouraged to consult Graves, Rinnooy Kan, Zipkin (1993); Hadley and Whitin (1963); Johnson and Montgomery (1974); McClain and Thomas (1985); Nahmias (1993); Peterson and Silver (1985); Sherbrooke (1992); and Zipkin (1998).

Although some of these models require data that may be difficult or impossible to obtain, they do offer some basic insights:

1. *There is a tradeoff between setups (replenishment frequency) and inventory.* The more frequently we replenish inventory, the less *cycle stock* we will carry.
2. *There is a tradeoff between customer service and inventory.* Under conditions of random demand, higher customer service levels (i.e., fill rates) require higher levels of *safety stock*.
3. *There is a tradeoff between variability and inventory.* For a given replenishment frequency, if customer service remains fixed (at a sufficiently high level), then the higher the variability (i.e., standard deviation of demand or replenishment lead time), the more inventory we must carry.

Despite the efforts of some just-in-time advocates to deny the existence of such tradeoffs, they are facts of manufacturing life. The commonly heard admonitions "Inventory is evil" or "Setups are bad" do little to guide the manager to useful policies.

In contrast, an understanding of the dynamics of inventory, replenishment frequency, and customer service enables a manager to evaluate which actions are likely to have the greatest impact. Such intuition can help address such questions as, Which setups are

TABLE 2.7 Classification of Inventory Models

Modeling Decision	Model					
	EOQ	**EPL**	**WW**	**NV**	**BS**	**(Q, r)**
Continuous (C) or discrete (D) time	C	C	D	D	C	C
Single (S) or multiple (M) products	S	S	S	S	S	S
Single (S) or multiple (M) periods	—	—	M	S	—	—
Backordering (B) or lost sales (L)	—	—	—	L	B	B
Setup or order cost [yes (Y) or no (N)]	Y	Y	Y	N	N	Y
Deterministic (D) or random (R) demand	D	D	D	R	R	R
Deterministic (D) or random (R) production	D	D	D	D	D	D
Constant (C) or dynamic (D) demand	C	C	D	—	C	C
Finite (F) or infinite (I) production rate	I	F	I	—	I	I
Finite (F) or infinite (I) horizon	I	I	F	F	I	I
Single (S) or multiple (M) echelons	S	S	S	S	S	S

most disruptive? How much inventory is too much? How much will an improvement in customer service cost? How much is a more reliable vendor worth? And so on. We will develop additional insights regarding inventory in Part II and will return to the practical considerations of inventory management in Chapter 17 of Part III.

The inventory models and insights discussed here also provide a framework for thinking about higher-level actions that can change the nature of these tradeoffs, such as increased system flexibility, better vendor management, and improved quality. Finding ways to alter these fundamental relationships is a key management priority that we will explore more fully in Parts II and III.

APPENDIX 2A
BASIC PROBABILITY

Random Experiments and Events

The starting point of the field of **probability** is the random experiment. A **random experiment** is any measurement or determination for which the outcome is not known in advance. Examples include measuring the hardness of a piece of bar stock, checking a circuit board for short circuits, or tossing a coin.

The set of all possible outcomes of the experiment is called the **sample space.** For example, consider the random experiment of tossing two coins. Let (a, b) denote the outcome of the experiment, where a is H if the first coin comes up heads or T if it comes up tails, with b defined similarly for the second coin. The sample space is then {(H, H), (H, T), (T, H), (T, T)}.

An **event** is a subset of the sample space. The individual elements in the sample space are called **elementary events.** A nonelementary event in our sample space is "at least one coin comes up heads," which corresponds to the set {(H, H), (H, T), (T, H)}. Events are used to make **probability statements.** For instance, we can ask, What is the probability that no tails appear?

Once the set of events has been defined, we can make statements concerning their probability.

Definitions of Probability

Over the years, three basic definitions of probability have been proposed: (1) classical or a priori probability, (2) frequency or a posteriori probability, and (3) subjective probability. The different definitions are useful for different types of experiments.

A priori probability is appropriate when the random experiment has a sample space composed of n mutually exclusive and equally likely outcomes. Under these conditions, if event A is made up of n_A of these outcomes, we define the probability of A occurring as n_A/n. This definition is useful in describing games of chance. For example, the question regarding the probability of no tails occurring when two coins are tossed can be interpreted in this way. Clearly, all the outcomes in the sample space are mutually exclusive. If the coins are "fair," then no particular outcome is "special" and therefore cannot be more likely to occur than any other. Thus, there are four mutually exclusive and equally likely outcomes. Only one of these contains no tails. Therefore the probability of no tails is $\frac{1}{4}$, or 0.25.

The second definition of probability, **frequency** or **a posteriori probability,** is also couched in terms of a random experiment, but *after* the experiment instead of *before* it. To describe this definition, we imagine performing a number of experiments, say N, of which M result in event E. Then we define the probability of E to be the number p to which the ratio of M/N converges as N becomes larger and larger. For instance, suppose $p = 0.75$ is the long-run fraction of good chips produced on a line in a wafer fabrication. Then we can consider p to be the probability of producing a good wafer on any given try.

Subjective probability can be used to describe experiments that are intrinsically impossible to replicate. For instance, the probability of rain at the company picnic tomorrow is a meaningful number, but is impossible to determine experimentally since tomorrow cannot be repeated. So when the weather forecaster says that the chance of rain tomorrow is 50 percent, this number represents a purely subjective estimate of likelihood.

Fortunately, regardless of the definition of probability used, the tools and techniques for analyzing probability problems are the same. The first step is to assign probabilities to events by means of a probability function. A **probability function** is a mathematical function that takes as input an event and produces a number between zero and one (i.e., a probability).

For example, consider again the two-coin toss experiment. Suppose P is the corresponding probability function. Since there is nothing unique about any of the outcomes listed above, they should be equally likely. Thus, we can write

$$P\{(\mathrm{H}, \mathrm{T})\} = \tfrac{1}{4}$$

Also, since the events (H, T) and (T, H) are mutually exclusive, their probabilities are additive, so

$$P\{(\mathrm{H}, \mathrm{T}) \text{ or } (\mathrm{T}, \mathrm{H})\} = \tfrac{1}{4} + \tfrac{1}{4} = \tfrac{1}{2}$$

Similarly, the probability of the "sure event" (i.e., that (H, H), (H, T), (T, H), or (T, T) will occur) must be one. Probability functions provide a useful shorthand for making statements regarding random events.

Random Variables and Distribution Functions

The majority of probability results turn on the concept of a **random variable.** Unfortunately, the term *random variable* is a misnomer since it is neither random nor a variable. Like a probability function, a random variable is a function. But instead of defining probabilities to events, it assigns numbers to *outcomes* of a random experiment. This greatly simplifies notation by replacing clumsy representations of outcomes like (H, T) with numbers.

For example, a random variable for the two coin experiment can be defined as

Outcome	Value of Random Variable
(H, H)	0
(H, T)	1
(T, H)	2
(T, T)	3

A random variable for the experiment to measure the hardness of bar stock might be the output of a device that applies a known pressure to the bar and reads out the Rockwell hardness index. A random variable for the circuit-board experiment might be simply the number of short circuits.

Random variables can be either **continuous** or **discrete.** Continuous random variables assign real numbers to their associated outcomes. The hardness experiment is one such example. Discrete random variables, on the other hand, assign outcomes to integers. Examples of discrete random variables are the random variable defined above for the coin toss experiment and the number of short circuits on a circuit board.

Random variables are also useful in defining events. For instance, all the outcomes of the circuit-board experiment with no more than five short circuits constitute an event. The linkage between the event referenced by a random variable and the probability of the event is given by its associated **distribution function,** which we will denote by G. For instance, let X denote the hardness of a piece of steel with an associated distribution function G. Then the probability that the hardness is less than or equal to some value x can be written as

$$P\{X \leq x\} = G(x)$$

If the event of interest is that the hardness is in some range of values, say from x_1 to x_2, we can write

$$P\{x_1 < X \leq x_2\} = G(x_2) - G(x_1)$$

Note that since X is continuous, it can take on values with an infinite number of decimal places of accuracy. Thus, the probability of X being *exactly* any number in particular (say, $X = 500.0000\ldots$) is zero. However, we can talk about the **probability density function** f as the probability of X lying in a small interval divided by the size of the interval, so that

$$g(x)\,\Delta x = P\{x \leq X \leq x + \Delta x\}$$

Of course, to be precise, $g(x)$ is defined only in the limit as Δx goes to zero. But for practical purposes, as long as Δx is small, this expression is almost exact. For instance,

$$P[4.9999 \leq X \leq 5.0001] \approx f(5) \cdot 0.0002$$

to a high degree of accuracy.

For continuous random variables defined for positive real numbers, g and G are related by

$$G(x) = \int_0^x g(x)\,dx$$

Analogous to the probability density functions of continuous random variables, discrete random variables have **probability mass** functions. We typically denote these functions by $p(x)$ to distinguish them from density functions. For instance, in the two-coin experiment, the event of two heads coming up is the same as the event $\{X = 0\}$. Its associated probability is

$$P\{\text{two heads}\} = P\{X = 0\} = p(0) = \tfrac{1}{4}$$

Notice that, unlike in the continuous case, in the discrete case there *is* a finite probability of particular values of the random variable.

In many cases, discrete random variables are defined from zero to positive infinity. For these discrete distributions, the relationship between p and G is given by

$$G(x) = \sum_{i=0}^{x} p(i)$$

Using the distribution function G for the two-coin experiment, we can write the probability of one or fewer tails as

$$P\{\text{one or fewer tails}\} = P[X \leq 2] = G(2) = p(0) + p(1) + p(2)$$

Expectations and Moments

The probability density and mass functions can be used to compute the **expectation** of a random variable, which is also known as the **first moment, mean,** or **average** and is often denoted by μ. For a discrete random variable X defined from zero to infinity with probability mass function p, the **expected value** of X, frequently written $E[X]$, is given by

$$\mu = E[X] = p(1) + 2p(2) + 3p(3) + \cdots = \sum_{x=0}^{\infty} xp(x)$$

For a continuous random variable with density g, the expected value is defined analogously as

$$\mu = E[X] = \int_0^{\infty} xg(x)\,dx$$

Note that it follows from these definitions that the mean of the sum of random variables is the sum of their means. For example, if X and Y are random variables of any kind (e.g., discrete or continuous, independent or not), then

$$E[X + Y] = E[X] + E[Y]$$

In addition to computing the expectation, one can compute the expected value of virtually any function of a random variable, although only a few are commonly used. The most important function of a random variable, which measures its **dispersion** or spread, is $(X - E[X])^2$. Its expectation is called the **variance,** usually denoted as σ^2, and is given by

$$\sigma^2 = E[(X - E[X])^2] = E[X^2 - 2XE[X] - E[X]^2] = E[X^2] - E[X]^2$$
$$= \sum_{x=0}^{\infty} x^2 p(x) - \mu^2$$

for the discrete case and by

$$\sigma^2 = E[(X - E[X])^2] = E[X^2] - E[X]^2$$
$$= \int_0^{\infty} x^2 g(x)\,dx - \mu^2$$

for the continuous case. The **standard deviation** is defined as the square root of the variance. Note that the standard deviation has the same units as the mean and the random variable itself.

In Chapters 8 and 9, both the mean and the standard deviation are used extensively to describe many important random variables associated with manufacturing systems (e.g., capacity, cycle time, and quality).

Conditional Probability

Beyond simply characterizing the likelihood of individual events, it is often important to describe the dependence of events on one another. For example, we might ask, What is the probability that a machine is out of adjustment *given* it has produced three bad parts in a row? Questions like these are addressed via the concept of **conditional probability.**

The conditional probability that event E_1 occurs, given event E_2 has occurred, written $P[E_1|E_2]$, is defined by

$$P[E_1|E_2] = \frac{P[E_1 \text{ and } E_2]}{P[E_2]}$$

To illustrate this concept, consider the following questions related to the experiment with two coins: What is the probability of two heads, given the first coin is a head? and What is the probability of two heads, given there is at least one head?

To answer the first question, let E_1 be the event "two heads" and let E_2 be the event "the first coin is a head." Note that the event "E_1 and E_2" is equivalent to the event E_1 (the only way to have two heads *and* the first coin to be a head is to have two heads). Hence,

$$P[E_1 \text{ and } E_2] = P[E_1] = \tfrac{1}{4}$$

Since there are two ways for the first coin to be a head [(H, H) and (H, T)], the probability of E_2 is one-half, so

$$\begin{aligned} P[E_1|E_2] &= \frac{P[E_1 \text{ and } E_2]}{P[E_2]} \\ &= \frac{P[E_1]}{P[E_2]} \\ &= \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2} \end{aligned}$$

One way to think about conditioning is that the information of knowing an event has occurred serves to reduce the "effective" sample space. In the above example, knowing that "the first coin is a head" eliminates the outcomes (T, H) and (T, T), leaving only (H, H) and (H, T). Since the event "two heads" [(H, H)] corresponds to one-half of the remaining outcomes, its probability is one-half.

To answer the second question, let E_2 be the event "at least one head." Again, the event "E_1 and E_2" is equal to the event E_1 and has probability of one-fourth. However, there are three ways to have at least one head [(H, H), (H, T), and (T, H)], so $P[E_2] = \frac{3}{4}$ and

$$\begin{aligned} P[E_1|E_2] &= \frac{P[E_1 \text{ and } E_2]}{P[E_2]} \\ &= \frac{P[E_1]}{P[E_2]} \\ &= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \end{aligned}$$

This time, knowing that "at least one head" occurred eliminates only the outcome (T, T), which leaves the outcome (H, H) as one of three equally likely outcomes, which therefore has a probability of one-third.

As another example, consider a random experiment involving the tossing of two dice. The sample space of the experiment is given by $\{(d_1, d_2)\}$, where $d_i = 1, 2, \ldots, 6$ is the number of dots on die i. There are 36 different points in the sample space; by symmetry, these are all equally likely.

Now let X be a random variable equal to the sum of the number of spots on the dice. Note that the number of possible values of X is 11 and that these do *not* have equal probability. To compute the probability of any particular value of X, we must count the number of ways it can result (i.e., the number of outcomes making up the event) and divide by the total number outcomes in the sample space. Thus, the probability of rolling a six is found by noting there are five outcomes that result in a 6—{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)}—out of 36 possible outcomes, so $P[X = 6] = \frac{5}{36}$.

Computing the conditional probability of rolling a six given that the first die is three or less is a bit more complicated. Let E_1 be the event "rolling a six" and E_2 be the event "the first die is three or less." The event corresponding to E_1 *and* E_2 corresponds to three outcomes in the sample

space—{(1, 5), (2, 4), (3, 3)}—so that $P[E_1 \text{ and } E_2] = \frac{3}{36} = \frac{1}{12}$. Event E_2 corresponds to 18 outcomes in the sample space

$$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)(2,1),(2,2),(2,3),$$
$$(2,4),(2,5),(2,6)(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\}$$

so $P[E_2] = \frac{18}{36} = \frac{1}{2}$. Thus, the conditional probability of rolling a six given that the first die is three or less is

$$P[E_1|E_2] = \frac{P[E_1 \text{ and } E_2]}{P[E_2]} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$$

Independent Events

Conditional probability allows us to define the notion of **stochastic independence** or, simply, independence. Two events E_1 and E_2 are defined to be **independent** if

$$P[E_1 \text{ and } E_2] = P[E_1]P[E_2]$$

Notice that this definition implies that if E_1 and E_2 are independent and $P(E_2) > 0$, then

$$P[E_1|E_2] = \frac{P[E_1 \text{ and } E_2]}{P[E_2]} = \frac{P[E_1]P[E_2]}{P[E_2]} = P[E_1]$$

Thus, events E_1 and E_2 are independent if the fact that E_2 has occurred does not influence the probability of E_1.

If two events are independent, then the random variables associated with these events are also independent. Independent random variables have some nice properties. One of the most useful is that the expected value of the product of two independent random variables is simply the product of the expected values. For instance, if X and Y are independent random variables with means of μ_x and μ_y, respectively, then

$$E[XY] = E[X]E[Y] = \mu_x\mu_y$$

This is not true in general if X and Y are not independent.

Independence also has important consequences for computing the variance of the sum of random variables. Specifically, if X and Y are independent, then

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$$

Again, this is not true in general if X and Y are not independent.

An important special case of this variance result occurs when random variables X_i, $i = 1, 2, \ldots, n$, are independent and identically distributed (i.e., they have the same distribution function) with mean μ and variance σ^2, and Y, another random variable, is defined as $\sum_{i=1}^{n} X_i$. Then since means are always additive, the mean of Y is given by

$$E[Y] = E\left[\sum_{i=1}^{n} X_i\right] = n\mu$$

Also, by independence, the variance of Y is given by

$$\text{Var}(Y) = \text{Var}\left(\sum_{i=1}^{n} X_i\right) = n\sigma^2$$

Note that the standard deviation of Y is therefore $\sqrt{n}\sigma$, which does not increase with the sample size n as fast as the mean. This result is important in statistical estimation, as we note later in this appendix.

Special Distributions

There are many different types of distribution functions that describe various kinds of random variables. Two of the most important for modeling production systems are the (discrete) Poisson distribution and the (continuous) normal distribution.

The Poisson Distribution. The **Poisson distribution** describes a discrete random variable that can take on values 0, 1, 2, ... The probability mass function (pmf) is given by

$$p(k) = \frac{e^{-\mu}\mu^k}{k!} \qquad k = 0, 1, 2, \ldots$$

and the cumulative distribution function (cdf) is given by

$$G(x) = \sum_{k=0}^{x} p(x)$$

The mean (expectation) of the Poisson is μ, and the standard deviation is $\sqrt{\mu}$. Notice that this implies that the Poisson is a "one-parameter distribution" because specifying the mean automatically specifies the standard deviation.

To illustrate the use of the Poisson pmf and cdf, suppose the number of customers who place orders to a particular plant on any given day is Poisson-distributed with a mean of two. Then the probability of zero orders being placed is given by

$$p(0) = \frac{e^{-2}2^0}{0!} = e^{-2} = 0.135$$

The probability of exactly one order on a given day is

$$p(1) = \frac{e^{-2}2^1}{1!} = e^{-2} \times 2 = 0.271$$

The probability of two or more orders on a given day is one minus the probability of one or fewer orders, which is given by

$$1 - G(1) = 1 - p(0) - p(1) = 1 - 0.135 - 0.271 = 0.594$$

Part of the reason that the Poisson distribution is so important is that it arises frequently in practice. In particular, **counting processes** that are composed of a number of independent counting processes tend to look Poisson. For example, in the situation used for the numerical calculations above, the underlying counting process is the number of customers who place orders. This is made up of the sum of the separate counting processes representing the number of orders placed by individual customers. To be more specific, if we let $N(t)$ denote the total number of orders that have been placed on the plant by time t, we let $N_i(t)$ denote the number of orders placed by customer i by time t (which may or may not be Poisson), and we let M denote the total number of potential customers, then clearly

$$N(t) = N_1(t) + \cdots + N_M(t)$$

As long as M is "large enough" (say 20 or more, the exact number depends on how close the $N_i(t)$ are to Poisson) and the times between counts for processes $N_i(t)$ are independent, identically distributed, random variables for each i, then $N(t)$ will be a Poisson process. (Note that the interarrival times between orders need only be identically distributed for each given customer; they do not need to be the same for different customers. So it is entirely permissible to have customers with different rates of ordering.)

If $N(t)$ is a Poisson process with a rate of λ arrivals per unit time, then the number of arrivals in t units of time is Poisson-distributed with mean λt. That is, the probability of exactly k arrivals in an interval of length t is

$$p(k) = \frac{e^{-\lambda t}(\lambda t)^k}{k!} \qquad k = 0, 1, 2, \ldots$$

This situation arises frequently. The historical application of the Poisson process was in characterizing the number of phone calls to an exchange in a given time interval. Since callers tend to space their phone calls independently of one another, the total number of phone calls received by the exchange over an interval of time tends to look Poisson. For this same reason, many other arrival processes (e.g., customers in a bank or a restaurant, hits on a Web site, demands experienced by a retailer) are well characterized by the Poisson distribution. A related situation of

importance to manufacturing is the number of failures that a machine experiences. Since complex machinery can fail for a wide variety of reasons (e.g., power loss, pump failure, jamming, loss of coolant, and component breakage) and since we do not replace *all* the components whenever one breaks, we end up with a set of components having different times to failure and different ages. Thus, we can think of the failures as "arriving" from a number of different sources. Since these different sources are often independent, the number of failures experienced during a given interval of operating time tends to look Poisson.

The Exponential Distribution

One additional important point about the Poisson distribution is that the times between arrivals in a Poisson process with arrival rate λ are **exponentially distributed.** That is, the time between the kth and $(k+1)$st arrival is a continuous random variable with density function

$$g(t) = \lambda e^{-\lambda t} \qquad \lambda \geq 0$$

and cumulative distribution function

$$G(t) = 1 - e^{-\lambda t} \qquad \lambda \geq 0$$

The mean of the exponential is $1/\lambda$, and the standard deviation is also $1/\lambda$; so, like the Poisson, the exponential is a one-parameter distribution.

To illustrate the relationship between the Poisson and exponential distributions, let us reconsider the previous example in which we had a Poisson process with an arrival rate of two orders per day. The probability that the time until the first order is less than or equal to one day is given by the exponential cdf as

$$G(1) = 1 - e^{(-2)(1)} = 0.865$$

Notice that the probability that the first order arrives within one day is exactly the same as the probability of one or more orders on the first day. This is 1 minus the probability of zero arrivals on the first day, which can be computed using the Poisson probability mass function as

$$1 - p(0) = 1 - 0.135 = 0.865$$

We see that there is a close relationship between the Poisson (which measures the number of arrivals) and exponential (which measures times between arrivals) distributions. However, it is important to keep the two distinct, since the Poisson distribution is discrete and therefore suited to counting processes, while the exponential is continuous and therefore suited to times.

A fascinating fact about the exponential distribution is that it is the *only* continuous distribution that possesses the **memorylessness** property. This property is defined through the **failure rate function,** which is also called the **hazard rate function** and is defined for any random variable X with cdf $G(t)$ and pdf $g(t)$ as

$$h(t) = \frac{g(t)}{1 - G(t)} \tag{2.59}$$

To interpret $h(t)$, suppose that the random variable X has survived for t hours. The probability that it will not survive for an additional time dt is given by

$$\begin{aligned} P[X \in (t, t+dt) | X > t] &= \frac{P[X \in (t, t+dt), X > t]}{P[X > t]} \\ &= \frac{P[X \in (t, t+dt)]}{P[X > t]} \\ &= \frac{g(t)\,dt}{1 - G(t)} \\ &= h(t)\,dt \end{aligned}$$

Hence, if X represents a lifetime, then $h(t)$ represents the conditional density that a t-year-old item will die (fail). If X represents the time until an arrival in a counting process, then $h(t)$ represents the probability density of an arrival given that no arrivals have occurred before t.

A random variable that has $h(t)$ increasing in t is called **increasing failure rate (IFR)** and becomes more likely to fail (or otherwise end) as it ages. A random variable that has $h(t)$ decreasing in t is called **decreasing failure rate (DFR)** and becomes less likely to fail as it ages. Some random variables (e.g., the life of an item that goes through an initial burn-in period during which it grows more reliable and then eventually goes through an aging period in which it becomes less reliable) are neither IFR nor DFR.

Now let us return to the exponential distribution. The failure rate function for this distribution is

$$h(t) = \frac{g(t)}{1 - G(t)} = \frac{\lambda e^{-\lambda t}}{1 - (1 - e^{-\lambda t})} = \lambda$$

which is constant! This means that a component whose lifetime is exponentially distributed grows neither more nor less likely to fail as it ages. While this may seem remarkable, it is actually quite common because, as we noted, Poisson counting processes, and hence exponential interarrival times, occur often. For instance, as we observed, a complex machine that fails due to a variety of causes will have failure events described by a Poisson process, and hence the times until failure will be exponential.

The Normal Distribution

Another distribution that is extremely important to modeling production systems, arises in a huge number of practical situations, and underlies a good part of the field of statistics is the normal distribution. The normal is a continuous distribution that is described by two parameters, the mean μ and the standard deviation σ. The density function is given by

$$g(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}$$

The cumulative distribution function, as always, is the integral of the density function

$$G(x) = \int_{-\infty}^{x} g(y)\, dy$$

Unfortunately, it is not possible to write $G(y)$ as a simple, closed-form expression. But it is possible to "standardize" normal random variables and compute $G(x)$ from a lookup table of the standard normal distribution, as we describe below.

A **standard normal distribution** is a normal distribution with mean zero and standard deviation of one. Its density function is virtually always denoted by $\phi(z)$ and is given by

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

The cumulative distribution function is denoted by $\Phi(z)$ and is given by

$$\Phi(z) = \int_{-\infty}^{z} \phi(y)\, dy$$

There is no closed-form expression for $\Phi(z)$ either, but this function is readily available in lookup tables, such as Table 1 at the end of the book and using functions built into scientific calculators and spreadsheet programs.

The reason that standard normal tables are so useful is that if a random variable X is normally distributed with mean μ and standard deviation σ, then the "standardized" random variable

$$Z = \frac{X - \mu}{\sigma}$$

is normally distributed with mean zero and standard deviation one.

To illustrate how this property can be exploited, suppose a casting process produces castings whose weights are normally distributed with mean 1,000 grams and standard deviation 150 grams.

Let X denote the (random) weight of a given casting. Then the probability that the casting will weigh less than or equal to 850 grams is

$$G(850) = P(X \leq 850) = P\left(\frac{X - 1{,}000}{150} \leq \frac{850 - 1{,}000}{150}\right) = P(Z \leq -1) = \Phi(-1)$$

From a standard normal table we find that $\Phi(-1) = 0.159$. Hence, we would expect 15.9 percent of the castings to have weights less than 850 grams. Similarly, the probability of the casting having a weight greater than 1,150 grams is

$$1 - G(1{,}150) = 1 - P(X \leq 1{,}150) = 1 - P\left(Z \leq \frac{1{,}150 - 1{,}000}{150}\right) = 1 - P(Z \leq 1) = \Phi(1)$$

From a standard normal table, $\Phi(1) = 0.841$, so $1 - \Phi(1) = 0.159$. Notice that this is the same as $\Phi(-1)$. The reason is that the standard normal distribution is symmetric (bell-shaped). Hence, the probability of a random sample one standard deviation or more below the mean is equal to the probability of a random sample one standard deviation or more above the mean.

The probability that a randomly chosen casting weighs between 850 and 1,150 grams is given by $1 - G(1{,}150) - G(850) = 1 - 0.159 - 0.159 = 0.682$. These kinds of calculations are central to **statistical quality control.** For instance, if we were to observe fewer than 68.2 percent of castings in the weight range between 850 grams and 1,150 grams, then this would be a sign that the process was no longer producing castings whose weights are normally distributed with mean 1,000 and standard deviation 150. This could be due to a change in either the mean or the standard deviation in the underlying process. This type of logic can be used to construct **process control charts** for monitoring the behavior of many different types of processes.

A major reason that the normal distribution is so important in practice is that it arises frequently in nature. This is due to the famous **central limit theorem,** which states (roughly) that the sum of a sufficiently large number (say, greater than 30) of random variables will be normally distributed.

To illustrate this, suppose we measure the times between arrivals of phone calls to an exchange. From our discussion of the Poisson distribution, we know that these times are likely to be exponentially distributed. The exponential is very different from the normal, as we can see from the density functions. The normal density is a symmetric, bell-shaped function with its peak at the mean value μ. The exponential density, on the other hand, is only defined above zero, takes on its maximum value at zero, and declines exponentially above zero. Also, because the exponential always has a standard deviation equal to its mean, while the normal generally has a standard deviation less than its mean, we typically say that exponential random variables are more *variable* than normal random variables. We define a measure of variability and discuss this concept in greater depth in Chapter 8.

But even though the interarrival times between calls are far from normal, the central limit theorem implies that the sum of these times will tend to look normal. That is, if we add 40 interarrival times, which would represent the time until the 40th arrival and repeat this many times to create a histogram, the result will be a bell-shaped curve indistinguishable from that of a normally distributed random variable.

The central limit theorem is fundamental to statistics because in statistics we frequently compute means from data. For instance, if we select N individuals randomly from the population of the United States and measure their heights, then letting X_i represent the (random) height of the ith individual, we see the mean height of the selected group is

$$\bar{X} = \frac{X_1 + \cdots + X_N}{N}$$

If we were to repeat this experiment over and over, we would get different values for the N heights. Hence, the average $\bar{X}$ is itself a random variable. If N is large enough, $\bar{X}$ will be normally distributed. This fact allows us to use the normal distribution to compute the probability that $\bar{X}$ lies within a given interval (i.e., a confidence interval) and make a variety of statistical tests.

Parameters and Statistics

The true probabilities of events (e.g., the probability that a machine will run without breakdown for at least 100 hours) and moments of distributions (e.g., the mean time to process a job) are

parameters of the system. These are typically known only to God. We mere humans can only compute **estimates** of the true values of parameters. This is the basic task of the field of **statistics.**

To estimate a parameter, we take a **random sample,** which represents a collection of independent, identically distributed random variables from a given **population.**[12] For instance, since we cannot measure the hardness of every point on a piece of bar stock, we take a sample of measurements to give us an indication of the true hardness.

A **statistic** is simply a function of a random sample that can be computed (i.e., it has no unknown parameters). Two common statistics (also called **estimators**) are the **sample mean** and the **sample variance** of a random variable. Consider a sample of n independent and identically distributed random variables X_i, $i = 1, 2, \ldots, n$, each with mean μ and variance σ^2. The sample mean $\bar{X}$ is given by the average of the observations, computed as

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

Note that the sample mean is itself a random variable. The mean of $\bar{X}$ is also μ. Estimators, such as $\bar{X}$, whose expectation is equal to the value of the parameter being estimated are called **unbiased** estimators. Because the X_i are independent, the variance of $\bar{X}$ is given by

$$\text{Var}(\bar{X}) = \text{Var}\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}\text{Var}\left(\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}n\sigma^2 = \frac{\sigma^2}{n}$$

Hence, while the variance of any single observation is σ^2, the variance of the mean of n observations is σ^2/n (so the standard deviation is $\sigma/\sqrt{n}$). Since this variance decreases with n, the implication is that larger samples yield better (i.e., tighter) estimates of the true population mean.

This notion is formalized by the concept of a **confidence interval.** The $(1 - \alpha)$ percent confidence interval for the true mean of the population (i.e., the interval in which we expect the sample mean to lie $(1 - \alpha)$ percent of the time if we estimate it over and over) is given by

$$\bar{X} \pm \frac{z_{\alpha/2}\sigma}{\sqrt{n}}$$

where $z_{\alpha/2}$ is the value in the standard normal table such that $\Phi(z_{\alpha/2}) = 1 - \alpha/2$. Notice that as n grows larger, this interval becomes tighter, meaning that more data yield better estimates.

The above confidence interval assumes that the population variance is known with certainty. But in general the variance is also unknown and hence must itself be estimated. This is done by computing the **sample variance** s^2 which is an unbiased estimator for the true variance and is given by

$$s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

or, in a form that is easier to compute, by

$$s^2 = \frac{\sum_{i=1}^{n} X_i^2 - n\bar{X}^2}{n-1}$$

The confidence interval for the population mean becomes

$$\bar{X} \pm \frac{t_{\alpha/2;n-1}s}{\sqrt{n}}$$

where $t_{\alpha/2;n-1} > z_{\alpha/2}$, so that the confidence interval is wider due to the uncertainty introduced by having to estimate the variance. However, as n grows large, $t_{\alpha/2;n-1}$ converges to $z_{\alpha/2}$; so for large sample sizes the two confidence intervals are essentially the same.

For example, suppose we wish to characterize the process times of a new machine. The first job takes 90 minutes of run time, the second job 40 minutes, and the third job 110 minutes. Based

[12] In a sense, the job of the field of *statistics* is the reverse of that of the field of *probability*. In statistics we use samples to estimate properties of a population. In probability we use properties of the population to describe the likelihood of samples.

on these data, we estimate the mean process time to be $\bar{X} = (90 + 40 + 110)/3 = 80$ hours. Similarly, the estimate of the variance is $s^2 = 1{,}300$ (so $s = \sqrt{1{,}300} = 36.06$). For this particular case (assuming the run times are normally distributed), it turns out that $t_{\alpha/2;n-1} = t_{0.05;2} = 2.92$, so the 90 percent confidence interval for the true mean time between outages is given by

$$\bar{X} \pm \frac{t_{\alpha/2;n-1}s}{\sqrt{n}} = 80 \pm \frac{2.92(36.06)}{\sqrt{3}} = 80 \pm 60.78$$

Not surprisingly, with only three observations, we do not have much confidence in this estimate.

In this book we are primarily interested in how systems behave as a function of their parameters (e.g., mean process time, variance of process time) and thus will assume we know these exactly. We caution the reader, however, that in practice one must use estimates of the true parameters. Often, these estimates are not very good, so collecting more data is an important part of the analysis.

Appendix 2B
Inventory Formulas

Poisson Demand Case

If demand during replenishment lead time is Poisson-distributed with mean θ, then the probability mass function (pmf) and cumulative distribution function (cdf) are given by $p(x)$ and $G(x)$, respectively, where

$$p(x) = \frac{e^{-\theta}\theta^x}{x!} \qquad x = 0, 1, 2, \ldots \tag{2.60}$$

$$G(x) = \sum_{k=0}^{x} p(k) \qquad x = 0, 1, 2, \ldots \tag{2.61}$$

These are the basic building blocks of all the performance measures. They can be easily entered as formulas in a spreadsheet, or in some spreadsheets they are actually built in. For example, in Excel

$$p(x) = \text{POISSON}(x, \theta, \text{ FALSE})$$
$$G(x) = \text{POISSON}(x, \theta, \text{ TRUE})$$

Here θ represents the mean, and TRUE and FALSE are used to toggle between the cdf and the pmf. We caution the reader, however, that the Poisson functions in Excel are not always stable for large x, because the formula for $p(x)$ involves the ratio of two large numbers. When θ is large (and hence the reorder point r is likely to be large), it is often safer to use the normal distribution (formulas) with mean θ and standard deviation $\sqrt{\theta}$.

By using the $G(x)$ function, it is simple to compute the fill rate for the base stock model with base stock level R as

$$S(R) = G(R - 1) \tag{2.62}$$

Next we compute the loss function $B(R)$, which represents the average backorder level in a base stock model with base stock level R. Alternatively, $B(R)$ can be interpreted as the expected amount by which lead-time demand exceeds R. It can be written in various forms, including

$$\begin{aligned} B(R) &= \sum_{x=R}^{\infty}(x - R)p(x) \\ &= \theta - \sum_{x=0}^{R-1}[1 - G(x)] \\ &= \theta p(R) + [\theta - R)(1 - G(R)] \end{aligned} \tag{2.63}$$

The last form is the most convenient for use in spreadsheets, since it can be computed without the use of any sums but is correct only for the Poisson case.

Using $B(R)$, we can compute the average inventory level $I(R)$ for the base stock model as a function of the base stock level R as

$$I(R) = R - \theta + B(R) \tag{2.64}$$

Now we turn to the performance measures for the (Q, r) model under the assumption of Poisson demand. As we observed in Section 2.4.3, the inventory position in the (Q, r) model is uniformly spread over the values between $r + 1$ and $r + Q$, which enables us to compute the fill rate by averaging the base stock fill rates for these levels as follows:

$$\begin{aligned} S(Q, r) &= \frac{1}{Q} \sum_{x=r+1}^{r+Q} G(x-1) \\ &= \frac{1}{Q} \sum_{x=r}^{r+Q-1} G(x) \\ &= 1 - \frac{1}{Q}[B(r) - B(r+Q)] \end{aligned} \tag{2.65}$$

The last form, which expresses the fill rate in terms of the $B(x)$ function, is the most convenient for use in a spreadsheet, since it does not require compution of a sum.

We can use the same type of argument to compute the backorder level for the (Q, r) model as the average of the backorder levels of the base stock model over the inventory positions from $r + 1$ to $r + Q$:

$$B(Q, r) = \frac{1}{Q} \sum_{x=r+1}^{r+Q} B(x) \tag{2.66}$$

However, we can write this in a simpler form by defining the following function:

$$\begin{aligned} \beta(x) &= \sum_{k=x+1}^{\infty} B(k) \\ &= \tfrac{1}{2}\{[(x-\theta)^2 + x][1 - G(x)] - \theta(x-\theta)p(x)\} \end{aligned} \tag{2.67}$$

with the last expression holding only for the Poisson case. The function $\beta(x)$ is sometimes referred to as a **second-order loss function,** since it represents the sum of the first-order loss function $B(k)$ above level x. Using the second form for $\beta(x)$ makes this expression simpler to compute in a spreadsheet. Using $\beta(x)$, we can express the backorder level for the (Q, r) model as

$$B(Q, r) = \frac{1}{Q}[\beta(r) - \beta(r+Q)] \tag{2.68}$$

Finally, once we have $B(Q, r)$, it is simple to compute the average inventory level in the (Q, r) model as

$$I(Q, r) = \frac{Q+1}{2} + r - \theta + B(Q, r) \tag{2.69}$$

Normal Demand Case

If demand during replenishment lead time is approximated by a normal distribution with mean θ and standard deviation σ, then the probability density function and cumulative distribution function are given by $g(x)$ and $G(x)$, respectively, where

$$g(x) = \frac{1}{\sigma}\phi(z) \tag{2.70}$$

$$G(x) = \Phi(z) \tag{2.71}$$

and $$z = \frac{x - \theta}{\sigma}$$

and ϕ and Φ represent the pdf and cdf, respectively, of the standard normal distribution, which are tabulated in any standard normal table and are included as standard functions in most spreadsheet programs. For example, in Excel,

$$\phi(z) = \text{NORMDIST}(z, 0, 1, \text{FALSE})$$
$$\Phi(z) = \text{NORMDIST}(z, 0, 1, \text{TRUE})$$

Here, the zero and one represent the mean and standard deviation, respectively, of the standard normal distribution and the inputs TRUE and FALSE are used to toggle between the cdf and pdf.

We can compute the fill rate for the base stock model with base stock level R as

$$S(R) = G(R) \tag{2.72}$$

Notice that this differs from the fill rate expression in the Poisson demand case. The reason is that the normal distribution is continuous. So while the fill rate is still given by $P(X < R)$, where X represents the (random) demand during the replenishment lead time, $P(X < R) = P(X \leq R) = G(R)$, because there is no mass at the point $X = R$ for a continuous distribution. When demand is modeled using a discrete distribution, there *is* probability mass at $X = R$ and hence $P(X < R) = P(X \leq R - 1) = G(R - 1)$.

The expected backorder level in the base stock model with base stock level R is given by

$$\begin{aligned} B(R) &= \int_R^\infty (x - R)g(x)\,dx \\ &= (\theta - R)[1 - \Phi(z)] + \sigma\phi(z) \end{aligned} \tag{2.73}$$

where $z = (R - \theta)/\sigma$. The second form is obviously better suited to use in a spreadsheet since it does not have an integral.

Using $B(R)$, we can compute the average inventory level as

$$I(R) = R - \theta + B(R) \tag{2.74}$$

Now we turn to the performance measures for the (Q, r) model under the assumption of normal demand. As we did for the Poisson case, we can compute the fill rate and backorder level by averaging these measures from the base stock case. However, since the normal distribution is continuous, we must average over the range from r to $r + Q$ (instead of from $r + 1$ to $r + Q$) and we must use an integral instead of a sum. For the fill rate, this yields

$$\begin{aligned} S(Q, r) &= \frac{1}{Q}\int_r^{r+Q} G(x)\,dx \\ &= 1 - \frac{1}{Q}[B(r) - B(r + Q)] \end{aligned} \tag{2.75}$$

Since we have a simple form for the $B(x)$ function, the second form of the above is easily computed in a spreadsheet.

We can do the same type of averaging to get an expression for the average backorder level

$$B(Q, r) = \frac{1}{Q}\int_r^{r+Q} B(x)\,dx \tag{2.76}$$

However, this is not simple to evaluate in a spreadsheet, since it involves an integral. We can simplify it by defining the continuous analog to the second-order loss function $\beta(x)$ as

$$\begin{aligned} \beta(x) &= \int_x^\infty B(y)\,dy \\ &= \frac{\sigma^2}{2}\{(z^2 + 1)[1 - \Phi(z)] - z\phi(z)\} \end{aligned} \tag{2.77}$$

where $z = (x - \theta)/\sigma$. This allows us to simplify the expression for $B(Q, r)$ to

$$B(Q, r) = \frac{1}{Q}[\beta(r) - \beta(r + Q)] \tag{2.78}$$

Finally, we can express the average inventory level as

$$I(Q, r) = \frac{Q}{2} + r - \theta + B(Q, r) \tag{2.79}$$

Notice that this differs from the average inventory level in the Poisson case by a quantity of one-half. The reason for this is that we are using a continuous model of demand, which views the decline of inventory as smooth rather than in unit steps. Since almost all real-world systems involve discrete inventory, it generally makes sense to use the discrete inventory formula (2.69) even when using a continuous model to compute Q and r.

We conclude by reiterating that *all* the formulas are straightforward to compute in a spreadsheet. Therefore, once we have computed R for a base stock model or Q and r for a (Q, r) model using any heuristic—either one of those suggested in this chapter or another one—we can compute the exact performance that will result by using the formulas in this appendix. While approximate objective functions can be useful for purposes of computing the decision variables, there is *no excuse* for using approximate measures in reporting the resulting performance or for checking these against desired target values.

Study Questions

1. Harris, in the original 1913 paper on the EOQ model, suggested that "most managers, indeed, have a rather hazy idea as to just what this [setup] cost amounts to."
   a. Do you think that setup cost, as defined in the EOQ model, is more easily specified today than in 1913? Why or why not?
   b. Give some examples of costs that might make up this setup cost.
   c. What might setup cost in the model actually be serving as a surrogate for in the real system?
2. Analogous to item 1*c* above, what might inventory carrying cost in the EOQ model serve as a surrogate for in the real system? With this in mind, comment on the suggestion (once fairly common in textbooks) that "a charge of 10 percent on stock is a fair one to cover both interest and depreciation." What is another name for this "charge"?
3. Harris wrote that "higher mathematics" is required to solve the EOQ model. What is the name of this branch of mathematics? Who invented it and when? When do most Americans study this subject in the current educational system? Was this really "higher mathematics" in 1913?
4. Consider the following situations. Label them as either A for appropriate or L for less appropriate for application of the EOQ model.
   a. Automobile manufacturer ordering screws from a vendor
   b. Automobile manufacturer deciding on how many cars to paint per batch of a particular color
   c. A job shop ordering bar stock
   d. Office ordering copier paper
   e. A steel company deciding how many slabs to move at once between the casting furnace and the rolling mill

5. A basic modeling assumption underlying the EOQ model is constant and level demand over the infinite time horizon. Of course, this is never satisfied exactly in practice. What options does one have for lot sizing in the face of nonconstant demand?
6. What is the key difference in the modeling assumptions between the EOQ and the Wagner–Whitin models?
7. Does the Wagner–Whitin property offer a fundamental insight into plant behavior? If so, what is it? What problems are there with this property as a guide for manufacturing practice?
8. Give at least three criticisms of the validity of the Wagner–Whitin model.
9. What is the key difference between the EOQ model and the (Q, r) model? Between the base stock model and the (Q, r) model?
10. Why is the statement "The reorder point r affects customer service, while the replenishment quantity Q affects replenishment frequency" true in rough terms but not precisely true?
11. Why does increasing the variability of the demand process tend to require a higher level of safety stock (i.e., a higher reorder point)?
12. Suppose you are stocking parts purchased from vendors in a warehouse. How could you use a (Q, r) model to determine whether a vendor of a part with a higher price but a shorter lead time is offering a good deal? What other factors should you consider in deciding to change vendors?
13. In a multiproduct reorder point problem subject to an aggregate service constraint, what will be the effect of increasing the cost of one of the parts on the fill rate of that part? On the fill rates of the other parts?
14. A man was discovered trying to carry a bomb onto an airplane. When he was removed, his excuse was: "Everyone knows that the probability of there being a bomb on an airplane is extremely low. Imagine how low the probability of *two* bombs on the airplane must be! I had no intention of blowing up the plane. By carrying a bomb on board, I was only trying to make it safer!"

    What do you think of the man's reasoning? (*Hint:* Use conditional probability.)

Problems

1. Perform the two-coin toss experiment discussed in Appendix 2A by flipping two coins (a penny and a nickel) 50 times and recording the outcome (H or T for each coin) for each flip.
    a. Estimate the probability of two heads given at least one head by counting the number of (H, H) outcomes and dividing by the number of outcomes that have at least one head. How does this compare to the true value of one-third computed in Appendix 2A?
    b. Estimate the probability of two heads given that the penny is a head by counting the number of (H, H) outcomes and dividing by the number of outcomes for which the penny is a head. How does this compare to the true value of one-half computed in Appendix 2A?
2. Recall the game show "Let's Make a Deal." You are a contestant and there is a fabulous prize behind door number 1, door number 2, or door number 3. You have chosen door number 1. The host of the show opens door number 3 revealing a not-so-fabulous prize, and asks you if you want to change your mind. You have watched the show for a number of years and have noticed that the host always offers contestants the option of switching doors. Moreover, you know that when the host has a choice of doors to open (e.g., the prizes behind both doors 2 and 3 are duds), he chooses randomly. Should you switch to door 2 or stick with door 1 in order to maximize your chances of winning the fabulous prize?

3. A gift shop sells Little Lentils—cuddly animal dolls stuffed with dried lentils—at a very steady pace of 10 per day, 310 days per year. The wholesale cost of the dolls is $5, and the gift shop uses an annual interest rate of 20 percent to compute holding costs.
   a. If the shop wants to place an average of 20 replenishment orders per year, what order quantity should it use?
   b. If the shop orders dolls in quantities of 100, what is the implied fixed order cost?
   c. If the shop estimates the cost of placing a purchase order to be $10, what is the optimal order quantity?
4. Quarter-inch stainless-steel bolts, one and one-half inches long are consumed in a factory at a fairly steady rate of 60 per week. The bolts cost the plant two cents each. It costs the plant $12 to initiate an order, and holding costs are based on an annual interest rate of 25 percent.
   a. Determine the optimal number of bolts for the plant to purchase and the time between placement of orders.
   b. What is the yearly holding and setup cost for this item?
   c. Suppose instead of small bolts we were talking about a bulky item, such as packaging materials. What problem might there be with our analysis?
5. Reconsider the bolt example in Problem 4. Suppose that although we have estimated demand to be 60 per week, it turns out that it is actually 120 per week (i.e., we have a 100 percent forecasting error).
   a. If we use the lot size calculated in the previous problem (i.e., using the erroneous demand estimate), what will the setup plus holding cost be under the true demand rate?
   b. What would the cost be if we had used the optimum lot size?
   c. What percentage increase in cost was caused by the 100 percent demand forecasting error? What does this tell you about the sensitivity of the EOQ model to errors in the data?
6. Consider the bolt example from Problem 4 yet again, assuming that the demand of 30 per week is correct. Now, however, suppose the minimum reorder interval is one month and all order cycles are placed on a power-of-2 multiple of months (that is, one month, two months, four months, eight months, etc. in order to permit truck sharing with orders of other parts.
   a. What is the least-cost reorder interval under this restriction?
   b. How much does this add to the total cost?
   c. How is the effectiveness of powers-of-2 order intervals related to the result of the previous problem regarding the effect of demand forecasting errors?
7. Danny Steel, Inc., fabricates various products from two basic inputs, bar stock and sheet stock. Bar stock is used at a steady rate of 1,000 units per year and costs $200 per bar. Sheet stock is used at a rate of 500 units per year and costs $300 per sheet. The company uses a 20 percent annual holding cost rate, and the fixed cost to place an order is $50, of which $10 is the cost of placing the purchase order and $40 is the fixed cost of a truck delivery. The variable (i.e., per unit charge) trucking cost is included in the unit price. The plant runs 365 days per year.
   a. Use the EOQ formula with the full fixed order cost of $50 to compute the optimal order quantities, order intervals, and annual cost for bar stock and sheet stock. What fraction of the total annual (holding plus order) cost consists of fixed trucking cost?
   b. Using a week (seven days) as the base interval, round the order intervals for bar stock and sheet stock to the nearest power of 2. If you charge the fixed trucking fee only once for deliveries that coincide, what is the annual cost now?
   c. Leave the order quantity for bar stock as in part *b*, but reduce the order interval for sheet stock to match that of bar stock. Recompute the total annual cost and compare to part *b*. Explain your result.
   d. Based on your observation in part *c*, propose an approach for computing a replenishment schedule in a multiproduct environment like this, where part of the fixed order cost corresponds to a fixed trucking fee that is only paid once per delivery regardless of how many different parts are on the truck.

8. Consider the following table resulting from lot sizing by the Wagner–Whitin algorithm:

Month	Demand	Min. Cost	Order Period
1	69	85	1
2	29	114	1
3	36	186	1
4	61	277	3
5	61	348	4
6	26	400	4
7	34	469	5
8	67	555	8
9	45	600	8
10	67	710	10
11	79	789	10
12	56	864	11

   a. Develop the "optimal" ordering schedule.
   b. What will the schedule be if your planning horizon was only six months?

9. Nozone, Inc., a manufacturer of Freon recovery units (for automotive air conditioner maintenance), experiences a strongly seasonal demand pattern, driven by the summer air conditioning season. This year Nozone has put together a six-month production plan, where the monthly demands D_t for recovery units are given in the table below. Each recovery unit is manufacured from one chassis assembly plus a variety of other parts. The chassis assemblies are produced in the machining center. Since there is a single chassis assembly per recovery unit, the demands in the table below also represent demands for chassis assemblies. The unit cost, fixed setup cost, and monthly holding cost for chassis assemblies are also given in this table. The fixed setup cost is the firm's estimate of the cost to change over the machining center to produce chassis assemblies, including labor and materials cost and the cost of disruption of other product lines.

t	1	2	3	4	5	6
D_t	1,000	1,200	500	200	800	1,000
c_t	50	50	50	50	50	50
A_t	2,000	2,000	2,000	2,000	2,000	2,000
h_t	10	10	10	10	10	10

   a. Use the Wagner–Whitin algorithm to compute an "optimal" six-month production schedule for chassis assemblies.
   b. Comment on the appropriateness of using monthly planning periods. What factors should influence the choice of a planning period?
   c. Comment on the validity of using a fixed order cost to consider the capacity constraint at the machining center.

10. YB Sporting Apparel prints up novelty T-shirts commemorating major sports events (e.g., the Super Bowl, the World Series, Northwestern University winning the NCAA Basketball Tournament). The T-shirts cost $5 to make and distribute and sell for $20. Company policy is to dispose of any excess inventory after the event by discounting the T-shirts by 80 percent, that is, sell for $4. In 1994, YB printed shirts for the World Cup soccer playoffs in Chicago. It estimated demand at 12,000 shirts, with a significant amount of uncertainty. Because of this uncertainty, YB printed only 10,000 shirts. What do you think of this decision? What quantity would you have recommended printing?

11. Slaq Computer Company manufactures notebook computers. The economic lifetime of a particular model is only four to six months, which means that Slaq has very little time to make adjustments in production capacity and supplier contracts over the production run. For a soon-to-be-introduced notebook, Slaq must negotiate a contract with a supplier of motherboards. Because supplier capacity is tight, this contract will specify the number of motherboards in advance of the start of the production run. At the time of contract negotiation, Slaq has forecasted that demand for the new notebook is normally distributed with a mean of 10,000 units and a standard deviation of 2,500 units. The net revenue from a notebook sale is \$500 (note that this includes the cost of the motherboard, as well as all other material, production, and shipping costs). Motherboards cost \$200 and have no salvage value (i.e., if they are not used for this particular model of notebook, they will have to be written off).
    a. Use the news vendor model to compute a purchase quantity of motherboards that balances the cost of lost sales and the cost of excess material.
    b. Comment on the appropriateness of the news vendor model for this capacity planning situation. What factors are not considered that might be important?
12. Chairish-Is-The-Word, Inc., manufactures top-end hardwood chairs that are sold through a variety of retail outlets. The most popular model sells (wholesale) for \$400 per chair and costs \$300 to make. Past data show that average monthly demand is 1,000 chairs with a standard deviation of 200 chairs and that the normal distribution is a reasonable fit. CITW uses a 20 percent annual interest charge to estimate inventory carrying costs, so that the cost to carry one chair in stock for one month is $\$300(0.20)/12 = \$5$.
    a. If all orders are backlogged and the cost of lost customer goodwill from carrying a single chair on backorder is \$20, what order-up-to (base stock) level should CITW use?
    b. If any order not filled from stock is lost (i.e., the customer buys it from the competition), what order-up-to level should CITW use?
    c. Explain the reason for the difference between your answers in parts *a* and *b*.
13. Jill, the office manager of a desktop publishing outfit, stocks replacement toner cartridges for laser printers. Demand for cartridges is approximately 100 per year and is quite variable (i.e., can be represented using the Poisson distribution). Cartridges cost \$100 each. Jill uses a (Q, r) approach to control stock levels.
    a. If Jill wants to restrict replenishment orders to twice per year on average, what batch size Q should she use? If she wants to ensure a service level (i.e., probability of having the cartridge in stock when needed) of at least 98 percent, what reorder point r should she use? (*Hint:* Use Table 2.6.)
    b. If Jill is willing to increase the number of replenishment orders per year to six, how do Q and r change? Explain the difference in r.
    c. If the supplier of toner cartridges offers a quantity discount of \$10 per cartridge for orders of 50 or more, how does this affect the relative attractiveness of ordering twice per year versus six times per year? Try to frame your answer in definite economic terms.
14. Moonbeam-Musel (MM), a manufacturer of small appliances, has a large injection molding department. Because MM's CEO, Crosscut Sal, is a stickler for keeping machinery running, the company stocks quick-change replacement modules for the two most common types of failure. Type A modules cost \$150 each and have been used at a rate of about seven per month, while type B modules cost \$15 and have been used at a rate of about 30 per month, and for simplicity we assume a month is 30 days. Both modules are purchased from a supplier; replenishment lead times are one month and one-half month (15 days) for modules 1 and 2, respectively.
    a. Suppose MM wishes to follow a base stock policy. Assuming that demand is Poisson-distributed, what should the base stock levels be for type A and type B modules in order to ensure a fill rate of at least 98 percent for each module? What are the expected backorder level and the expected inventory level (in dollars)?
    b. Suppose MM estimates the cost to place a replenishment order (regardless of type) to be \$5 and the holding cost interest rate to be three percent per month. Use the EOQ model to compute order quantities (where the EOQ values are rounded to the nearest integer to get

Q). Using these order quantities, what should the reorder points be to achieve a 98 percent fill rate for both modules? How do these reorder points and the resulting average backorder level and inventory level compare to those in part *a*? Explain any difference.

c. Suppose MM estimates the cost per month per unit of backorder to be \$15. Use approximation (2.49) to compute reorder points for type A and type B modules (again rounding to the nearest integer). Using the order quantities from part *b* along with these new reorder points, compare the average total inventory, backorder level, and fill rate with those in part *b*. Comment on any difference. (Note that the average fill rate is computed by $(D_1S_1 + D_2S_2)/(D_1 + D_2)$, where D_1, D_2 are the monthly demand rates and S_1, S_2 are the fill rates for type A and type B components, respectively.)

d. Recompute the reorder points as in part *c*, but this time assume that replenishment lead times are variable with standard deviations of 7 and 15 days for type A and type B modules, respectively. How much of an effect does this have on the reorder points?

15. Walled-In Books stocks the novel *War and Peace*. Demand averages 15 copies per month, but is quite variable (i.e., is well represented by a Poisson distribution). Replenishments from the publisher require a two-week lead time. The wholesale cost is \$12, and Walled-In uses a weekly holding cost rate of one-half percent. It also estimates that the fixed cost of placing and receiving a replenishment order is \$5.

a. Compute the approximately optimal order quantity, using the EOQ formula and rounding to the nearest integer. Using this order quantity, find the reorder point that makes the fill rate at least 90 percent. Compute the resulting average inventory (in dollars).

b. Using the order quantity computed in part *a*, find the reorder point that makes the type I approximation of fill rate at least 90 percent. Compute the true fill rate and inventory level resulting from this reorder point and compare to the values in part *a*. What does this say about the accuracy of the type I service approximation?

c. Using the order quantity computed in part *a*, find the reorder point that makes the type II approximation of fill rate at least 90 percent. Compute the true fill rate and inventory level resulting from this reorder point, and compare to the values in part *a*. What does this say about the accuracy of the type II service approximation? How does the value of Q affect the accuracy of the type II approximation?

d. Cut the order quantity from part *a* in half, and compute the reorder point needed to make fill rate at least 90 percent. How does the resulting inventory compare to that from part *a*? Does this imply that the EOQ approximation is poor? Why or why not?

C H A P T E R

3 THE MRP CRUSADE

Unlike many other approaches and techniques, material requirements planning "works," which is its best recommendation.

Joseph Orlicky, 1974

3.1 Material Requirements Planning—MRP

By the early 1960s, many companies were using digital computers to perform routine accounting functions. Given the complexity and tedium of scheduling and inventory control, it was natural to try to extend the computer to these functions as well. One of the first experimenters in this area was IBM, where Joseph Orlicky and others developed what came to be called **material requirements planning (MRP).** Although it started slowly, MRP got a tremendous boost in 1972 when the American Production and Inventory Control Society (APICS) launched its "MRP Crusade" to promote its use. Since that time, MRP has become the principal production control paradigm in the United States. By 1989, sales of MRP software and implementation support exceeded $1 billion.

Because it is so prevalent, any well-trained manufacturing manager must have some familiarity with how MRP works. Therefore, in this chapter we describe the MRP paradigm and that of its immediate successor, **manufacturing resources planning (MRP II),** as well as its current incarnation, **enterprise resources planning (ERP).** We also highlight the basic insights represented by MRP as well as some difficulties it leaves unresolved.

3.1.1 The Key Insight of MRP

As we noted in Chapter 2, before MRP, most production control systems were based on some variant of statistical reorder points. Essentially this meant that production of any part, finished product, or component was triggered by inventory for that part falling below a specified level. Orlicky and the other originators of MRP recognized that this approach is much better suited to final products than components. The reason is that demand for final products originates outside the system and is therefore subject to uncertainty. However, because components are used to produce final products, demand for components is a function of demand for final products and is therefore *known* for

any given final assembly schedule. Treating the two types of demand equivalently, as is done in a statistical reorder point system, ignores the dependence of component demand on final product demand and therefore leads to inefficiencies in scheduling production.

Any demand that originates outside the system is called **independent demand.** This includes all demand for final products and possibly some demand for components (e.g., when they are sold as replacement parts). **Dependent demand** is demand for components that make up independent demand products. Using these terms, the key insight of MRP can be stated as follows:

> Dependent demand is different from independent demand. Production to meet dependent demand should be scheduled so as to explicitly recognize its linkage to production to meet independent demand.

As we will see, the basic mechanics of MRP do exactly this. By working backward from a production schedule of an independent-demand item to derive schedules for dependent-demand components, MRP adds the link between independent and dependent demand that is missing from statistical reorder point systems. MRP is therefore called a **push** system since it computes schedules of what should be started (or *pushed*) into production based on demand. This is in contrast to **pull** systems, such as Toyota's **kanban** system, that authorize production as inventory is consumed. We will discuss kanban in greater detail in Chapter 4 and provide a more complete comparison of push and pull systems in Chapter 10.

3.1.2 Overview of MRP

The basic function of MRP is revealed by its name—to plan material requirements. MRP is used to coordinate orders from within the plant and from outside. Outside orders are called **purchase orders,** while orders from within are called **jobs.** The main focus of MRP is on scheduling jobs and purchase orders to satisfy material requirements generated by external demand.

MRP deals with two basic dimensions of production control: quantities and timing. The system must determine appropriate production quantities of all types of items, from final products that are sold, to components used to build final products, to inputs purchased as raw materials. It must also determine production timing (i.e., job start times) that facilitates meeting order due dates.

In many MRP systems, time is divided into **buckets,** although some systems use continuous time. A bucket is an interval that is used to break time and demand into discrete chunks. The demand that accumulates over the time interval (bucket) is all considered due at the beginning of the bucket. Thus, if the bucket length is one week and during the third week (bucket) there is demand for 200 units on Monday, 250 on Tuesday, 100 on Wednesday, 50 on Thursday, and 350 on Friday, then demand for the third bucket is 950 units and is due on Monday morning. In the past, when data processing was more expensive, typical bucket sizes were one week or longer. Today, most modern MRP systems use daily buckets, although there are still many systems using weeks.

MRP works with both finished products, or **end items,** and their constituent parts, called **lower-level items.** The relationship between end items and lower-level items is described by the **bill of material (BOM),** as shown in Figure 3.1. Demand for end items generates dependent demand for lower-level items. As we noted above, all demand for end items is independent demand, while typically most demand for lower-level items is dependent demand. However, there can be independent demand for lower-level items in the form of spare parts, parts for research and quality tests, and so on.

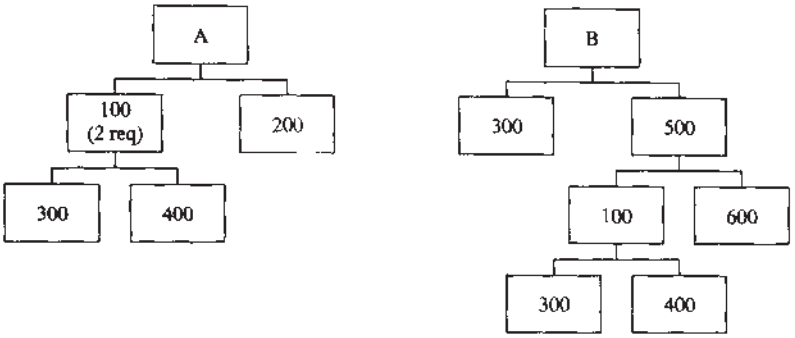


FIGURE 3.1
Two bills of material

To facilitate the MRP processing, each item in the BOM is given a **low-level code (LLC).** This code indicates the lowest level in a bill of material that a particular part is ever used.[1] End items (that are not a part of any other item) have LLCs of zero. A subassembly that is used only by end items has an LLC of one. A component that is used only by subassemblies having an LLC of one will have an LLC of two, and so on. For example, in Figure 3.1 parts A and B are end items with LLCs of zero. Requirements for these parts come from independent demand. At first glance, it might appear that part 100 should have an LLC of one since it is used directly in part A. However, because it is also a component part for part 500 (whose LLC is one), it is assigned an LLC of two. Similarly, since part 300 is required to make part B with an LLC of zero, but is also required to make part 100 that has an LLC of two, it is given an LLC of three.

Most commercial MRP packages include a **BOM processor** that is used to maintain the bills of material and automatically assign low-level codes. Other functions of the BOM processor include generating "goes-into" lists (where parts are used) and BOM printing.

In addition to the BOM information, MRP requires information concerning independent demand, which comes from the **master production schedule (MPS).** The MPS contains **gross requirements,** the current inventory status known as **on-hand** inventory, and the status of outstanding orders (both purchased and manufacturing) known as **scheduled receipts.**

The basic MRP procedure is simple. We will discuss each of the steps in detail. But briefly, for each level in the bill of material, beginning with end items, MRP does the following for each part:

1. *Netting:* Determine **net requirements** by subtracting on-hand inventory and any scheduled receipts from the gross requirements. The gross requirements for level-zero items come from the MPS, while those for lower-level items are the result of previous MRP operations.
2. *Lot sizing:* Divide the netted demand into appropriate **lot sizes** to form jobs.
3. *Time phasing:* Offset the due dates of the jobs with **lead times** to determine start times.
4. *BOM explosion:* Use the start times, the lot sizes, and the BOM to generate gross requirements of any required components at the next level(s).
5. *Iterate:* Repeat these steps until all levels are processed.

[1] Unfortunately, *low*-level codes have the property that the *lower* a part is in the bill of material, the *higher* its *low*-level code.

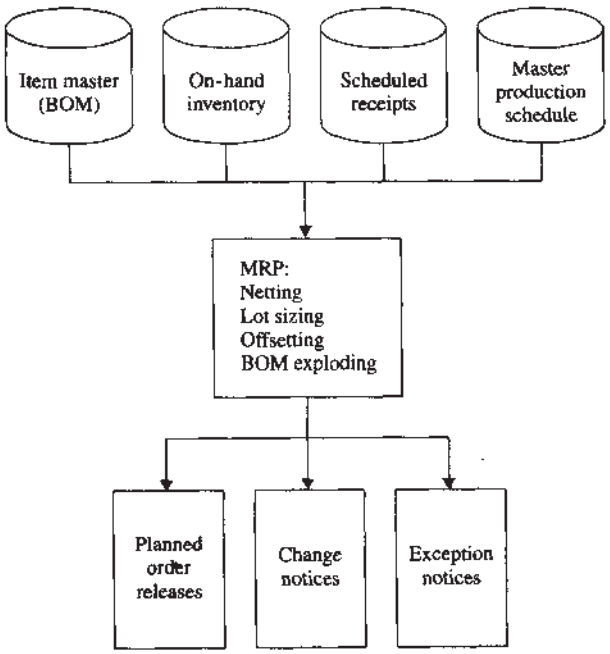


FIGURE 3.2
Schematic of MRP

As each part in the bill of material is processed, requirements are generated for lower levels. MRP processes all parts for one level before beginning the next level. Because of the way low-level codes are defined, doing this generates all the gross demand for a lower-level part before it is processed. We will describe each of these steps in detail in Section 3.1.4. The basic outputs of an MRP system are planned order releases, change notices, and exception reports. These we will define in Section 3.1.3. Figure 3.2 presents a schematic of the overall process.

We now illustrate this procedure with a simple example. Suppose the demand for part A is given by the gross requirements from the following master production schedule:

Part A	1	2	3	4	5	6	7	8
Gross requirements	15	20	50	10	30	30	30	30

Suppose further that there are no scheduled receipts (these are a bit tricky and we will discuss them later) and there are 30 units on hand in inventory. We assume that the lot size for part A is 75 units and the lead time is one week. The MRP processing goes as follows.

Netting. The 30 units on hand will cover all the demand in week one and 15 units left over. The remaining 15 leave five units of the demand of 20 in week two uncovered. Thus, net requirements are as follows:

Part A		1	2	3	4	5	6	7	8
Gross requirements		15	20	50	10	30	30	30	30
Projected on-hand	30	15	−5	—	—	—	—	—	—
Net requirements		0	5	50	10	30	30	30	30

Lot Sizing. The first uncovered demand is in week two. Therefore, the first **planned order receipt** will be in week two for 75 units (the lot size). Since only five units are needed in week two, 70 units are carried over to week three, which has a demand of 50. This leaves 20 for week four, which has a demand of 10. After covering week four, the remainder is insufficient to cover the demand of 30 units in week five. Thus, we need another lot of 75 to arrive at the beginning of week five. After subtracting 30 units, we have 55 available for week six, which also has a demand of 30, leaving 25 for week seven. The 25 units are not sufficient to cover the demand of 30, and so we need another lot of 75 to arrive in week seven. This lot covers both the remaining demand in week seven (five) and the 30 needed in week eight. We show the results of these calculations in the following tableau:

Part A		1	2	3	4	5	6	7	8
Gross requirements		15	20	50	10	30	30	30	30
Projected on-hand	30	15	−5	—	—	—	—	—	—
Net requirements		0	5	50	10	30	30	30	30
Planned order receipts			75			75		75	

Time Phasing. To determine when to release the jobs (if made in-house) or purchase orders (if bought from someone else), we simply subtract the lead time from the time of the planned order receipts to obtain the planned order releases. The result using planned lead times of one week is shown below:

Part A		1	2	3	4	5	6	7	8
Gross requirements		15	20	50	10	30	30	30	30
Projected on-hand	30	15	−5	—	—	—	—	—	—
Net requirements		0	5	50	10	30	30	30	30
Planned order receipts			75			75		75	
Planned order releases		75			75		75		

BOM Explosion. Once we have determined start times and quantities for part A, it is a simple matter to generate demand requirements for all its components. For instance, each unit of part A requires two units of part 100. Therefore, gross requirements for part 100 to produce part A are computed by simply doubling the planned order releases for part A. The gross requirements for part 100 generated by part A must be added to those generated by other parts (e.g., part 500) in order to compute the total gross requirements for part 100. As long as we process parts in order (low to high) of their low-level code, we will have accumulated all the gross requirements for each part before processing it.

3.1.3 MRP Inputs and Outputs

The basic inputs to MRP are a forecast of demand for end items, the associated bills of material, and the current inventory status, plus any data needed to specify production policies. These data come from three sources: (1) the item master file, (2) the master production schedule, and (3) the inventory status file.

The Master Production Schedule. The master production schedule is the source of demand for the MRP system. It gives the quantity and due dates for all parts that have independent demand. This will include demand for all end items as well as external demand for lower-level parts (e.g., demand for spare parts).

The minimum information contained in the master production schedule is a set of records containing a part number, a need quantity, and a due date for each purchase order. This information is used by MRP to obtain the gross requirements that initiate the MRP procedure. The MPS typically uses the part number to link to the item master file where other processing information is located.

The Item Master File. The item master file is organized by part number and contains, at a minimum, a description of the part, bill-of-material information, lot-sizing information, and planning lead times.

The BOM data for a part typically list the components and quantities that are directly required to make only that part. The bill-of-material processor uses this information to display complete bills of material for any item, although such detailed information is not needed for MRP processing.

By using low-level codes, MRP accumulates all the demand of a part *before* it processes that part. To see why this is necessary, suppose it were not done. In our example, MRP might process part 100 after processing parts A and B but before processing part 500. If so, it would not have the demand for part 100 generated by part 500. If we go back and schedule more production of part 100, we may wind up with many small jobs of part 100 instead of a few large ones. Several small jobs could easily have the same due date. The result would be a failure to exploit any economies of scale from sharing setups on critical equipment. The use of low-level codes prevents this from happening.

Two other pieces of information needed to perform MRP processing are the **lot-sizing rule (LSR)** and the **planning lead time (PLT).** The LSR determines how the jobs will be sized in order to balance the competing desires of reducing inventory (by using smaller lots) and increasing capacity (by using larger lots to avoid frequent setups). EOQ and Wagner–Whitin, as discussed in Chapter 2, are possible lot-sizing rules. We discuss the use of these and other rules later in this chapter.

The PLT is used to determine job start times. In MRP, this procedure is simple: The start time is equal to the due date minus the PLT. Thus, if the lead times were always precisely equal to the PLTs, MRP would result in parts being ready exactly when needed

(i.e., just in time). However, actual lead times vary and are never known in advance. Thus, deciding what planned lead times to use in an MRP system can be a difficult question and one that we will discuss further, in this chapter and in Chapter 5.

On-Hand Inventory. On-hand inventory data are stored by part number and contain information describing the part, where it is located, and how many are currently on hand. On-hand inventory includes raw material stock, "crib" stock (i.e., inventory that has been processed since being raw material and kept within the plant), and assembly stock. On-hand inventory may also contain information about **allocation** that indicates how many parts are reserved for jobs to be released.

Scheduled Receipts. This file contains all previously released orders, either **purchase orders** or manufacturing **jobs.** A **scheduled receipt (SR)** is a **planned order release** that has actually been released. For purchased parts, this involves executing a purchase order (PO) and sending it to a vendor. For manufactured parts, this entails gathering all necessary routing and manufacturing information, allocating the necessary inventory for the job, and releasing the job to the plant. Once the PO or job has been released, the planned order release is deleted in the database and the scheduled receipt is created. Thus, SRs are jobs and orders resulting from previous MRP runs and either are currently in process or have not yet been received from the vendor. Jobs that have not yet arrived at an inventory location are considered part of **work in process (WIP).** When the job is completed (i.e., it has finished its routing and goes into stock), the scheduled receipt is deleted from the database and the on-hand inventory is updated to reflect the amount of the part that was completed. A corresponding procedure follows the receipt of a purchased part from a vendor.

The minimum information contained for each scheduled receipt is an identifier (PO number or job number), due date, release date, unit of measure, quantity needed, and current quantity. Other information may include price or cost, routing data, vendor data, material requirements, special handling, anticipated ending quantity, anticipated completion date, etc.

Knowledge of on-hand inventory and scheduled receipts is important to determining net requirements. This procedure is often called **coverage analysis,** and it involves determining how much demand is "covered" by current inventory, purchase orders, and manufacturing jobs.

If demands never changed and jobs always finished on time, all existing scheduled receipts would correspond exactly to subsequent requirements. Unfortunately, demands do change and jobs do not always finish on time, and so scheduled receipts sometimes need to be adjusted. Such adjustments are indicated in **change notices,** described below.

MRP Outputs. The output of an MRP system includes planned order releases, change notices, and exception reports. Planned order releases eventually become the jobs that are processed in the plant.

A **planned order release (POR)** contains at least three pieces of information: (1) the part number (there can be only one per POR), (2) the number of units required, and (3) the due date for the job. A job or a POR need not correspond to an individual customer order and, in most cases, will not. Indeed, in a situation where there are many common parts, PORs for common components will often be for many different assemblies, not to mention customers. However, if all jobs finish on their due dates, all customer orders will be filled on time. This is accomplished automatically in the MRP processing that we discuss in detail next.

Change notices indicate modifications of existing jobs, such as changes in due dates or priorities. Moving a due date earlier is called **expediting** while making a due date later is known as **deferring.**

Exception reports, as in any large management information system, are used to notify the users that there are discrepancies between what is expected and what will transpire. Such reports might indicate job count differences, inventory discrepancies, imminently tardy jobs, and the like.

3.1.4 The MRP Procedure

While the basic ideas of MRP are simple, the details can get very messy. In this section we go through the MRP procedure in enough detail to give the reader an idea of the basic workings of most commercial MRP systems. To do this, we make use of the following notation. For each part, define:

D_t = gross requirements (demand) for period t (e.g., a week)

S_t = quantity currently scheduled to complete in period t (i.e., a scheduled receipt)

I_t = projected on-hand inventory for end of period t, where current on-hand inventory is given by I_0

N_t = net requirements for period t

With these we will now describe the four basic steps of MRP: netting, lot sizing, time phasing, and BOM explosion.

Netting. Netting, or coverage analysis, provides two important functions. (1) It adjusts scheduled receipts by expediting those that are currently scheduled to arrive too late and deferring those currently scheduled to arrive too soon, and (2) it computes net demand.

Most MRP systems assume that all SRs will be received before any newly created job can be completed. This makes sense; since SRs are already "on the way," it is unlikely that any new planned order release would be able to "pass" the SR to become available sooner. If an SR is outstanding with a vendor, it should be easier to expedite the existing order than to start a new one. Likewise an SR that is currently in the shop should finish before one that we start now. Therefore, we will assume that coverage will come first from on-hand inventory, second from SRs (regardless of their due date), and finally from new PORs. To compute when the first SR should arrive, we first determine how far into the future the on-hand inventory will cover demand. We compute

$$I_t = I_{t-1} - D_t$$

starting with $t = 1$ and with I_0 equal to current on-hand inventory. We increment t and continue to compute I_t until it becomes less than zero. The period in which this occurs is when the first scheduled receipt should arrive. If the current due date of the first SR is different from this, it should be changed. This will give rise to a change notice indicating a deferral if the SR is to be pushed back and an expedite if it is to be moved forward.[2] Once the SR is changed, the projected on-hand inventory should reflect the change; that is,

$$I_t(\text{after change in SR}) = I_t(\text{before change in SR}) + S_t$$

[2]Of course, this automatic changing of due dates occurs only within the database unless someone acts. The change notices are used to propagate this information to the "expediter" who is responsible for ensuring that a job finishes on its due date. This is all very easy in theory, but many times a job may be expedited to a point where it is impossible to finish on time. Such instances lead to occasions when the data in the MRP database do not reflect the true situation on the shop floor.

where S_t is the quantity of SR that is moved into period t. If I_t remains less than zero, the next SR should also be moved to period t. This is repeated until either I_t becomes nonnegative or there are no more scheduled receipts.

Once the projected on-hand inventory is made nonnegative in period t, we continue the procedure by moving forward in time, computing

$$I_t = I_{t-1} - D_t$$

until again I_t becomes less than zero. We repeat this procedure until either we exhaust the scheduled receipts or we have reached the end of the time horizon. If this happens while there are remaining scheduled receipts, a change notice should be issued to either cancel those orders/jobs or defer them to a very late date, since there is no demand for them at this time. More often we will run out of on-hand inventory and SRs before we have exhausted demand. The demands beyond what the on-hand inventory and the scheduled receipts can cover are the **net requirements.**

Once scheduled receipts have been adjusted, the net requirements are easily determined. Let t^* be the first period with a negative projected on-hand inventory *after* the SRs have been properly adjusted.[3] Then the net requirements will be zero for all the periods prior to t^*, equal to the magnitude of the first negative projected on-hand inventory for period t^*, and equal to the gross requirements for the periods beyond t^*. Using our notation,

$$N_t = \begin{cases} 0 & \text{for } t < t^* \\ -I_t & \text{for } t = t^* \\ D_t & \text{for } t > t^* \end{cases}$$

The net requirements are then used in the lot-sizing procedure.

Before we move on to lot sizing, consider an example to illustrate these coverage analysis procedures. Table 3.1 contains the gross requirements from the master production schedule for part A, three scheduled receipts, and the current on-hand inventory count.

TABLE 3.1 Input Data for Example

Part A		1	2	3	4	5	6	7	8
Gross requirements		15	20	50	10	30	30	30	30
Scheduled receipts		10	10		100				
Adjusted SRs									
Projected on-hand	20								
Net requirements									
Planned order receipts									
Planned order releases									

[3]Notice that if we did not adjust SRs first, this could happen in more than one time period. Then we would have an early net requirement, followed by a scheduled receipt, followed by more net requirements.

We begin by computing the projected on-hand inventory. Starting with 20 units in stock, we subtract 15 for the gross requirements in period 1, leaving five remaining on-hand. Notice we do not consider the SR of 10 in period 1 since we always use on-hand inventory before using scheduled receipts.

Moving to the second period, we see that the gross requirement of 20 exceeds the five in stock, and so we issue a change notice to defer the SR with 10 from period 1 to period 2. However, this still provides only a total of 15 units, five less than what is needed. Therefore we add the second SR to period 2, bringing the total to 25 units. Notice that since this SR is already scheduled for period 2, we do not need to generate a change notice. After adjusting the first two SRs to period 2 and subtracting the gross requirements, we have an on-hand inventory of five. Since this quantity is insufficient to cover the third demand of 50, we issue an expedite notice to change the due date of the third SR from period 4 to period 3, yielding an on-hand inventory of 55. In some systems the job could be split, expediting only that portion which is needed at the earlier date. In this example, however, we expedite the entire job. This more than covers the 10 units in period 4, leaving 45, as well as the 30 in period 5, leaving 15 units. The demand in period 6 exceeds the projected on-hand inventory, and there are no more SRs to be adjusted. Thus, the first uncovered demand occurs in period 6 and is equal to 15. Table 3.2 summarizes the coverage analysis calculations used to generate projected on-hand inventory.

The net requirements are now easily computed, as shown in Table 3.2. For periods 1 through 5 they are zero because projected on-hand inventory is greater than zero. For period 6 they are 15, simply the negative of projected on-hand inventory. For periods 7 and 8 the net requirements are equal to the gross requirements, both of which are 30.

Lot Sizing. Once we have computed the net requirements, we must schedule production quantities to satisfy them. Because MRP assumes demands are deterministic but nonconstant over time, this is exactly the same problem we addressed in Chapter 2 and solved "optimally" using the Wagner–Whitin algorithm. We will discuss this and other lot-sizing techniques in Section 3.1.6. For clarity and to illustrate the basic MRP computations, we restrict our attention at this point to two very simple lot-sizing rules.

TABLE 3.2 Adjusted Scheduled Receipts, Projected On-Hand, and Net Requirements

Part A		1	2	3	4	5	6	7	8
Gross requirements		15	20	50	10	30	30	30	30
Scheduled receipts		10	10		100				
Adjusted SRs			20	100					
Projected on-hand	20	5	5	55	45	15	−15	—	—
Net requirements							15	30	30
Planned order receipts									
Planned order releases									

TABLE 3.3 Planned Order Receipts and Releases

Part A		1	2	3	4	5	6	7	8
Gross requirements		15	20	50	10	30	30	30	30
Scheduled receipts		10	10		100				
Adjusted SRs			20	100					
Projected on-hand	20	5	5	55	45	15	−15	—	—
Net requirements							15	30	30
Planned order receipts							45		30
Planned order releases					45		30		

The simplest lot-sizing rule, known as **lot for lot,** states that the amount to be produced in a period is equal to that period's net requirements. This policy is easier to use than the fixed quantity policy in the example in Section 3.1.2, and is consistent with just-in-time philosophy (see Chapter 4) of making only what is needed.

Another simple rule is known as **fixed order period (FOP),** also sometimes called **period order quantity.** This rule attempts to reduce the number of setups by combining the net requirements of P periods. Note that when $P = 1$, FOP is equivalent to lot-for-lot.

Returning to our example, assume that the lot-sizing rule for parts A and B is fixed order period with $P = 2$ and for all other parts we use lot-for-lot. Then, for part A, we plan on receiving 45 units in period 6 (combining net demand from periods 6 and 7) and 30 units in period 8 (we cannot combine beyond our planning horizon). The results of these lot-sizing calculations are shown in Table 3.3

Time Phasing. Almost universally, MRP systems assume that the time to make a part is fixed, although a few systems do allow for the planned lead time to be a function of the job size. Regardless of the specifics, however, MRP treats lead times as attributes of the part and possibly the job, but *not* of the status of the shop floor. This can cause problems, as we will see later.

If we return to our example and assume that the planned lead time for part A is two periods, we are able to compute the planned order releases as shown in Table 3.3.

BOM Explosion. Table 3.3 shows the final result of processing part A. Recall that part A is made up of two units of part 100 and one unit of part 200 (see Figure 3.1). Thus, the planned order releases generated for part A create gross requirements for parts 100 and 200. Specifically, we need 90 units of part 100 in period 4 (two are needed for each unit of A) and 60 units in period 6. Similarly, we require 45 units of part 200 in period 4 and 30 units in period 6. These demands must be added to any requirements already accumulated for these parts (e.g., if we have already processed other parts that require them as subcomponents). To illustrate this, we will pursue our example a bit further.

The next step is to process any other parts having a low-level code of zero. In this example, we would process part B next. Suppose that the master production schedule for part B is as follows:

t	1	2	3	4	5	6	7	8
Demand	10	15	10	20	20	15	15	15

Furthermore, assume the following inventory and part data for parts B, 100, 300, and 500 (for brevity, we will not treat part 200, 400, or 600).

Part Number	Current On-Hand	SRs		Lot-Sizing Rule	Lead Time
		Due	Quanity		
B	40	0		FOP, 2 weeks	2 weeks
100	40	0		Lot-for-lot	2 weeks
300	50	2	100	Lot-for-lot	1 week
500	40	0		Lot-for-lot	4 weeks

Since there are no scheduled receipts for part B, the MRP calculations for this part are simple. Table 3.4 shows the completed tableau.

We have now completed processing all parts with an LLC of zero (i.e., parts A and B). Of the remaining parts we are considering, only part 500 has an LLC of one. Therefore we treat it next.

The only source of demand for part 500 is from part B (i.e., part A does not require part 500, and there is no external demand for part 500). Because each unit of B requires one unit of part 500, the planned order releases for part B become the gross requirements for part 500. Again, there are no scheduled receipts. The MRP processing is shown in Table 3.5.

Because the lead time for part 500 is four weeks, there is not enough time to finish the first 25 units before week four. Therefore, a planned order release is scheduled

TABLE 3.4 MRP Processing for Part B

Part B		1	2	3	4	5	6	7	8
Gross requirements		10	15	10	20	20	15	15	15
Scheduled receipts									
Adjusted SRs									
Projected on-hand	40	30	15	5	−15	—	—	—	—
Net requirements					15	20	15	15	15
Planned order receipts					35		30		15
Planned order releases			35		30		15		

TABLE 3.5 MRP Calculations for Part 500

Part 500		1	2	3	4	5	6	7	8
Gross requirements			35		30		15		
Scheduled receipts									
Adjusted SRs									
Projected on-hand	40	40	5	5	–25	—	—	—	—
Net requirements					25		15		
Planned order receipts					25		15		
Planned order releases		25*	15						

*Indicates a late start

for week one (as soon as possible) with an indication on an exception report that it is expected to be late.

We now turn to level 2 and part 100. Part 100 has two sources of demand, two units for each unit of part A and one unit for each unit of part 500. There are no scheduled receipts. The MRP processing is shown in Table 3.6

The only part at level 3 we consider is part 300. It has requirements from parts B and 100. Also, there is a scheduled receipt of 100 units in week two. Since it arrives at the time of the first uncovered requirement, no adjustments are necessary. The MRP processing is shown in Table 3.7.

We have now completed the MRP processing for all the parts of interest (processing for parts 200 and 400 is entirely analogous to that done for the other parts). Table 3.8

TABLE 3.6 MRP Calculations for Part 100

Part 100		1	2	3	4	5	6
Required from A					90		60
Required from 500		25	15				
Gross requirements		25	15		90		60
Scheduled receipts							
Adjusted SRs							
Projected on-hand	40	15	0	0	–90	—	—
Net requirements					90		60
Planned order receipts					90		60
Planned order releases			90		60		

TABLE 3.7 MRP Calculations for Part 300

Part 300		1	2	3	4	5	6	7	8
Required from B			35		30		15		
Required from 100			90		60				
Gross requirements			125		90		15		
Scheduled receipts			100						
Adjusted SRs			100						
Projected on-hand	50	50	25	25	−65	—	—	—	—
Net requirements					65		15		
Planned order receipts					65		15		
Planned order releases				65		15			

TABLE 3.8 Summary of MRP Output

Transaction	Part Number	Old Due Date or Release Date	New Due Date	Quantity	Notice
Change notice	A	1	2	10	Defer
Change notice	A	4	3	100	Expedite
Planned order release	A	4	6	45	OK
Planned order release	A	6	8	30	OK
Planned order release	B	2	4	35	OK
Planned order release	B	4	6	30	OK
Planned order release	B	6	8	15	OK
Planned order release	100	2	4	90	OK
Planned order release	100	4	6	60	OK
Planned order release	300	3	4	65	OK
Planned order release	300	5	6	15	OK
Planned order release	500	1	4	25	Late
Planned order release	500	2	6	15	OK

gives a summary of the outputs that an MRP system would generate from the above calculations. For each change notice, the system reports the quantity and part number affected, old due date, new due date, and whether it is an expedite or deferral. For each new planned order release, it reports the release date, the (new) due date, the release quantity, and whether it is anticipated to be late.

3.1.5 Special Topics in MRP

Up to now, we have focused on the mechanics of MRP processing. We now consider several technical issues that affect MRP performance. In particular, we address the question of what can be done to improve performance when things do not go as planned.

Updating Frequency. A key determinant of the effectiveness of an MRP system is the frequency of updating. If we update too frequently, the shop can be inundated with exception reports and constantly changing planned order releases.[4] If, on the other hand, we update too infrequently, we can end up with old plans that are often out of date. In designing an MRP system, one must balance the need for timeliness against the need for stability.

Firm Planned Orders. Changing the production schedule frequently can cause it to become very unstable. This makes it difficult for managers to shift workers effectively and prepare for setups. Therefore, it is desirable to minimize schedule disruption due to changes. One way to do this is by using **firm planned orders.** A firm planned order is a planned order release that is held fixed; that is, it will be released regardless of changes in the system. Consequently, firm planned orders are treated in MRP processing as if they were scheduled receipts (i.e., they must be included in the coverage analysis). By converting all planned order releases within a specified time interval to firm planned orders, the production plans become more stable. This is particularly important in the short term for managerial control purposes. Firm planned orders are also useful for reducing system **nervousness,** which is discussed in greater detail below.

Troubleshooting in MRP. A wise man named Murphy once said, "If something can go wrong, it will go wrong." In an MRP system, there are many things that can go wrong. Jobs can finish late, parts can be scrapped, demands can change, and so on. As a result, over the years MRP systems have acquired features to assist the planner as conditions change. Examples include the techniques of pegging and bottom-up replanning.

Pegging allows the planner to see the source of demand that results in a given planned order release. It is facilitated by providing a link from the gross requirements of an item to all its sources of demand. For example, consider the planned order release of 65 units of part 300 in week three shown in Table 3.7. Pegging would link this to the individual requirements of 60 units of part 100 and 30 units of part B in week four. These, in turn, could be linked to their demand sources, namely, part B to the master production schedule and part 100 to the 60 units needed to make part A in week six (see Table 3.6).

One of the uses of pegging is in **bottom-up replanning.** This is best illustrated with an example. Suppose we discover that the scheduled receipt of 100 units of part 300 due in week two will not be coming in (someone found the purchase order that was supposed to be sent to the vendor behind a file cabinet). Of course, the appropriate action would be to place the order immediately, call the vendor, and see if the order can be expedited. If this is not possible, we can use bottom-up replanning to investigate the impact of the late delivery.

From Table 3.7, we see that the gross requirements affected are the 125 required in week two. If the scheduled receipt will not be coming in, then we have only the 50 that are on-hand to cover demand, leaving 75 units uncovered. Of the 125 demanded, 35 are for part B, a level 0 item, and 90 are for part 100, a level 2 item. If we attempt to cover the lowest-level items first (reasoning that these have the potential for causing the greatest disruption), then we see that we can cover only 50 of the 90 units of part 100 needed in period 2. Further pegging shows that these requirements are from 90 units of demand for part A, for which we can now cover only 50 units. At this point we might

[4] In the past, when computer systems were small in memory and slow in processing, the cost of computer processing could also be prohibitive. However, with the dramatic increases in computer power in recent years, this is much less a factor in choosing a regeneration frequency.

want to contact the customer for the 90 units of part A and see if we can deliver 50 when requested and the other 40 later.

Alternatively, we might use the 50 units on hand to cover the demand for part B first (the idea here is to cover the items that generate revenue). If we do this, we can cover the 35 units of demand for B and are left with 15 units to cover the 90 required for part 100. Again pegging these to their original demand shows that 75 of the 90 units of part A required in period 4 would not be covered. If the demand for part B in the MPS is for an actual customer, while that for part A is only a forecast, we might want to cover B first. Of course, a different option is to split the 50 on hand to cover some of the demand for part B and some for part 100. The "correct" choice depends on the customers involved, their willingness to accept late orders, and so on.

Instead of pegging, we could have eliminated the scheduled receipt of 100 units of part 300 and made a complete regeneration of MRP. This would have resulted in a planned order release in week one with an exception notice that it is expected to be late. However, a regeneration of MRP cannot determine which customer orders will be late as a result of this delay. Bottom-up replanning and pegging provide the planner with this ability. The use of firm planned orders allows the planner to remedy a schedule by overriding standard MRP processing.

3.1.6 Lot Sizing in MRP

To demonstrate basic MRP processing, we have described two simple lot-sizing rules—fixed order period and lot-for-lot. In this section, we will discuss issues surrounding the lot-sizing problem and describe other, more complex lot-sizing rules.

The lot-sizing problem deals with the basic tradeoff between having many small jobs, which tend to increase setup costs (materials, tracking costs, labor, etc.) and/or decrease capacity, versus having a few large jobs, which tend to increase inventory.

Recall that in Chapter 2 we formulated the Wagner–Whitin (WW) approach to the lot-sizing problem by assuming infinite capacity and known setup and inventory carrying costs. Under these assumptions, we showed that the lot-sizing problem can be solved optimally using the WW algorithm. Of course, the questions with this approach are whether anyone *can* know the setup and inventory carrying costs and whether capacities will be binding. As one wag remarked about setup costs, "I have yet to write out a check to a machine." In many instances, setup "cost" is used as proxy for limited *capacity*. The idea is to design lot-sizing rules so that higher setup costs result in larger lots (e.g., the EOQ). Since larger lots require fewer setups, less capacity is consumed. Conversely, when capacity is not tight, smaller setup costs can be used to reduce lot sizes (and thereby inventory) at the expense of more setups. Thus, by adjusting setup costs, the planner can trade inventory for capacity.

Unfortunately, the so-called Wagner–Whitin property of producing only when inventory levels reach zero is *not* optimal when capacity is a constraint. Nonetheless, many of the lot-sizing rules that have been suggested possess the WW property and are typically compared to the WW algorithm when their performance is assessed. Thus, although many of the assumptions may be invalid in realistic situations, it would appear that most lot-sizing rule designers have accepted the Wagner–Whitin paradigm. Interestingly, we know of no commercial MRP package that actually uses the WW algorithm. The reasons usually given are that it is too complicated or that it is too slow. But with the advent of fast computers, speed is no longer an issue—an efficient WW algorithm runs quickly on a modern personal computer. A more likely reason may be found in the observation that "People would rather live with a problem they cannot solve than accept a solution they do not understand." Regardless of the reason, a host of alternative lot-sizing

algorithms have been suggested and are offered in various forms in most commercial MRP systems. We will discuss here some of the more commonly used methods.

Lot-for-Lot. As we have already noted, lot-for-lot (LFL) is the simplest of the lot-sizing rules—simply produce in period t the net requirements for period t. Since this leaves no inventory at the end of *any* period (given the assumptions of MRP), this method minimizes inventory (assuming that it is possible to produce the demand in each period). However, under the Wagner–Whitin paradigm, since there is a "setup" in every period with demand, this method also maximizes total setup cost. Despite this, lot-for-lot is attractive in several respects. First, it is simple. Second, it is consistent with the just-in-time philosophy (see Chapter 4) of making only what is needed when it is needed. Finally, since the procedure does not lump requirements together in some periods and produce nothing in others, it tends to generate a smoother production schedule. In situations where setup times (costs) are minimal, it is probably the best policy to use.

Fixed Order Quantity and EOQ. A second very simple policy is to order a predetermined quantity whenever an order is placed. We use this rule, fixed order quantity, in our first example. It is commonly used for two simple reasons.

First, when there are certain sized totes, carts, or other fixtures used to transport jobs in the shop, it makes sense to create jobs only in these sizes. In some cases, different sized totes are used at different points in the shop. For instance, fenders are usually carried in smaller quantities than spark plugs. To avoid leftovers, it makes sense to coordinate the sizes of the quantities. One way to do this is to choose power-of-2 (1, 2, 4, 8, 16, etc.) lot sizes.

Second, fixing the job size influences the number of setups. Since the basic tradeoff is between setup cost and inventory carrying cost, the problem of choosing an appropriate fixed order quantity is very similar to that of the economic order quantity problem discussed in Chapter 2. The primary difference is that the EOQ model assumed a constant demand rate. In MRP, demand need not be constant. However, we can make use of the EOQ model by replacing the constant demand of that model with an estimate of the average demand $\bar{D}$. Then, using A to represent the setup cost and h to denote the inventory carrying cost per annum, we can use the EOQ formula we derived in Chapter 2

$$Q = \sqrt{\frac{2A\bar{D}}{h}}$$

to compute the fixed order quantity Q. As discussed previously, we may want to round this quantity to the nearest power of 2. The ratio of A/h can be adjusted to achieve a desired setup frequency. Making A/h larger will reduce the setup frequency, while reducing A/h will increase the setup frequency. After some experience, a value that is compatible with the capacity of the line can be found. Of course, since this value will depend on the actual orders, it may change frequently.

Unlike the lot-for-lot rule, the fixed order quantity method (whether or not one uses the EOQ to obtain the order size) will *not* have the Wagner–Whitin property of producing only when inventory reaches zero. This means that it can result in incurring cost to carry inventory that does not eliminate a setup—an obvious inefficiency (under the assumptions of Wagner–Whitin).[5]

[5]Of course, as a practical measure, we will probably not plan to run out of inventory exactly when receiving the next order. Nonetheless, we can use safety stock (discussed in the next section) to provide some cushion and then insist on the Wagner–Whitin property for the cycle stock (i.e., the stock that is intended to be used).

However, we can modify the rule slightly to consider only job sizes that are equal to the exact demand of one or more periods, and then choose the one that is closest to the desired fixed job size. This practice recovers the Wagner–Whitin property. Consider the following example. Suppose our fixed order quantity is 50 units and the net requirements are these:

Net requirements	15	15	60	65	55	15	20	10

Then, to preserve the Wagner–Whitin property, our planned order receipts would be

Planned order receipts	30		60	65	55	45		

In period 1, 30 is closer to 50 than is 15, so we ordered two periods' worth of demand instead of one. In period 3, 60 is closer than 125, so we ordered one period's worth instead of two, and so on.

Fixed Order Period. The fixed order period (FOP) rule was used in the MRP processing example in Section 3.1.4. Its operation is simple: If you are going to produce in period t, then produce all the demand for period $t, t+1, \ldots, t+P-1$, where P is a parameter of the policy. If $P = 1$, the policy is lot-for-lot, since we only produce for the current period. Since each production quantity is for the exact amount required in a given set of periods, the policy has the Wagner–Whitin property.

While simple, the policy does have some subtlety. The policy *does not* state that production will occur once every P periods. If there are periods with no demand, they are skipped. Consider the following example with $P = 3$.

Period	1	2	3	4	5	6	7	8	9
Net requirements		15	45			25	15	20	15
Planned order receipts		60				60			15

We skip the first period since there is no demand. The first demand occurs in period 2 and so we accumulate the demand for periods 2, 3, and 4 (note there is no demand in period 4) and therefore order 60 units for period 2. We again skip period 5, as it has no demand, and accumulate periods 6, 7, and 8 with a planned order receipt of 60 units in period 6. Finally, we order 15 units for period 9 and look no farther out since we are at the end of our time horizon.

One way to determine an "optimal" value for P is to use the EOQ formula and the average demand in a fashion similar to that used for the fixed order quantity rule. In the preceding example, the total demand for nine periods is 135 units, so the average demand is 15 units per period. Suppose the setup cost is \$150 and the carrying cost per period is \$2. We can then compute the EOQ as

$$Q = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2 \times 150 \times 15}{2}} = 47.4$$

We can then compute the order period P as

$$P = \frac{Q}{D} = \frac{47.4}{15} = 3.16 \approx 3 \text{ periods}$$

Of course, the validity of computing P using this method has all the limitations of the EOQ method that were noted in Chapter 2.

Part-Period Balancing. Part-period balancing (PPB) is a policy that combines the assumptions of the Wagner–Whitin paradigm with the mechanics of the EOQ. One of the properties of the EOQ solution to the lot-sizing problem is that it sets the average inventory carrying cost equal to the setup cost.

The idea of PPB is to balance (i.e., set equal) the inventory carrying cost and setup cost. To describe this, we need to define the notion of a **part-period** as the product of the number of parts in a lot times the number of periods they are carried in inventory. For instance, 1 part carried for 10 periods, 5 parts carried for 2 periods, and 10 parts carried for 1 period all represent 10 part-periods and incur the same inventory carrying cost. Part-period balancing seeks to make the carrying cost as close to the setup cost as possible. We can demonstrate this by using the data of the previous example.

By considering only those quantities that preserve the Wagner–Whitin property, we reduce our choices to a relative few. Since there are no requirements in period 1, there will be no production in period 1. The choices for period 2 are 15 (produce for period 2 only), 60 (produce for periods 2 and 3), 85 (produce for periods 2, 3, and 6), and so on. The following table shows the part-periods and the costs involved.

Quantity for Period 2	Setup Cost (\$)	Part-Periods	Inventory Carrying Cost (\$)
15	150	0	0
60	150	$45 \times 1 = 45$	90
85	150	$45 + 25 \times 4 = 145$	290

Since \$90 is the closest to \$150 of the options available, we elect to make 60 units in period 2. Since there are no requirements, we will make nothing in periods 3, 4, and 5. For period 6 the choices are 25, 40, 60, and 75 units. Again we present the computations in a table.

Quantity for Period 6	Setup Cost ($)	Part-Periods	Inventory Carrying Cost ($)
25	150	0	0
40	150	$15 \times 1 = 15$	30
60	150	$15 + 20 \times 2 = 55$	110
75	150	$55 + 15 \times 3 = 100$	200

The inventory carrying cost closest to $150 results from making 60 units in period 6. This covers requirements for periods 6, 7, and 8, leaving 15 for period 9. Note that this is exactly the same schedule that resulted from the FOP policy.

Other Methods. A host of other methods for lot sizing have been proposed by researchers. Most of these attempt to provide a near-optimal solution according to the Wagner–Whitin criteria. Whether these criteria are appropriate is a matter of debate, as we have discussed. Baker (1993) gives a good review of many of the lot-sizing methods that have been suggested.

Finally, we note that although the Wagner–Whitin algorithm is optimal under certain conditions, other rules may perform better in practice. For instance, Bahl et al. (1987) report in a review of the lot-sizing literature that the fixed order quantity method, *without* modification to give it the Wagner–Whitin property, tends to work better than rules that do possess the Wagner–Whitin property in multilevel production systems with capacity limitations. They conclude that the often-imposed Wagner–Whitin property may not be practical in real settings, since "the remnants avoided by almost all (other lot-sizing rules) become an asset in terms of on-time delivery of end items." This makes sense, since these remnants become a form of safety stock, an issue that we explore in the next section.

3.1.7 Safety Stock and Safety Lead Times

Operations management researchers have long debated the role of safety stock and safety lead times in MRP systems. Orlicky felt that these had no place in the system except, possibly, for end items. Lower-level items, he believed, were more than adequately covered by the workings of the system. Since Orlicky's time, many researchers have disagreed. Because MRP is deterministic, the logic goes, something should be done to account for uncertainty and randomness.

There are several sources of uncertainty. First, in all except pure make-to-order systems, neither the demand quantity nor the timing of the demand is known exactly. Second, production timing is almost always subject to variation, due to machine breakdowns, quality problems, fluctuations in staffing, and so on. Third, production quantities are uncertain because the number of good parts that finish can be less than the quantity that start due to **yield loss** or **fallout.**

Safety stock and **safety lead time** can be used as protection against these problems. Vollmann et al. (1992) suggest that *safety stock* should be used to protect against uncertainties in production and demand *quantities,* while *safety lead time* should be used to protect against uncertainties in production and demand *timing.*

Providing safety stock (SS) in an MRP system is fairly straightforward. Suppose we wish to maintain a safety stock level of 10 units for part B (refer to Table 3.4). This time we compute the first net requirement as we did before, but we subtract an additional 10 units for the desired safety stock. The projected on-hand *minus safety stock* first becomes negative in period 3 (as opposed to period 4 before), as we see in Table 3.9.

Thus, our first planned order release is for five units needed to bring the inventory to the desired safety stock level, plus 20 units for actual demand.

Introducing safety lead time into the MRP calculations is a bit different. If the nominal lead time is two weeks and we desire a safety lead time of one week, we perform the offsetting in two stages: the first for the safety lead time regarding the planned order *receipt* date (i.e., the due date) and the second using the usual MRP method, to obtain the planned order *release* date. We demonstrate the use of a safety lead time of one week, using the same data as in the previous example in Table 3.10.

TABLE 3.9 MRP Computations for Part B with Safety Stock

Part B		1	2	3	4	5	6	7	8
Gross requirements		10	15	10	20	20	15	15	15
Scheduled receipts									
Adjusted SRs									
Projected on-hand	40	30	15	5	—	—	—	—	—
Projected on-hand—SS	30	20	5	−5	—	—	—	—	—
Net requirements				5	20	20	15	15	15
Planned order receipts				25		35		30	
Planned order releases		25		35		30			

TABLE 3.10 MRP Calculations for Part B with Safety Lead Time

Part B		1	2	3	4	5	6	7	8
Gross requirements		10	15	10	20	20	15	15	15
Scheduled receipts									
Adjusted SRs									
Projected on-hand	40	30	15	5	−15	—	—	—	—
Net requirements					15	20	15	15	15
Planned order receipts					35		30		15
Adjusted planned order receipts				35		30		15	
Planned order releases		35		30		15			

The one additional step beyond the usual MRP calculation is shown in the "Adjusted planned order receipts" line, which backs up these receipts according to the one week safety lead time. Notice that the effect on planned order releases is identical to simply inflating the planned lead times. However, the due dates on the jobs are earlier in a system using safety lead times than in one without it. The effect of safety lead times on a single part is fairly simple. Bringing parts in a week early means they will be available unless delivery is late by more than a week. However, things are more subtle when we consider multiple parts and assemblies.

For instance, suppose a plant manufactures a part that requires 10 components to come together at assembly. Suppose also that the actual manufacturing lead times can be well approximated using a normal distribution with a mean of three weeks and a standard deviation of one week. To maintain good customer service, we want assemblies to start on time at least 95 percent of the time. If s is the service level (i.e., the probability of on-time delivery) for each component, then the probability that all 10 components are available on time (assuming independent deliveries) is given by

$$\Pr\{\text{on-time start of assembly}\} = s^{10}$$

Since we want this probability to equal 0.95, we can solve for s as follows:

$$s = (0.95)^{1/10} = 0.9949$$

Since the manufacturing lead times are normally distributed, this represents approximately 2.6 standard deviations above the mean, or around 5.6 weeks—about twice the mean lead time for the planned lead time.

Of course, this analysis assumes that the 10 items are arriving to the assembly operation independently of one another, an assumption that may not be true if they are all being fabricated in the same plant. Nonetheless, the point is made—if we are to try to guarantee any level of service for an assembly, the service for the component parts must be *much* greater.

In conclusion, although safety stock and safety lead times can be useful in an MRP system, we must be cognizant of the fact that both procedures *lie* to the system. Safety stock requires the intentional production of quantities for which there is no customer need, while safety lead times set due dates earlier than are really required. Both situations will make **available-to-promise** calculations (used to quote deliveries to customers, discussed below) less accurate. Excess safety stocks and long safety lead times will result in customers being turned away due to perceived schedule infeasibility even though the schedule is actually feasible. In addition, there is always the risk that once safety stock and/or lead times are discovered by the users, an informal system of "real" quantities and due dates will appear. Such behavior can lead to a subversion of the formal system and can degrade its performance.

3.1.8 Accommodating Yield Losses

The above discussion and examples illustrate the use of hedges against uncertainties in demand and timing. However, hedging against random scrapping of parts during production—yield loss—involves an additional computation. Suppose the net demand is N_t units and the average yield fraction is y. Also suppose, for this example, that N_t is a large number, so that we do not have to worry about integer quantities. Thus, if we start $N_t(1/y)$ units, we will, on average, finish with N_t units, the net demand. However, if $N_t(1/y)$ is a large number, it is very unlikely that we will finish with exactly N_t. We will, with roughly equal probability, finish with either more or less than the net demand.

Finishing with more means that we will carry the extra parts in inventory until they are netted from future demand. If the product is highly customized, this can be a problem. On the other hand, if we finish with less, a *new* job will be required to make up the difference, and it is unlikely that the order will ship on time.

Safety stock can improve customer service and responsiveness in this case. We inflate the size of the job to $N_r(1/y)$ as before and carry safety stock to accommodate instances when production is less than the average yield. Another strategy is to carry no safety stock but to inflate the job by more than $1/y$. In this case, it is likely that the job will finish with more than the net demand and that the extra stock will be carried in inventory. The two procedures are essentially equivalent since both result in better service at the expense of additional inventory.

Lastly, we should point out that the effectiveness of any yield strategy depends on the *variability* of the yields themselves. For instance, if a job starts with 100 units, each unit having an independent probability of 0.9 of being completed, then the mean and standard deviation of the number of units finishing will be 90 and 3, respectively. Thus, by starting 120 (that is, $100/0.9 + 3 \times 3$) units, we have a probability of greater than 0.99 (three standard deviations above the mean) that we will finish with at least 100 units. However, if the yield situation is more of an all-or-nothing type, so that either all the units that start finish properly or none of them do (as in a batch process), then we need to release two separate jobs of 100 each to obtain a 0.99 probability of finishing 100 on time. In the first (independent) case, the average increase in inventory would be eight units ($120 \times 0.9 - 100$). In the second (batch) case, it would be 80 units ($200 \times 0.9 - 100$). The moral is that *average* yield rate is not enough to determine an effective yielding strategy. The mechanism and variability of the processing causing the yield fallout must also be considered.

3.1.9 Problems in MRP

Despite enthusiastic support of MRP by early proponents—Orlicky's book was subtitled *A New Way of Life*—several problems were recognized early on. Three of the most severe were (1) capacity infeasibility of MRP schedules, (2) long planned lead times, and (3) system "nervousness." These and other problems first led to new MRP procedures and spawned a new generation of MRP, called **manufacturing resources planning** or **MRP II,** which, in turn evolved into **enterprise resources planning (ERP),** as we will discuss in the next section.

Capacity Infeasibility. The basic working model of MRP is a production line with a fixed lead time. Since this lead time does not depend on how much work is in the plant, there is an implicit assumption that the line will always have sufficient capacity regardless of the load. In other words, MRP assumes all lines have infinite capacity. This can create problems when production levels are at or near capacity.

One way to address this problem is to make sure that the master production schedule that supplies demand to the system is capacity-feasible. A check of this is provided by a procedure called rough-cut capacity planning (RCCP), as we will see later. As its name implies, RCCP is an approximation. A more detailed capacity assessment of the resulting MRP plans can be made by using a procedure known as **capacity requirements planning (CRP).** Both RCCP and CRP are modules that are often found in MRP II.

Long Planned Lead Times. As we saw in our earlier discussion of safety lead times, there are many pressures to increase planned lead times in an MRP system. In Part II,

we will see that long lead times invariably lead to large inventories. However, as long as the penalty for a late job is greater than that for excess inventory (which is typically the case, since inventory does not scream but dissatisfied customers do!), production control managers will tend toward long planned lead times.

The problems caused by long planned lead times are further exacerbated by the fact that MRP uses *constant* lead times when, in fact, actual manufacturing times vary continually. To compensate, a planner will typically choose pessimistic (long) estimates for the planned lead times. Suppose for example, the average manufacturing lead time is three weeks, with a standard deviation of one week. To maintain good customer service, the planned lead time is set to five weeks. Since the actual lead times are random, some will be less than the mean of three weeks and others will be greater. If these follow an approximately normal distribution, then the most likely lead time will be three weeks, so the most likely holding time in inventory will be two weeks. The result can be a large amount of inventory.

The longer the planned lead times, the longer parts will wait for the next operation, and so the more inventory there will be in the system. Since setting planned lead times equal to the average manufacturing time yields a service level of only 50 percent for each component (and therefore much worse service for finished assemblies), managers will virtually always choose lead times that are much longer than average manufacturing times. Such behavior results in a lack of responsiveness as well as high inventory levels.

System Nervousness. Nervousness in an MRP system occurs when a small change in the master production schedule results in a large change in planned order releases. This can lead to strange effects. For instance, as we demonstrate with the following example, it is actually possible for a *decrease* in demand to cause a formerly feasible MRP plan to become infeasible.

The following example is taken from Vollmann et al. (1992). We consider two parts. Item A has a lead time of two weeks and uses the fixed order period (FOP) lot-sizing rule with an order period of five weeks. Each unit of A requires one unit of component B, which has a lead time of four weeks and uses the FOP rule with an order period of five weeks. Tables 3.11 and 3.12 give the MRP calculations for both parts.

TABLE 3.11 MRP Calculations for Item A before Change in Demand

Item A		1	2	3	4	5	6	7	8
Gross requirements		2	24	3	5	1	3	4	50
Scheduled receipts									
Adjusted SRs									
Projected on-hand	28	26	2	−1	—	—	—	—	—
Net requirements				1	5	1	3	4	50
Planned order receipts				14					50
Planned order releases		14					50		

TABLE 3.12 MRP Calculations for Component B before Change in Demand

Component B		1	2	3	4	5	6	7	8
Gross requirements		14					50		
Scheduled receipts		14							
Adjusted SRs		14							
Projected on-hand	2	2	2	2	2	2	−48	—	—
Net requirements							48		
Planned order receipts							48		
Planned order releases			48						

TABLE 3.13 MRP Calculations for Item A after Change in Demand

Item A		1	2	3	4	5	6	7	8
Gross requirements		2	23	3	5	1	3	4	50
Scheduled receipts									
Adjusted SRs									
Projected on-hand	28	26	3	0	−5	—	—	—	—
Net requirements					5	1	3	4	50
Planned order receipts					63				
Planned order releases			63						

We now *reduce* the demand in period 2 from 24 to 23. It would seem obvious that any schedule that is feasible for 24 parts in period 2 should also be feasible for 23 parts in the same period. But notice what happens to the calculations in Table 3.13. The aggregation of demand during lot sizing causes a drastically different set of planned order releases. In the case of component B (Table 3.14), the planned order releases are no longer even feasible.

There have been several remedies offered to reduce nervousness. One is the proper use of lot-sizing rules. Clearly, if we use lot-for-lot, the magnitude of the change to the planned order releases will be no larger than the changes to the MPS. However, lot-for-lot may result in too many setups, so we need to look for other cures.

Vollmann et al. (1992) recommend the use of different lot-sizing rules for different levels in the BOM, with fixed order quantity for end items, either fixed order quantity or lot-for-lot for intermediate levels, and fixed order period for the lowest levels. Since order sizes do not change at the higher levels, this tends to dampen nervousness due to

TABLE 3.14 MRP Calculations for Component B after Change in Demand

Component B		1	2	3	4	5	6	7	8
Gross requirements			63						
Scheduled receipts		14							
Adjusted SRs			14						
Projected on-hand	2	2	−47	—	—	—	—	—	—
Net requirements			47				48		
Planned order receipts			47						
Planned order releases		47*							

*Indicates a late start

changes in lot size. Of course, care must be taken when establishing the magnitude of the fixed lot size.

While the use of proper lot-sizing rules can reduce system nervousness, other measures can alleviate some of its effects. One obvious way is to reduce changes in the input itself. This can be done by freezing the early part of the master production schedule. This reduces the amount of change that can occur in the MPS, thereby reducing changes in planned order releases. Since early planned order releases are the ones in which change is most disruptive, a **frozen zone,** an initial number of periods in the MPS in which changes are not permitted, can dramatically reduce the problems caused by nervousness.

In some companies the first X weeks of the MPS are considered frozen. However, in most real systems, the term *frozen* may be too strong, since changes are resisted but not strictly forbidden. (Perhaps *slushy zone* would be a more accurate metaphor.) The concept of **time fences** formalizes this type of behavior. The earliest time fence, say for four weeks out, is absolutely frozen—no changes can be made. The next fence, maybe five to seven weeks out, is restricted but less rigid. Changes might be accepted in model options if the options are available, and possibly resulting in a financial penalty to the customer. The next fence, perhaps 8 to 12 weeks out, is less rigid still. In this case, changes in part number might be accepted if all components are on hand. In the final fence, 13 weeks and beyond, anything goes.

Another way to reduce the consequences of nervousness is to make use of **firm planned orders.** Unlike frozen zones or time fences, firm planned orders fix planned order releases. By converting early planned order releases to firm planned orders, we eliminate all system nervousness early in the schedule, where it is most disruptive. Consider what would happen if the first planned order release in Table 3.11 were made into a firm planned order before the change in demand. This would result in its being treated just like a scheduled receipt in the MRP processing. With this change there is no nervousness, as is shown in Tables 3.15 and 3.16.

Of course, the use of firm planned orders and time fencing means that the frozen part of the schedule will be less responsive to changes in demand. Another drawback is that the firm planned orders represent tedious manual entries that must be managed by planners.

TABLE 3.15 MRP Calculations for Item A with FPO

Item A		1	2	3	4	5	6	7	8
Gross requirements		2	23	3	5	1	3	4	50
Scheduled receipts									
Firm planned orders				14					
Projected on-hand	28	26	3	14	9	8	5	1	−49
Net requirements									49
Planned order receipts				[14]					49
Planned order releases		[14]					49		

TABLE 3.16 MRP Calculations for Component B with FPO

Component B		1	2	3	4	5	6	7	8
Gross requirements		14					49		
Scheduled receipts		14							
Adjusted SRs									
Projected on-hand	2	2	2	2	2	2	−47	—	—
Net requirements							47		
Planned order receipts							47		
Planned order releases			47						

3.2 Manufacturing Resources Planning—MRP II

Material requirements planning offered a systematic method for planning and procuring materials to support production. The ideas were relatively simple and easily implemented using a computer. However, some problems remained.

As we have mentioned, issues such as capacity infeasibility, long planned lead times, system nervousness, and others can undermine the effectiveness of an MRP system. Over time, additional procedures were developed to address some of these problems. These were incorporated into a larger construct known as **manufacturing resources planning,** or **MRP II.**

Beyond simply addressing deficiencies of MRP, MRP II also brought together other functions to make a truly integrated manufacturing management system. The additional functions subsumed by MRP II included demand management, forecasting, capacity planning, master production scheduling, rough-cut capacity planning, capacity requirements planning, dispatching, and input/output control. In this section we describe the

MRP II hierarchy into which these functions fit and discuss some of the associated modules. Our presentation is somewhat abbreviated for two reasons. First, MRP and MRP II are subjects that can occupy an entire volume themselves. We recommend Vollmann et al. (1992) as an excellent comprehensive reference. Second, we take up the issue of hierarchical production planning (in the context of pull systems) in Chapter 13. There we will address generic issues associated with any planning hierarchy such as time scales, forecasting, demand management, and so forth in greater detail.

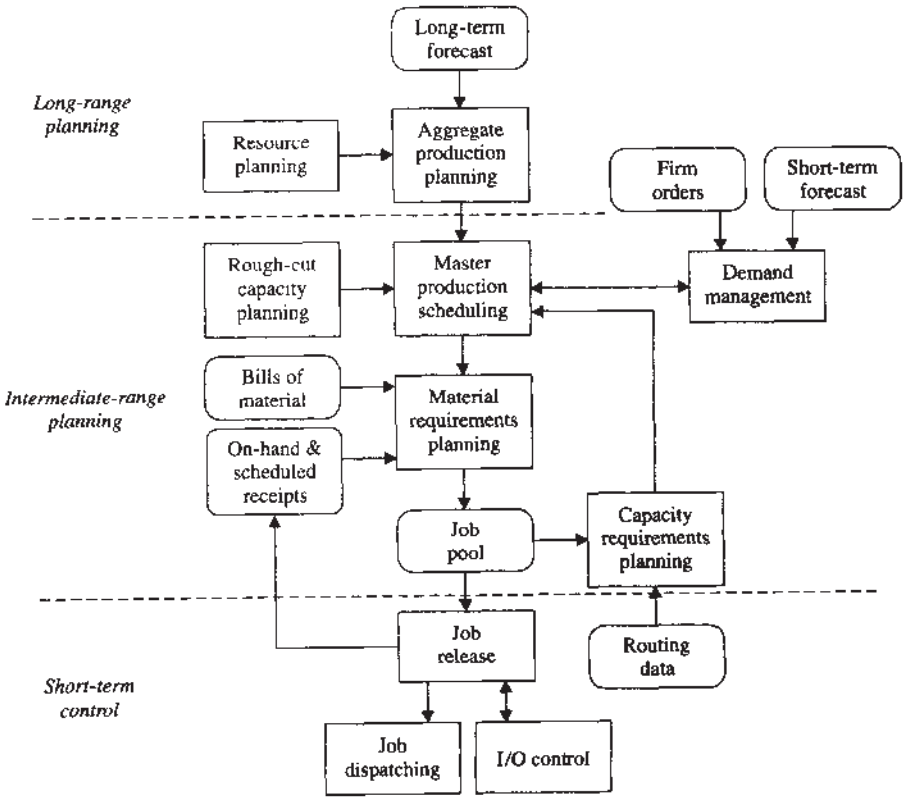
3.2.1 The MRP II Hierarchy

Figure 3.3 depicts an instance of the MRP II hierarchy. We use the word *instance* because there are probably as many different hierarchies for MRP II as there are MRP II software vendors (and there are many such vendors, although most call themselves ERP or "enterprise" software vendors now).

3.2.2 Long-Range Planning

At the top of the hierarchy we have **long-range planning.** This involves three functions: resource planning, aggregate planning, and forecasting. The length of the time horizon for long-range planning ranges from around six months to five years. The frequency for replanning varies from once per month, to once per year, with two to four times per year being typical. The degree of detail is typically at the part family level (i.e., a grouping of end items having similar demand and production characteristics).

FIGURE 3.3
MRP II hierarchy



The **forecasting** function seeks to predict demands in the future. Long-range forecasting is important to determining the capacity, tooling, and personnel requirements. Short-term forecasting converts a long-range forecast of part families to short-term forecasts of individual end items. Both kinds of forecasts are input to the intermediate-level function of **demand management.** We describe specific forecasting techniques in detail in Chapter 13.

Resource planning is the process of determining capacity requirements over the long term. Decisions such as whether to build a new plant or to expand an existing one are part of the capacity planning function. An important output of resource planning is projected available capacity over the long-term planning horizon. This information is fed as a parameter to the aggregate planning function.

Aggregate planning is used to determine levels of production, staffing, inventory, overtime, and so on over the long term. The level of detail is typically by month and for part families. For instance, the aggregate planning function will determine whether we build up inventories in anticipation of increased demand (from the forecasting function), "chase" the demand by varying capacity using overtime, or do some combination of both. Optimization techniques such as linear programming are often used to assist the aggregate planning process. We discuss aggregate planning and models for supporting it in greater detail in Chapter 16.

3.2.3 Intermediate Planning

At the intermediate level, we have the bulk of the production planning functions. These include demand management, rough-cut capacity planning, master production scheduling, material requirements planning, and capacity requirements planning.

The process of converting the long-term aggregate forecast to a detailed forecast while tracking individual customer orders is the function of **demand management.** The output of the demand management module is a set of actual customer orders plus a forecast of anticipated orders. As time progresses, the anticipated orders should be "consumed" by actual orders.

This is accomplished with a technique known as **available to promise (ATP).** This feature allows the planner to know which orders on the MPS are already committed and which are available to promise to new customers. ATP combined with a capacity-feasible MPS facilitates negotiation of realistic due dates. If more orders than expected are received, so that quoted lead times become excessive, additional capacity (e.g., overtime) might be required. On the other hand, if fewer than expected orders arrive, sales might want to offer discounts or some other incentives to increase demand. In either case, the forecast and possibly the aggregate plan should be revised.

Master production scheduling takes the demand forecast along with the firm orders from the demand management module and, using aggregate capacity limits, generates an anticipated build schedule at the highest level of planning detail. These are the "demands" (i.e., part number, quantity, and due date) used by MRP. Thus, the master production schedule contains an order quantity in each time bucket for every item with independent demand, for every planning date. For most industries, these are given at the **end item** level. However, in some cases, it makes more sense to plan for groups of items or models instead of end items. An example of this is seen in the automobile industry where the exact make and specification of a car are not determined until the last minute on the assembly line. In these situations, a **final assembly schedule** determines when the exact end items are produced while the master production schedule is used to schedule models. A key input to this type of planning is the **superbill of material** that contains forecast percentages for the different options of each particular model. For a

complete discussion of superbills in final assembly scheduling, the reader is referred to Vollman et al. (1992).

Rough-cut capacity planning (RCCP) is used to provide a quick capacity check of a few critical resources to ensure the feasibility of the master production schedule. Although more detailed than aggregate planning, RCCP is less detailed than capacity requirements planning (CRP), which is another tool for performing capacity checks after the MRP processing. RCCP makes use of a **bill of resources** for each end item on the MPS. The bill of resources gives the number of hours required at each critical resource to build a particular end item. These times include not only the end item itself but all the exploded requirements as well. For instance, suppose part A is made up of components A_1 and A_2. Part A requires one hour of process time in process center 21 while components A_1 and A_2 require one-half hour and one hour, respectively. Thus the bill of resource for part A would show two and one-half hours for process center 21 for each unit of A. Suppose we also have part B with no components that requires two hours in process center 21.

To continue the example, suppose we have the following information regarding the master production schedule for parts A and B:

Week	1	2	3	4	5	6	7	8
Part A	10	10	10	20	20	20	20	10
Part B	5	25	5	15	10	25	15	10

The bills of resources for parts A and B are given by

Process Center	Part A	Part B
21	2.5	2.0

Then the RCCP calculations for parts A and B at process center 21 are as follows:

Week	1	2	3	4	5	6	7	8
Part A (hour)	25	25	25	50	50	50	50	25
Part B (hour)	10	50	10	30	20	50	30	10
Total (hour)	35	75	35	80	70	100	80	35
Available	65	65	65	65	65	65	65	65
Over(+)/under(−)	30	−10	30	−15	−5	−35	−15	30

If we had considered only the sum of the eight periods in aggregate, we would have concluded that there was sufficient capacity—520 hours versus a requirement of 510 hours. However, after performing RCCP, we see that several periods have insufficent

capacity while others have an excess. It is now up to the planner to determine what can be done to remedy the situation. Her options are to (1) adjust the MPS by changing due dates or (2) adjust capacity by adding or taking away resources, using overtime, or subcontracting some of the work.

Notice that RCCP does not perform any offsetting. Thus, the periods used must be long enough that the part, its subassemblies, and its components can all be completed within a single period. RCCP also assumes that the demand can be met without regard to how the work is scheduled within the process center (i.e., without any induced idle time). In this way, RCCP provides an optimistic estimate of what can be done.

On the other hand, RCCP does not perform any netting. While this may be acceptable for end items (demand for these can be netted against finished goods inventory relatively easily), it is less acceptable for subassemblies and components, particularly when there are many shared components and WIP levels are large. This aspect of RCCP tends to make it conservative.

These two effects make the behavior of RCCP difficult to gauge. Usually the first approximation tends to dominate the second, making RCCP an optimistic estimation of what can be done, but not always. Consequently, rough-cut capacity planning can be very rough indeed.

Capacity requirements planning (CRP) provides a more detailed capacity check on MRP-generated production plans than RCCP. Necessary inputs include all planned order releases, existing WIP positions, routing data, as well as capacity and lead times for all process centers. In spite of its name, capacity requirements planning does *not* generate finite capacity analysis. Instead, CRP performs what is called **infinite forward loading.** CRP predicts job completion times *for each process center,* using given *fixed* lead times, and then computes a predicted loading over time. These loadings are then compared against the available capacity, but no correction is made for an overloaded situation.[6]

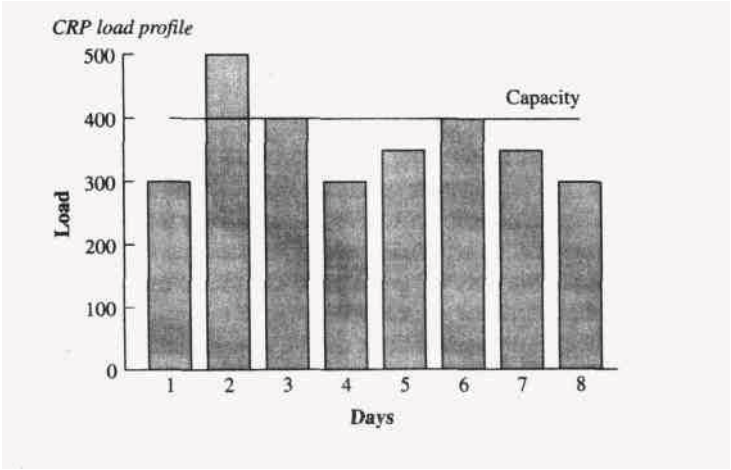
To illustrate how CRP works, consider a simple example for a process center that has a three-day lead time and a capacity of 400 parts per day. At the start of the current day, 400 units have just been released into the process center, 500 units have been there for one day, and 300 have been there for two days. The planned order releases for the next five days are as follows:

Day	1	2	3	4	5
Planned order releases	300	350	400	350	300

Using the three-day lead time, we can compute when the parts will depart the process center. If we ever predict more than 400 units departing in a day, the process center is considered to be overloaded. The resulting **load profile** is shown in Figure 3.4. The first day shows the load to be 300 (these are the same 300 units that have been in the process center for two days and depart at the end of day one). The second day shows 500; again these are the same 500 that were in for one day at the start of the procedure. Since 500 is greater than the capacity of 400 per day, this represents an overloaded condition.

[6]Unlike MRP and CRP, true finite capacity analysis does not assume a fixed lead time. Instead the time to go through a manufacturing operations depends on how many other jobs are already there and their relative priority. Most finite capacity analysis packages do some sort of deterministic simulation of the flow of the jobs through the facility. As a result, finite capacity analysis is much more complex than CRP.

FIGURE 3.4
CRP load profile



Note that even when load exceeds capacity, CRP assumes that the time to go through the process center does not change. Of course, we know that it will take longer to get through a heavily loaded process center than a lightly loaded one. Hence, all the estimates of CRP beyond such an overloaded condition will be in error. Therefore, CRP is typically not a good predictor of load conditions except in the very near term. Another problem with CRP is that it only tells the planner that there is a problem; it offers nothing about what caused the problem or what can be done to alleviate it. To determine this, the planner must first obtain a report that disaggregates the load to determine which jobs are causing the problem, and then must use pegging to track the cause back to demand on the MPS. This can be quite tedious.

A fundamental flaw with CRP is that, like MRP itself, it implicitly assumes an infinite capacity. This assumption comes from the assumption of fixed lead times that do not depend on the load of the process center. Consider the same process center having no work in it at the start and the following planned order releases, produced with a lot-sizing rule that tends to group demand to avoid setups:

Day	1	2	3	4	5
Planned order releases	1,200	0	0	1,200	0

Using CRP, the load profile will show an overloaded condition on day three and day six. If we were to perform *finite* capacity loading, we would see a very different picture. There would be no output for two days (the first release needs to work its way through), and then we would see 400 units output each day for the next six days. The second release on day four would arrive just as the last of the first release was being pulled into the process center. The basic relations between capacity, work in process, and the time to traverse a process center are the subject of Chapter 7.

Thus, in spite of its hopeful introduction and worthy goals, there are fundamental problems with CRP. First, there are enormous data requirements, and the output is voluminous and tedious. Second is the fact that it offers no remedy to an overloaded situation. Finally, since the procedure uses infinite loading and many modern systems can perform true finite capacity loading, fewer and fewer companies are seriously using CRP.

The **material requirements planning** module of all early versions of MRP II and many modern ERP systems is identical to the MRP procedure described earlier. The output of MRP is the **job pool,** consisting of planned order releases. These are released onto the shop floor by the **job release** function.

3.2.4 Short-Term Control

The plans generated in the long- and intermediate-term planning functions are implemented in the short-term control modules, of **job release, job dispatching,** and **input/output control.**

Job release converts planned order releases to scheduled receipts. One of the important functions of job release is **allocation.** When there are several high-level items that use the same lower-level part, a conflict can arise when there is an insufficient quantity on hand. By allocating parts to one job or another, the job release function can rationalize these conflicts. Suppose there are two planned order releases that require component A. Suppose further that there is enough stock on hand of component A for either job to be released but not for both. The first POR also requires component B for which there is plenty of stock, while the other POR requires component C for which there is insufficient stock. The job release function will allocate the available stock to the first POR since there is enough stock of both components A and B to start the job. A shortage notice would be generated for the second POR, which would remain in the job pool until it could be released.

Once a job or purchase order is released, some control must be maintained to make sure it is completed on time with the correct quantity and specification. If the job is for purchased components, the purchase order must be tracked. This is a straightforward practice of monitoring when orders arrive and tracking outstanding orders. If the job is for internal manufacture, this falls under the function known as **shop floor control (SFC)** or **production activity control (PAC).** Throughout this book we use the term SFC, as it is more traditional and more widely used. Within SFC are two main functions: **job dispatching** and **input/output control.**

Job Dispatching. The basic idea behind job dispatching is simple: Develop a rule for arranging the queue in front of each workstation that will maintain due date integrity while keeping machine utilization high and manufacturing times low. Many rules have been proposed for doing this.

One of the simplest dispatching rules is known as **shortest process time,** or **SPT.** Under SPT, jobs at the process center queue are sorted with the shortest jobs first in line. Thus, the job in the queue having the shortest processing time will always be performed next. The effect is to clear out small jobs and get them through the plant quickly. Use of SPT typically decreases average manufacturing times and increases machine utilization. *Average* due date performance is also generally quite good, even though due dates are not considered in the ordering.

Problems with SPT occur whenever there are particularly *long* jobs. In such cases, jobs can sit for a long time without ever being started. Thus, while average due date performance of SPT is good, the variance of the lateness can be quite high. One way to avoid this is to use a rule known as SPT^x, where x is a parameter. By this rule, the next job to be worked will be the one with the shortest processing time *unless* a job has been waiting x time units or longer, in which case it becomes the next job. This rule seems to yield reasonably good performance in many situations.

If jobs are all approximately the same size and routings are fairly consistent, a good dispatching rule is **earliest due date,** or **EDD.** Under EDD, the job closest to its due date is worked on next. EDD exhibits reasonably good performance under the above conditions, but typically does not work better than SPT under more general conditions.

Here are three other common rules.

Least slack: The slack for a job is its due date minus the remaining process time (including setups) minus the current time. The highest priority is the job with the lowest slack value.

Least slack per remaining operation: This is similar to the least slack rule except we take the slack and divide it by the number of operations remaining on the routing. Again, the highest-priority job has the smallest value.

Critical ratio: Jobs are sorted according to an index computed by dividing the time remaining (i.e., due date minus the current time) by the number of hours of work remaining. If the index is greater than one, the job should finish early. If it is less than one, the job will be late; and if it is negative, it is already late. Again, the highest-priority job has the smallest value of the critical ratio.

There are at least 100 different dispatching rules that have been offered in the operations management literature. A good survey of many of these is found in Blackstone et al. (1982), where the authors test various rules by using a simulated factory under a broad range of conditions.

Of course, no dispatching rule can work well all the time, because, by their very nature, dispatching rules are myopic. The only consistent way to achieve good schedules is to consider the shop as a whole. The problem with doing this is that (1) the shop scheduling problem is extremely complex and can require an enormous amount of computational time and (2) the resulting schedules are often not intuitive. We will address the scheduling problem more fully in Chapter 15.

Input/Output Control. Input/output (I/O) control was first suggested by Wight (1970) as a way to keep lead times under control. I/O control works in the following way:

1. Monitor the WIP level in each process center.
2. If the WIP goes above a certain level, then the current release rate is too high, so reduce it.
3. If it goes below a specified lower level, then the current release rate is too low, so increase it.
4. If it stays between these control levels, the release rate is correct for the current conditions.

The actions—reduce and increase—must be done by changing the MPS.

I/O control provides an easy way to check releases against available capacity. However, by waiting until WIP levels have become excessive, the system has, in many respects, already gone out of control. This may be one reason that so-called pull systems (e.g., Toyota's kanban system) may work better than push systems such as MRP, MRP II, and ERP. While these systems control releases (via the MPS) and measure WIP levels (via I/O control), kanban systems control WIP directly and measure output rates daily. Thus, kanban does not allow WIP levels to become excessive and detects problems (i.e., production shortfalls) quickly. Kanban is discussed in greater detail in Chapter 4, while the basics of push and pull are explored more fully in Chapter 10.

3.3 Beyond MRP II—Enterprise Resources Planning

In the years following the development of MRP II, a number of would-be successors were offered by vendors and consultants. MRP III never quite caught on, nor did the indigestibly acronymed BRP (business requirements planning). Finally, in spite of its gastronomically unpleasant acronym, enterprise resources planning (ERP) has emerged victorious.

This is due largely to the success of a few vendors, notably SAP, who have targeted not only manufacturing operations but *all* operations (e.g., manufacturing, distribution, accounting, financial, and personnel) of a company. Hence, the system offered is designed to control the entire *enterprise*.

SAP's R/3 software is typical of an interwoven comprehensive ERP system. The system can "act as a powerful network that can speed decision-making, slash costs, and give managers control over global empires at the click of a mouse," according to *Business Week* (Edmonson 1997). Within such "trade hype" is a kernel of truth. ERP systems are linking information together in ways that make it much easier for upper management to have a more global picture of operations in almost real time.

Advantages of this integrated approach include

1. Integrated functionality
2. Consistent user interfaces
3. Integrated database
4. Single vendor and contract
5. Unified architecture and tool set
6. Unified product support

But there are also disadvantages, including

1. Incompatibility with existing systems
2. Long and expensive implementation
3. Incompatibility with existing management practices
4. Loss of flexibility to use tactical point systems
5. Long product development and implementation cycles
6. Long payback period
7. Lack of technological innovation

In spite of any of these perceived drawbacks, ERP has enjoyed remarkable success in the marketplace, as we discuss below.

3.3.1 History and Success of ERP

The success of ERP is at least partly due to three coincident undercurrents preceding its development. The first is recognition of a field that has come to be called **supply-chain management (SCM).** In many ways, SCM extends traditional inventory control methods over a broader scope to include distribution, warehousing, and multiple production locations. Importantly, defining a function called supply-chain management has led to an appreciation of the importance of logistical issues. We see the importance of this area reflected in the growth of trade organizations such as the Council of Logistics Management, which grew from 6,256 members in 1990 to almost 14,000 in 1997.

The second trend that spurred acceptance of ERP was the **business process reengineering (BPR)** movement (see Hammer and Champy 1993). Prior to the 1990s, few companies would have been willing to radically change their management structures to support a new software package. But BPR has taught managers to think in terms of radical change. Today, many managers feel that one of the benefits of ERP implementation is the chance to reengineer their operations.

The third trend is the explosive growth in distributed processing and the power of smaller computers. An MRP run that took a weekend to run on a million-dollar computer in the 1960s can now be done on a laptop in a few seconds. Instead of a central repository for all corporate data, information is now stored where used on a personal computer or a workstation. These are linked via an intracompany network, and the data are shared by all functions. The latest offerings of ERP vendors are designed with exactly this architecture in mind (Parker 1997).

The growth of ERP sales indicates the degree of its acceptance. In 1989 total sales for MRP II at $1.2 billion accounted for just under one-third of the total software sales in the United States (*Industrial Engineering* 1991). Worldwide sales for the top 10 vendors of ERP alone were $2.8 billion in 1995, $4.2 billion in 1996, and $5.8 billion in 1997 (Michel 1997). One company, SAP, alone sold more than $3.2 billion in ERP software in 1997 (Edmonson 1997).

However, large sales of software are not the whole picture. Many companies are disenchanted at the sometimes staggering cost of implementation. In a survey of *Fortune* 1000 firms that had implemented ERP, 44 percent reported they spent at least four times as much on implementation help (e.g., consultants) as on the software itself. We are aware of several companies that have canceled projects after sepending millions, not wanting to "throw good money after bad."

Nonetheless, in spite of the high cost, some companies report enormous productivity improvements. Bob Barett, vice-president at Monsanto Co., finished installing the accounting module of SAP in July 1996. He cited the software as responsible for a reduction in the planning cycle from six weeks to three, lower inventories, less working capital, an increase in its bargaining power with suppliers, all of which led to an estimated savings of $200 million per year to the company (Edmonson 1997).

3.3.2 An Example: SAP R/3

The software offering of SAP known as R/3 is a typical ERP system. R/3 utilizes client/server computer technology to provide what SAP calls a **data warehouse.** This allows common access by all applications to a single data set. It also provides the capability to share data with other software via a general interface.

Like most ERP systems, R/3 is a large, transaction-oriented software package. SAP has organized it into four application suites: financial, human resource, manufacturing and logistics, and sales and distribution. Each of these has numerous application programs. Among them, interestingly, is a simple material requirements planning module that is almost logically identical to that written by Orlicky 30 years ago.

SAP's R/3 is constantly being updated with additional modules, including modules for specific industries. A key desire is to establish what are best practices and then to incorporate these into the software. Many companies using SAP have radically changed their management procedures to conform to the software and these best practices. Indeed, this radical reduction in individuality on the part of corporate managers may ultimately prove to be the greatest social consequence of the SAP success. But since codifying practices that are several years old is hardly the best strategy for maintaining a competitive advantage, this may leave some ERP users vulnerable to more creative competitors.

3.3.3 Manufacturing Execution Systems

A **manufacturing execution system (MES)** is an automated implementation of what MRP II called *shop floor control.* Unlike SFC, however, MES tracks work in process automatically; records process, yield, and quality data; executes a schedule; releases new jobs into the system; etc. Whether the MES is part of ERP or not can generate a hot debate among consultants and software purveyors. Nonetheless, with the increasing integration of more and more business functions by software offerings like the SAP R/3, it is doubtful that MES will remain an independent entity for long.

3.3.4 Advanced Planning Systems

While ERP systems integrate company data, **advanced planning systems (APS)** are used to analyze the data up and down the organization. The capabilities of APS are as varied as the vendors supplying the software. Most APS applications are memory-based algorithms that perform functions. These include finite capacity scheduling, forecasting, available to promise, demand management, warehouse management, distribution and traffic management, etc. In many cases, ERP vendors partner with more specialized software developers to provide these functions. Interestingly, this add-on approach has frequently resembled the earlier MRP II approach to "fixing" the MRP problem of infinite backward scheduling of reworking the schedule *after* it has been generated.

3.4 Conclusions

Material requirements planning evolved from the fundamental recognition of the difference between dependent and independent demand. It was also the first major application of modern computers in production control. MRP provides a simple method for ordering materials based on needs, as established by a master production schedule and bills of material. As such, it is well suited for use in controlling the purchasing of components. However, in the control of production, there are still problems.

Manufacturing resources planning, or MRP II, was developed to address the problems of MRP and to further integrate business functions into a common framework. MRP II has provided a very general control structure that breaks the production control problem into a hierarchy based on time scale and product aggregation. Without such a hierarchical approach, it would be virtually impossible to address the huge problem of coordinating thousands of orders with hundreds of tools for thousands of end items made up of additional thousands of components. More recently, ERP has integrated this hierarchical approach into a formidable management tool that can consolidate and track enormous quantities of data.

Despite the important contributions of MRP, MRP II, and ERP to the body of manufacturing knowledge, there are fundamental problems with the basic model underlying these systems (i.e., the assumptions of infinite capacity and fixed lead times that are found even in some of the most sophisticated ERP systems). As we will discuss further in Chapter 5, a critical issue for the long term is how to resolve the basic difficulties of MRP while retaining its simplicity and broad applicability. We will address this problem in Part III, after we have taken note of the insights offered by the just-in-time (JIT) movement in Chapter 4 and have developed some basic relationships concerning factory behavior in Part II.

Study Questions

1. What is the difference between raw material inventory, work-in-process (WIP) inventory, and finished goods inventory?
2. What is the difference between independent demand and dependent demand? Give several examples of each.
3. What level is an end item in a bill of material? What is a low-level code? What is the low-level code for an end item? Draw a bill of material for which component 200 occurs on two different levels and has a low-level code of three.
4. What is the master production schedule, and what does it provide for an MRP system?
5. How do you convert gross requirements to net requirements? What is this procedure called?
6. Why are scheduled receipts adjusted before any net requirements are computed?
7. Which lot-sizing rule results in the least inventory?
8. What are the tradeoffs considered in lot sizing?
9. In what respect is the Wagner–Whitin algorithm optimal? How is it sometimes impractical (i.e., what does it ignore)?
10. Which of the following lot-sizing rules possess the so-called Wagner–Whitin property?
    a. Wagner–Whitin
    b. Lot-for-lot
    c. Fixed order quantity (e.g., all jobs have size of 50)
    d. Fixed order period
    e. Part-period balancing
11. How do planned lead times differ from actual lead times? Which is typically bigger, the planned lead time or the average actual lead time? Why?
12. What assumption in MRP makes the implicit assumption of infinite capacity? What is the impact of this assumption on planned lead times? On inventory?
13. What is the difference between a planned order receipt and a planned order release? How does a scheduled receipt differ from a planned order release?
14. What is the difference between a scheduled receipt and a firm planned order? How are they similar?
15. Why do we perform all the MRP processing for one level before going to the next-lower level? What would happen if we did not?
16. What is the bill-of-material explosion?
17. What is pegging? How does it help in bottom-up replanning?
18. What is the effect of having safety stock when computing net requirements?
19. What is the difference between having a safety lead time of one period and simply adding one period to the planned lead time? What is the same?
20. What is nervousness in an MRP system? How is it caused? Why is it bad? What are some things that can be done to prevent it?
21. What is MRP II? Why was it created?
22. Why might rough-cut capacity planning be optimistic? Why might it be pessimistic?
23. Why is capacity requirements planning not very accurate? What assumptions are made in CRP that are the same as those in MRP?
24. What is the purpose of dispatching? What are dispatching rules? Why does shortest process time seem to work pretty well? When does earliest due date work well?
25. What is the purpose of input/output control? Why is it often "too little, too late"?

Problems

1. Suppose an assembly requires five components from five different vendors. To guarantee starting the assembly on time with 90 percent confidence, what must the service level be for each of the five components? (Assume the same service level for each component.)
2. End item A has a planned lead time of two weeks. There are currently 120 units on hand and no scheduled receipts. Compute the planned order releases using lot-for-lot and the MPS shown here:

Week	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Demand	41	44	84	42	84	86	7	18	49	30

3. Using the information in Problem 2, compute the planned order releases using part-period balancing where the ratio of setup cost to the holding cost is 200.
4. *(Challenge)* With the information in Problem 2, compute the planned order releases using Wagner–Whitin, where the ratio of setup cost to holding cost is 200. How much lower is the cost of the plan than in the previous case?
5. Rework Problem 2 with 50 units of safety stock. What is different from Problem 2?
6. Rework Problem 2 with a planned lead time of two periods and a safety lead time of one period. What is different from Problem 2?
7. Suppose demand for a power steering gear assembly is given by

Gear	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Demand	45	65	35	40	0	0	33	0	32	25

   Currently there are 150 parts on hand. Production is planned using the **fixed order period** method and two periods. The lead time is three periods. Determine the planned order release schedule.
8. Consider the previous problem, but assume that a scheduled receipt for 50 parts is scheduled to arrive in period five.
   a. What changes, if any, need to be made to the scheduled receipt?
   b. Using the same lot-sizing rule and lead time, compute the planned order release schedule.
9. Demand for a power steering gear assembly is given by

Gear	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Demand	14	12	12	13	5	90	20	20	20	20

   Currently there are 50 parts on hand. The lot-sizing rule is, again, **fixed order period** using two periods. Lead time is four periods.
   a. Determine the planned order release schedule for the gear.
   b. Suppose each gear assembly requires two pinions. Currently there are 175 pinions on hand, the lot-sizing rule is lot-for-lot, and the lead time is one period. Determine the gross requirements and then the planned order release schedule for pinions.

c. Suppose management decreases the demand forecast for the first period to 12. What happens to the planned order release schedule for gears? What happens to the planned order release schedule for pinions?

10. Consider an end item composed of a single component. Demand for the end item is 20 in week one, four in week two, two in week three, and zero until week eight when there is a demand of 50. Currently there are 25 units on hand and no scheduled receipts. For the component there are 10 units on hand and no scheduled receipts.

Planned order releases for all items are computed using the Wagner–Whitin algorithm with a setup cost of $248 and a carrying cost of $1 per week. The planned lead time for the end item is one week, and for the component it is three weeks.

a. Compute the planned order releases for the end item and the component. Are there any problems?

b. The forecast for demand in week eight has been changed to 49. Recompute the planned order releases for the end item and the component. Are there any problems?

c. Suppose the first two weeks' planned order releases from part *a* had been converted to *firm planned orders*. Do the computation again after changing the demand in week 8 to 49. Are there any problems? Comment on nervousness and the use of firm planned orders.

11. Generate the MRP output for items A, 200, 300, and 400 using the following information. (*Note:* End item A is the same as in Problem 3.)

- Bills of material:

  A: Two 200 and one 400
  200: Raw material
  300: Raw material
  400: One 200 and one 300

- Master production schedule:

Week	1	2	3	4	5	6	7	8	9	10
Demand (A)	41	44	84	42	84	86	7	18	49	30

- Item Master and Inventory Data:

Item	Amount on Hand	Amount on Order	Due	Lead Time (Weeks)	Lot Sizing Rule (Setup/Hold)
A	120	0		2	PPB (200)
200	300	200 100	3 5	2	Lot-for-lot
300	140	100 100	4 7	2	Lot-for-lot
400	200	0		3	Lot-for-lot

12. Consider a circuit-board plant that makes three kinds of boards: Trinity, Pecos, and Brazos. The bills of material are shown here:

Trinity: 1 subcomposite 111 and 1 subcomposite 112
Pecos: 1 subcomposite 211 and 1 subcomposite 212
Brazos: 1 subcomposite 311 and 1 subcomposite 312
Subcomposite 111: Core 1
Subcomposite 112: Core 2
Subcomposite 211: Core 1
Subcomposite 212: Core 1
Subcomposite 311: Core 1
Subcomposite 312: Core 2
All cores: raw material

Recently, the Lamination and the Core Circuitize operations have been bottlenecks. The unit hours (i.e., time for a single board on the bottleneck tools) in these areas are given below. These times are in hours and include inefficiencies such as operator unavailability, downtime, setups, and so forth.

Board	Trinity	Pecos	Brazos
Lam	0.020	0.022	0.020
Core Cir	0.000	0.000	0.000

Board	S111	S112	S211	S212	S311	S312
Lam	0.015	0.013	0.015	0.013	0.015	0.015
Core Cir	0.025	0.023	0.028	0.023	0.027	0.028

Board	Core 1	Core 2
Lam	0.008	0.008
Core Cir	0.000	0.000

The anticipated demand for the next six weeks is as follows:

Week	1	2	3	4	5	6
Trinity	7,474	2,984	5,276	5,516	3,818	3,048
Pecos	6,489	5,596	7,712	7,781	3,837	4,395
Brazos	3,898	3,966	3,858	6,132	5,975	6,051
Total	17,861	12,546	16,846	19,429	13,630	13,494

a. Construct bills of capacity for Trinity, Pecos, and Brazos at Lamination and Core Circuitize.
b. Use these bills to determine the load for each of the next six weeks at both Lamination and Core Circuitize. The process centers operate five days per week for three shifts per day (24 hours per day). Breaks and lunches are included in the unit hour data. There are six Lamination presses and eight expose machines (the bottleneck) in Core Circuitize. Which weeks are over- or underloaded? What should be done?

13. The Wills and Duncan parts must pass through process center 22. Wills is released to process center 22 while Duncan must first pass through process center 21 before going to process center 22. The planned lead time for going through process center 22 is three days, while the time to go through process center 21 is two days. There are 16 hours of capacity at process center 22 per day. Each Wills takes 0.04 hour while a Duncan takes 0.025 hour at process center 22. Currently there are 300 Wills units that have been in process center 22 for one day and 200 units that have been there for two days. Releases to the process center (i.e., Wills to 22 and Duncan to 21) are shown below. There are also 225 of the Duncan parts that have been in the process center for one day and 200 that have been there for two days. There are also 250 units in process center 21 that have been there for one day and 200 units that have been there for two days. The releases are as follows:

Day	Today	1	2	3	4	5
Wills	250	300	350	300	300	300
Duncan	250	150	150	150	150	150

a. Determine how many Wills parts will leave process center 22 on each day.
b. Determine how many Duncan parts will leave process center 22 on each day.
c. Compute the load profile for process center 22.

CHAPTER

4 THE JIT REVOLUTION

I tip my hat to the new constitution
Take a bow for the new revolution
Smile and grin at the change all around
Pick up my guitar and play
Just like yesterday
Then I get on my knees and pray
WE DON'T GET FOOLED AGAIN!

The Who

4.1 The Origins of JIT

In the 1970s and 1980s, while American manufacturers were (or were not) joining the MRP crusade, something entirely different was afoot in Japan. Much like the Americans had done in the 19th century, the Japanese were evolving a distinctive style of manufacturing that would eventually spark a period of huge economic growth. The manufacturing techniques behind the phenomenal Japanese success have become collectively known as just-in-time (JIT). They represent an important chapter in the history of manufacturing management.

The roots of JIT undoubtedly extend deep into Japanese cultural, geographic, and economic history. Because of their history of living with space and resource limitations, the Japanese are inclined toward conservation. This has made tight material control policies easier to accept in Japan than in the "throw-away society" of America. Eastern culture is also more systems-oriented than Western culture with its reductionist scientific roots. Policies that cut across individual workstations, such as cross-trained floating workers and *total* quality management, are more natural in this environment. Geography has also certainly influenced Japanese practices. Policies involving delivery of materials from suppliers several times per day are simply easier in Japan, where industry is spatially concentrated, than in America with its wide-open spaces. Many other structural reasons for the Japanese success have been advanced. However, since a manufacturing firm has no control over these factors, they are of limited interest to us here.

Of greater relevance are the JIT practices themselves. The most direct source for many of the ideas represented by JIT is the work of Taiichi Ohno at Toyota Motor Company. According to Ohno, Toyota began its innovative journey in 1945 when Toyoda Kiichiro, president of Toyota, demanded that his company "catch up with America in three years. Otherwise, the automobile industry of Japan will not survive" (Ohno 1988, 3). At the time, Japan's economy was shattered by the war, labor productivity was one-ninth that of the United States, and automotive production was at minuscule levels. Obviously, Toyota did not catch up to the Americans in three years, but it set in motion an effort that would eventually achieve Toyoda's goal and would spark the most fundamental changes in manufacturing management since the scientific management movement of the 1920s.

Ohno, who moved to Toyota Motor from Toyoda Spinning and Weaving in 1943, recognized that the only way to become competitive with America would be to close the huge productivity gap between the two countries. This, he argued, could only be done through waste elimination aimed at lowering costs. But unlike the American automobile companies, Toyota could not reduce costs by exploiting economies of scale in giant mass production facilities. The market for Japanese automobiles was simply too small. Thus, the managers at Toyota decided that their manufacturing strategy had to be to produce many models in small numbers.

The principal challenge from a production control standpoint was to maintain a smooth production flow in the face of a varied product mix. Moreover, to avoid waste, this had to be accomplished without large inventories. Ohno described the system evolved at Toyota to address this challenge as resting on two pillars:

1. **Just-in-time.**
2. **Autonomation,** or automation with a human touch.

He attributed the motivation for the just-in-time idea to Toyoda Kiichiro, who used the words to describe the ideal automobile assembly process. Ohno's model for JIT was the American-style supermarket, which appeared in Japan in the mid-1950s. In a supermarket, customers get what is needed, at the time needed, and in the amount needed. In Ohno's factory analogy, a workstation is a customer that gets materials from an upstream workstation that acts as a sort of store. Of course, in a supermarket, stock is replenished from a storeroom or by means of deliveries, while in a factory, replenishment requires production by an upstream workstation. His goal was to have each workstation acquire the required materials from upstream workstations precisely as needed, or **just in time.**

Just-in-time flow requires a very smoothly operating system. If materials are not available when a workstation requires them, the entire system may be disrupted. As we discuss in the next section, this has serious implications for the production environment. One means for avoiding disruptions is Ohno's concept of **autonomation,** which refers to machines that are both *automated,* so that one worker can operate many machines, and *foolproofed,* so that they automatically detect problems. Ohno received his inspiration for the idea of autonomation from Toyoda Sakichi, inventor of the automatically activated loom used at Toyoda Spinning and Weaving. Automation was essential for achieving the productivity improvements necessary to catch up with Americans. Foolproofing, which helps operators intervene in an automated process at the right time, is primarily what Ohno meant by "automation with a human touch." He viewed the combination as necessary to avoid disruptions in a JIT environment.

Between the late 1940s and the 1970s, Toyota instituted a host of procedures and systems for implementing just-in-time and autonomation. These included the now famous kanban system, which we will discuss in detail later, as well as a variety of systems related to setup reduction, worker training, vendor relations, quality control, and many

others. While not all the efforts were successful, many were, and the overall effect was to raise Toyota from an inconsequential player in the automotive market in 1950 to one of the largest automobile manufacturers in the world by the 1990s.

4.2 JIT Goals

To achieve Ohno's goal of workstations acquiring materials just in time, a pristine production environment is necessary. Perhaps as a result of the Japanese propensity to speak metaphorically,[1] or perhaps because of the difficulty of translating Japanese descriptions to English (the words translate, but the cultural context does not), this need has often been stated in terms of absolute ideals. For example, Robert Hall, one of the first American authors to describe JIT, used terms like **stockless production** and **zero inventories.** However, he did not literally mean that firms should operate without inventory. Rather, he wrote

> *Zero Inventories* connotes a level of perfection not ever attainable in a production process. However, the concept of a high level of excellence is important because it stimulates a quest for constant improvement through imaginative attention to both the overall task and to the minute details. (Hall 1983, 1)

Edwards (1983) pushed the use of absolute ideals to its limit by describing the goals of JIT in terms of the **seven zeros,** which are required to achieve **zero inventories.** These, along with the logic behind them, are summarized as follows:

1. **Zero defects.** To avoid disruption of the production process in a JIT environment where parts are acquired by workstations only as they are needed, it is essential that the parts be of good quality. Since there is no excess inventory with which to make up for the defective part, a defect will cause a delay. Thus, it is essential that every part be made correctly the first time. The only acceptable defect level is zero, and it is not possible to wait for inspection points to check quality. Quality must occur at the source.
2. **Zero (excess) lot size.** In a JIT system, the goal is to replenish stock taken by a downstream workstation as it is taken. Since the downstream workstations may take parts of many types, maximum responsiveness is maintained if each workstation is capable of replacing parts one at a time. If, instead, the workstation can only produce parts in large batches, then it may not be possible to replenish the stocks of all parts quickly enough to avoid delays. This goal is more frequently stated as a **lot size of one.**
3. **Zero setups.** The most common reason for large batch sizes in production systems is the existence of significant setup times. If it takes several hours to change a die on a machine to produce a different part type, then it only makes sense that large batches of each part will be run between setups. Small lot sizes would lead to frequent setups and thereby seriously degrade capacity. Hence, eliminating setups is a precondition for achieving lot sizes of one.
4. **Zero breakdowns.** Without excess WIP in the system to buffer machines against outages, breakdowns will quickly bring production to a halt throughout the line. Therefore, an ideal JIT environment cannot tolerate unplanned machine failures (or operator unavailability, for that matter).
5. **Zero handling.** If parts are made exactly in the quantities and at the times required, then material must not be handled more than is absolutely necessary. No extra

[1] Shigeo Shingo, who along with Ohno was influential in developing the Toyota system, writes such things as "the Toyota production wrings water out of towels that are already dry" (Shingo 1990, 54) and "there is nothing more important than planting 'trees of will' " (Shingo 1990, 172).

moves to and from storage can be tolerated. The ideal is to feed the material directly from workstation to workstation with no intermediate pauses. Any additional handling will move the system away from just-in-time operation, since parts will have to be produced early to accommodate the additional time spent in handling.

6. **Zero lead time.** When perfect just-in-time parts flow occurs, a downstream workstation requests parts and they are provided immediately. This requires zero lead time on the part of the upstream workstation. Of course, lot sizes of one go a long way toward reducing the effective lead time required to produce parts, but the actual processing time per part is also important, as is waiting (queuing) time. The goal of zero lead time is very close to the core of the zero inventories objective.

7. **Zero surging.** In a JIT environment where parts are produced only as needed, the flow of material through the plant will be smooth as long as the production plan is smooth. If there are sudden changes (surges) in the quantities or product mix in the production plan, then, since no excess WIP in the system can be used to level these changes, the system will be forced to respond. Unless there is substantial excess capacity in the system, this will be impossible and the result will be disruptions and delays. A level production plan and a uniform product mix are thus important inputs to a JIT system.

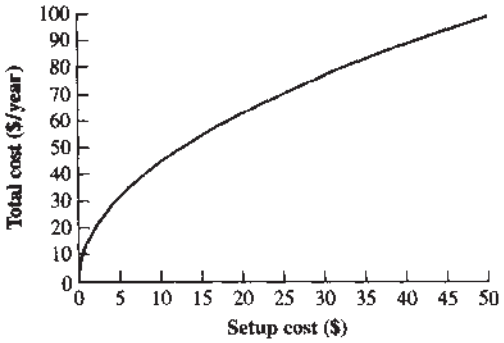
Obviously, the seven zeros are no more achievable in practice than is zero inventory. Zero lead time with no inventory literally means instantaneous production, which is physically impossible. The purpose of such goals, according to the JIT proponents who make use of them, is to inspire an environment of continual improvement. No matter how well a manufacturing system is running, there is always room for improvement. Gauging progress against absolute ideals provides both an incentive and a measure of success.

4.3 The Environment as a Control

The JIT ideals suggest an aspect of the Japanese production techniques that is truly revolutionary: the extent to which the Japanese have regarded the production environment as a control. Rather than simply reacting to such things as machine setup times, vendor deliveries, quality problems, production schedules, and so forth, they have worked proactively to shape the environment. By doing this, they have consciously made their manufacturing systems easier to manage.

In contrast, Americans, with their scientific management roots and reductionist tendencies, have been prone to isolating individual aspects of the production problem and working to "optimize" them separately. Americans took setup times (or costs) as fixed and tried to come up with optimal lot sizes (e.g., the EPL model). The Japanese tried to eliminate—or at least reduce—setups and thereby eliminate the lot-sizing problem. Americans took due dates as exogenously provided and attempted to optimize the production schedule (e.g., the Wagner–Whitin model). The Japanese realized that due dates are negotiated with customers and worked to integrate marketing and manufacturing to provide production schedules that do not require precise optimization or abrupt changes. Americans took infrequent, expensive deliveries from vendors as given and tried to compute optimal order sizes (e.g., the EOQ model). The Japanese worked to set up long-term agreements with a few vendors to make frequent deliveries feasible. Americans took quality defects as given and set up elaborate inspection procedures to find them. The Japanese worked to ensure that both vendors outside the plant and operators inside the plant were aware of quality requirements and equipped with the necessary tools to main-

FIGURE 4.1
Total cost versus setup cost in EOQ model



tain them. American manufacturing engineers got product specifications "thrown over the wall" from design engineers and did their best to adapt the manufacturing process to accommodate them. Japanese manufacturing and design engineers worked together to ensure designs that are practical to manufacture.

These distinctions between America and Japan are not a direct indictment of American models themselves. Indeed, as we highlighted in Chapter 2, models can offer valuable insights. For instance, the EOQ model suggests that total cost (i.e., setup plus inventory carrying cost) depends on the cost per setup according to the formula

$$\text{Annual cost} = \sqrt{2ADh}$$

where A is the setup cost (in dollars), D is the demand rate (in units per year), and h is the unit carrying cost (in dollars per unit per year). If we let $D = 100$ and $h = 1$ for purposes of illustration, then we can plot the relationship between total cost and setup cost as in Figure 4.1. This figure, and hence the model, clearly indicates that there are benefits to be gained from reducing the cost per setup. Since this cost presumably decreases with setup time, the EOQ model *does* point up the value of setup time reduction. However, while the insight is there, the sense of its strategic importance is not. Consequently, serious setup time reduction methodologies were evolved not in America, but in Japan.

In setups and many others areas, the Japanese have taken a holistic, systems view of manufacturing. Consequently, they have been able to identify policies that cut across traditional functions and to manage the interfaces between functions. Thus, while the specific techniques of JIT (which we shall discuss below) are important, the *systems approach* to transforming the manufacturing environment and the *constant attention to detail* over an extended period of time are fundamental. Ohno was urging just this with his admonition to "ask why five times," by which he meant that one should iteratively seek out and remove obstacles to the primary objective. A typical sequence of what Ohno had in mind might go as follows: A workstation becomes starved for work. Why? An upstream machine went down. Why? A pump failed. Why? It ran out of lubricant. Why? A leaky gasket was not detected. Why? And so on. This type of relentless pursuit of understanding and improvement may well be the real reason for Japan's remarkable success.

4.4 Implementing JIT

As the previous discussion makes clear, JIT is more than a system of frequent materials delivery or the use of **kanban** to control work releases. At the heart of the manufacturing

systems developed by Toyota and other Japanese firms is a careful restructuring of the production environment. Ohno (1988, 3) was very clear about this:

> Kanban is a tool for realizing just-in-time. For this tool to work fairly well, the production process must be managed to flow as much as possible. This is really the basic condition. Other important conditions are leveling production as much as possible and always working in accordance with standard work methods.

Only when the environmental changes have been made can the specific JIT techniques be effective. We now turn to the key environmental issues that must be addressed in order to implement JIT.

4.4.1 Production Smoothing

As called for by the zero surging ideal, JIT requires a relatively smooth production plan. If either the volume or product mix varies greatly over time, it will be very difficult for workstations to replenish stock just in time. To return to the supermarket analogy, if all customers decided to do their shopping on Tuesday, or if all shoppers decided to buy canned tomatoes at the same time, stockouts would be very likely. However, because customers are spread over time and buy different mixes of products, the supermarket is able to replenish the shelves a little at a time and, for the most part, avoid stockouts.

In a manufacturing system, requirements are ultimately generated by customer demand. However, the sequence in which products are manufactured need not match the sequence in which they will be purchased by customers. Indeed, since customer demands are almost never completely known by the manufacturer in advance, this is not even possible. Instead, plants make use of a **master production schedule (MPS)** that specifies which products are to be produced in each time interval. As we noted in the previous chapter, MRP systems typically make use of time intervals (buckets) of a week or longer for their MPS.

A first condition for JIT, therefore, is to ensure that the MPS is reasonably level over time. As we noted in Chapter 3, many ERP systems contain MPS modules for facilitating the smoothing process. This development was stimulated in part by the Japanese JIT movement.

But even a smoothed MPS that specifies only weekly or monthly requirements could allow surges within the week or month that exceed the system's ability to meet the demands in a just-in-time fashion. Hence, the Toyota system and virtually all other JIT systems make use of a **final assembly schedule (FAS),** which specifies daily, or even hourly, requirements. Developing a level FAS from the MPS involves two steps:

1. Smoothing aggregate production requirements.
2. Sequencing final assembly.

Smoothing aggregate production is straightforward. If the MPS calls for monthly production of 10,000 units and there are 20 working days in the month, then the FAS will call for 500 units per day. If there are two shifts, this translates into 250 units per shift. If each shift is 480 minutes long, then the average time between outputs will have to be $480/250 = 1.92$ minutes per unit. In a perfect situation, this means we should produce at a rate of exactly one unit every 1.92 minutes. A system in which discrete parts are produced at a fairly steady flow rate is called a **repetitive manufacturing** environment. The kanban system developed by Toyota, which we will discuss later, is well suited only to repetitive manufacturing environments.

In reality, we are unlikely to produce exactly one unit every 1.92 minutes. Small deviations are not a problem; if the line falls behind during one hour but catches up dur-

ing the next, fine. However, if the system departs from the specified rate over a period exceeding a shift or a day, corrective action (e.g., overtime) is typically required. Maintaining a steady, predictable output stream is the only means by which a JIT system can consistently meet customer due dates. Hence, JIT systems generally include measures to promote maintenance of a steady flow (e.g., incentives for making production quotas).

Once the aggregate requirements of the MPS have been translated to daily rates, we must translate the product-specific requirements to a production sequence. We do this by breaking out the daily requirements according to the product proportions from the MPS. For instance, if the 10,000 units to be produced during the month consist of 50 percent (5,000 units) product A, 25 percent (2,500 units) product B, and 25 percent (2,500 units) product C, then this means that the daily production of 500 units should consist of

$$0.5 \times 500 = 250 \text{ units of A}$$
$$0.25 \times 500 = 125 \text{ units of B}$$
$$0.25 \times 500 = 125 \text{ units of C}$$

Furthermore, the products should be sequenced on the line such that these proportions are maintained as uniformly as possible. Thus, the sequence

$$\text{A–B–A–C–A–B–A–C–A–B–A–C–A–B–A–C} \cdots$$

will maintain a 50-25-25 mix of A, B, and C over time. Obviously, this requires a line that is flexible enough to support this type of **mixed model production** (i.e., producing several products at once on the same line), which is impossible unless setups between products are very short or nonexistent. Furthermore, since the production rate is one unit every 1.92 minutes, this sequence implies that the times between outputs of product A will be $2 \times 1.92 = 3.84$ minutes. Times between outputs of products B and C will be $4 \times 1.92 = 7.68$ minutes. The assembly line, as well as the rest of the plant, must be physically capable of handling these times.

Of course, most production requirements will not lend themselves to such simple sequences. In that case, it may be reasonable to slightly adjust the demand figures (e.g., when demands are actually rough forecasts) to accommodate a simple sequence; or it may be reasonable to depart slightly from a simple sequence by spreading leftover units as evenly as possible throughout the daily schedule. The objective, however, remains as level a flow as possible. This is in sharp contrast with the traditional American practice of producing a large batch of one product before shifting to the next and emphasizing attainment of production quotas only at the end of the month.

4.4.2 Capacity Buffers

An apparent difficulty with JIT lies in coping with unexpected disruptions, such as order cancellations or machine failures. In an MRP system, when production requirements change, the schedule is simply regenerated, some jobs may be expedited, and things continue. However, in a JIT system, where great pains have been taken to ensure a constant flow, another approach is required. Similarly, if a machine failure causes production to fall behind, the netting operation in MRP will include the unmet requirements in the next pass. The JIT system with its level production quotas has no intrinsic way to keep track of such shortages.

This rigidity is certainly a problem with "ideal" JIT. But ideal JIT only works in an ideal environment—as does almost anything. (If demand is absolutely level, perfectly predictable, and within capacity capabilities, then MRP will work extremely well and will result in just-in-time production.) However, real-world JIT systems are never ideal

and out of necessity contain measures for dealing with unanticipated disruptions. An approach commonly used by the Japanese is that of a capacity buffer. By scheduling the facility to less than 24 hours per day, the line can catch up if it falls behind. If production gets ahead of the desired rate, then workers are either sent home or directed to other tasks. If production falls behind the desired rate, either because of problems in the line or because of changes in the requirements, then the extra time is used. One way to allow for this is **two-shifting,** in which two shifts are scheduled per day, separated by a down period (Schonberger 1982, 137). The down period can be used for preventive maintenance or catch-up, if necessary. A popular approach is to schedule shifts "4–8–4–8," in which two eight-hour shifts are separated by four-hour down periods.

The capacity buffer offered by the availability of overtime serves as an alternative to the WIP buffers found in most MRP systems. If an unexpected occurrence, such as a machine outage, causes production to fall behind at a workstation, then WIP buffers can prevent other workstations from starving. In a JIT system where the WIP buffers are very small, a failure is very likely to cause starvation somewhere in the system. Thus, to keep the production rate constant, overtime will be needed. In effect, the Japanese have reduced WIP, so that production occurs just-in-time, but they have maintained excess capacity, just-in-case.

4.4.3 Setup Reduction

A work sequence like that suggested earlier, A–B–A–C–A–B–A–C–A–B–A–C–, is probably not workable if there are significant setup times required to switch production from one product to another. For instance, if each of the three products requires a different die that takes several hours to change over, there is no way to achieve the desired daily rate of 500 units while using a sequence that requires a die change after each part. In America these setups were traditionally regarded as given, and large lot sizes were used to keep the number of changeovers to a manageable level. In Japan, reducing the setup times to the point where changeovers no longer prevent a uniform sequence became something of an art form. Ohno reported setups at Toyota that were reduced from three hours in 1945 to three minutes in 1971 (Ohno 1988).

A number of good references provide specifics on the many clever techniques that have been used to speed machine changeovers (Hall 1983; Monden 1983; Shingo 1985), so we will not go deeply into details here. Instead, we will make note of some general principles that have been invoked to guide setup reduction efforts.

The key to a general approach to setup reduction is the distinction between an **internal setup** and an **external setup.** Internal setup operations are those tasks that take place when the machine is stopped (i.e., not producing product), while external setup operations are those tasks that can be completed while the machine is still running. For instance, removing a die is an internal task, while collecting the necessary tools to remove it is an external task. It is the internal setup that is disruptive to the production process, and hence this is the portion of the overall setup process that deserves the most intense attention. With this distinction in mind, Monden (1983) identifies four basic concepts for setup reduction:

1. *Separate the internal setup from the external setup.* The fact that current practice has the machine stopped while certain tasks are being completed does not guarantee that they are internal tasks. The setup reduction process must start by asking which tasks *must* be done with the machine stopped.
2. *Convert as much as possible of the internal setup to the external setup.* For example, if some components can be preassembled before shutting down the machine,

or if a die casting can be preheated before installing it, the internal setup time can be substantially reduced.

3. *Eliminate the adjustment process.* This frequently accounts for 50 to 70 percent of the internal setup time and is therefore critical. Jigs, fixtures, or sensors can greatly speed or even eliminate adjustments.

4. *Abolish the setup itself.* This can be done by using a uniform product design (e.g., the same bracket for all products), by producing various parts at the same time (e.g., stamping parts A and B in a single stroke and separating them later), or by maintaining parallel machines, each set up for a different product.

The references cited offer a host of techniques for implementing these concepts, ranging from quick-release bolts, to standardized tools and procedures, to parallel operations (e.g., two workers performing the setup in parallel), to color coding schemes, and so on. The real lesson from this diversity of ideas is, perhaps, the old maxim "Necessity is the mother of invention." The uniform production sequences used in JIT *demanded* quick changeovers, and the diligent efforts of Japanese engineers provided them.

4.4.4 Cross-training and Plant Layout

Ohno interpreted productivity improvement as a crucial goal for Toyota very early on. However, because of his concern with ensuring smooth material flow without excess WIP, productivity improvements could not be achieved by having workers produce large lots on individual machines. It rapidly became clear that a JIT system is much better served by multifunctional workers who can move where needed to maintain the flow. Furthermore, having workers with multiple skills adds flexibility to an inherently inflexible system, greatly increasing a JIT system's ability to cope with product mix changes and other exceptional circumstances.

To cultivate a multiskilled workforce, Toyota made use of a worker rotation system. The rotations were of two types. First, workers were rotated through the various jobs in the shop.[2] Then, once a sufficient number of workers were cross-trained, rotations on a daily basis were begun. Daily rotations served the following functions:

1. To keep multiple skills sharp.
2. To reduce boredom and fatigue on the part of the workers.
3. To foster an appreciation for the overall picture on the part of everyone.
4. To increase the potential for new idea generation, since more people would be thinking about how to do each job.

These cross-training efforts did indeed help the Japanese catch up with the Americans in terms of labor productivity. But they also fostered a great deal of flexibility, which Americans, with their rigid job classifications and history of confrontational labor relations, found difficult to match.

With cross-training and autonomation, it becomes possible for a single worker to operate several machines at once. The worker loads a part into a machine, starts it up, and moves on to another machine while the processing takes place. But remember, in a JIT system with very little WIP, it is important to keep parts flowing. Hence, it is not practical to have a worker staffing a number of machines that perform the same operation in a large, isolated process center. There simply will not be enough WIP to feed such an operation.

[2] It is interesting to note that managers were also rotated through the various jobs, in order to prove their abilities to the workers.

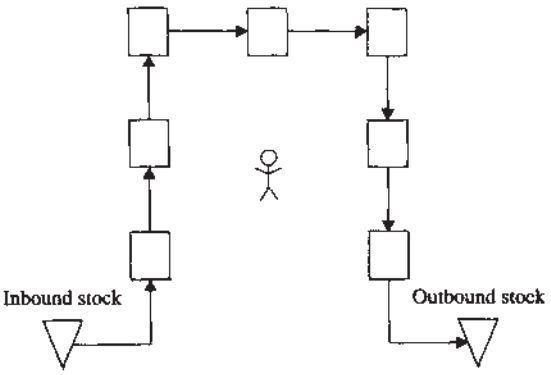


FIGURE 4.2
U-shaped manufacturing cell

A better layout is to have machines that perform successive operations located close to one another, so that the products can flow easily from one to another. A linear arrangement of machines, traditionally common to American facilities,[3] accommodates the product flow well, but is not well suited to having workers tend multiple machines because they must walk too far from machine to machine. To facilitate material flow and reduce walking time, the Japanese have tended toward U-shaped lines, or cells, as shown in Figure 4.2.

The advantages of U-shaped cells are as follows:

1. One worker can see and attend all the machines with a minimum of walking.
2. They are flexible in the number of workers they can accommodate, allowing adjustments to respond to changes in production requirements.
3. A single worker can monitor work entering and leaving the cell to ensure that it remains constant, thereby facilitating just-in-time flow.
4. Workers can conveniently cooperate to smooth out unbalanced operations and address other problems as they surface.

The use of cellular layouts in JIT systems precipitated a trend that gathered steam in the United States during the 1980s. One now sees U-shaped manufacturing cells in a variety of production environments, to the point where cellular manufacturing has become much more prevalent than the JIT systems that spawned it.

4.4.5 Total Quality Management

Although the basic techniques of quality control were developed and espoused long ago by Americans, particularly Shewhart (1931), Feigenbaum (1961), Juran (1965), and Deming (1950a, 1950b, 1960), it was within the Japanese JIT systems that quality was lifted to new and strategic importance. Schonberger (1983, 50) offers two possible reasons for why quality control "took" in Japan so much more readily than in America:

1. The Japanese historical abhorrence for wasting scarce resources (i.e., by making bad products).

[3]Linear layouts were essential in colonial water-powered plants, where machines were driven by belts from a central driveshaft. By the time steam and electricity replaced water power, straight production lines had become the norm in America.

2. The Japanese innate resistance to specialists, including quality control experts, which made it more natural to ensure quality at the point of production than to check it later at a quality control station.

Beyond these cultural factors is the simple fact that JIT *requires* a high level of quality to function. Under JIT, a machine operator does not have a large batch of parts to sift through to find one suitable for use. He or she may have only one to choose from; if it is bad, the line stops. If this were to happen often enough, the consequences would be devastating. The analogy that many JIT writers have used is that of water in a stream with rocks on the bottom. The water represents WIP, the rocks are problems. As long as the water is high, the rocks are covered. However, when the water level is lowered, the rocks are exposed. Similarly, when the WIP level in a plant is reduced, problems, such as defects, become very noticeable.

Notice that JIT not only highlights the fact that there are quality problems, but also facilitates identification of their source. If WIP levels are high and quality inspections are made at separate stations, operators may get relatively little feedback about their own quality levels. Moreover, what they get will not be timely. In contrast, in a JIT environment, the parts made by an operator will be used rapidly by a downstream operator, who will have a strong incentive to notify the upstream operator of defects. This will serve to alert the operator of a potential problem while there is still time to do something about it. It also induces substantial psychological motivation to "do it right the first time." JIT advocates claim that this results in an overall increase in quality awareness and improved quality to the customer.

Analogous to the effect it had on setup reduction techniques, the pressure exerted by JIT fostered a burst of creativity in quality improvement methodologies. A huge volume of literature has detailed these over the past decade (see, e.g., DeVor 1992; Garvin 1988; Juran 1988; Shingo 1986), and so we will not go into great detail here. Instead, we will summarize seven principles identified by Schonberger (1983, 55) as essential to the quality practices of the Japanese:

1. **Process control.** The Japanese devoted a great deal of effort to enable the workers themselves to make sure their production processes were operating properly. This included use of statistical process control (SPC) charts and other statistical methods, but also involved simply giving workers responsibility for quality and the authority to make changes when needed.

2. **Easy-to-see quality.** As they were urged to do by Juran and Deming in the 1950s, the Japanese made use of extensive visual displays of quality measures. Display boards, gauges, meters, plaques, and awards were used to "put quality on display." These practices were aimed partly at providing feedback to the workforce and partly at proving that quality level is high to inspectors from customer plants.

3. **Insistence on compliance.** Japanese workers were encouraged to demand compliance with quality standards at every level in the system. If materials from a supplier did not measure up, they were sent back. If a part in the line was defective, it was not accepted. The attitude was that quality comes first and output second.

4. **Line stop.** The Japanese emphasized the "quality first" ideal to the extent that each worker had the authority to stop the line to correct quality problems. At some plants, yellow (for a problem) and red (for a line-stopping problem) lights were used to signal quality problems to the entire line. Where these techniques were used, quality really did come before throughput.

5. **Correcting one's own errors.** In contrast to the rework lines often found in American plants, the Japanese typically required the worker or work group that produced a defective item to fix it. This gave the workers full responsibility for quality.

6. **The 100 percent check.** The long-range goal was to inspect every part, not just a random sample. Simple or automated inspection techniques are desirable; foolproof (autonomous) machines that monitor quality during production are even better. However, in some situations where true 100 percent inspection was not feasible, the Japanese made use of the $N = 2$ method, in which the first and last parts of a production run are inspected. If both are good, then it is assumed that the machine was not out of adjustment and therefore that the intermediate parts are also good.

7. **Continual improvement.** In contrast to the Western notion of an acceptable defect level, the Japanese looked toward the ideal of zero defects. In this context, there is always room for further quality improvements.

Like the impact it had on cellular plant layout, JIT has engendered a revolution in quality that has grown far beyond its role in kanban and other JIT systems. The 1980s have been labeled the *quality decade* and have seen the emergence of such high-visibility initiatives as the Malcolm Baldrige Award and the ISO 9000 standards. The current heightened awareness of quality around the world is directly rooted in the JIT revolution.

4.5 Kanban

The single technique most closely associated with the JIT practices of the Japanese is the kanban system developed at Toyota. The word *kanban* is Japanese for *card,*[4] and in the Toyota kanban system, cards were used to govern the flow of materials through the plant.

To describe the Toyota kanban system, it is useful to distinguish between **push** and **pull** production control systems.[5] In a push system, such as MRP, work releases are *scheduled.* In a pull system, releases are *authorized.* The difference is that a schedule is prepared in advance, while an authorization depends on the status of the plant. Because of this, a push system directly accommodates customer due dates, but has to be forced to respond to changes in the plant (e.g., MRP must be regenerated). Similarly, a pull system directly responds to plant changes, but must be forced to accommodate customer due dates (e.g., by matching a level production plan against demand and using overtime to ensure that the production rate is maintained).

Figure 4.3 gives a schematic comparison of MRP and kanban. In the MRP system, releases into the production line are triggered by the schedule. As soon as work on a part is complete at a workstation, it is "pushed" to the next workstation. As long as machine operators have parts, they continue working under this system.

In the kanban system, production is triggered by a demand. When a part is removed from the final inventory point (which may be finished goods inventory) the last workstation in the line is given authorization to replace the part. This workstation then sends an authorization signal to the upstream workstation to replace the part it just used. Each station does the same thing, replenishing the downstream void and sending authorization to the next workstation upstream. In the kanban system, an operator requires both parts *and* an authorization signal (kanban) to work.

The kanhan system developed at Toyota made use of two types of cards to authorize production and movement of product. This **two-card** system is illustrated in Figure 4.4.

[4] Ohno translates *kanban* as *sign board,* but we will use the more common translation of *card.*

[5] See Chapter 10 for a more detailed discussion and comparison of push and pull systems.

FIGURE 4.3

Comparison of MRP and kanban

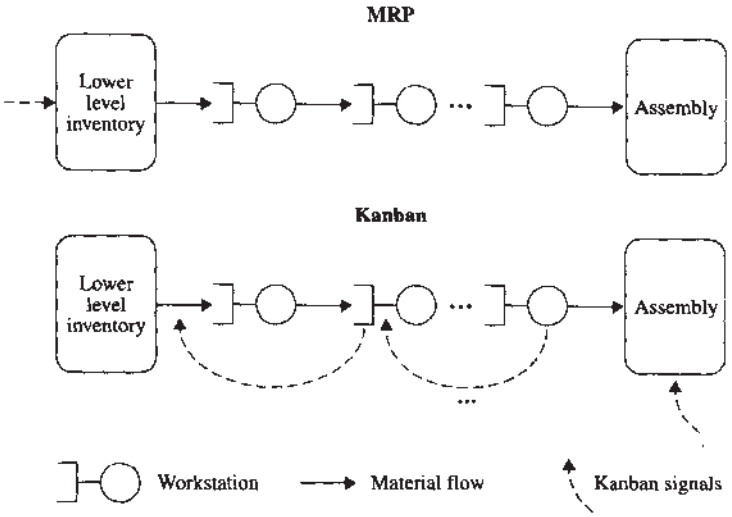
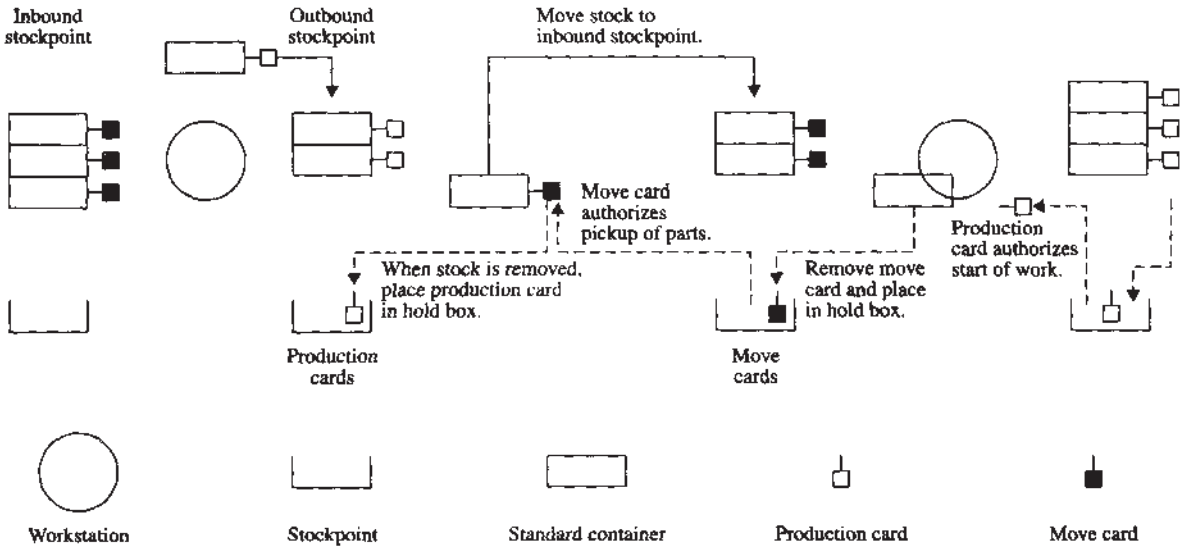


FIGURE 4.4

Toyota-style two-card kanban system



The basic mechanics are as follows. When a workstation becomes available for a new task, the operator takes the next **production card** from a box. This card tells the operator that a particular part is required at a downstream workstation. He or she looks to the inbound stockpoint for the materials required to make that part. If they are there, the operator removes the **move cards** attached to them and places them in another box. If the materials are not available, the operator chooses another production card. Whenever the operator finds both a production card and the necessary materials, he or she processes the part, attaches the production card, and places it in the outbound stockpoint.

Periodically, a **mover** will check the box containing move cards and will pick up the cards. He or she will get the materials indicated by the cards from their respective

outbound stockpoints, replace their production cards with the move cards, and move them to the appropriate inbound stockpoints. The removed production cards will be deposited in the boxes of the workstations from which they came, as signals to replenish the inventory in the outbound stock points.

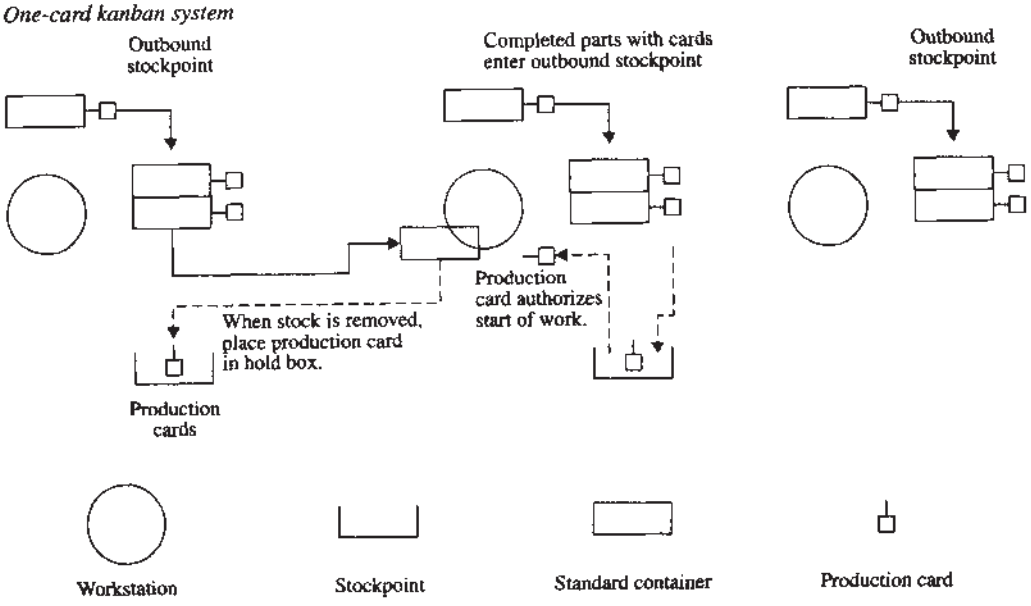
The rationale for the two-card system used by Toyota is that when workstations are spatially distributed, it is not feasible to achieve instantaneous movement of parts from one station to the next. Therefore, in-process inventory will have to be stored in two places, namely, an outbound stockpoint, when it has just finished processing on a machine, and an inbound stockpoint, when it has been moved to the next machine. The move cards serve as signals to the movers that material needs to be transferred from one location to another.

In a system with workstations close to one another, WIP can effectively be "handed" from one process to the next. In such settings, two inventory storage points are not necessary, and a **one-card** system, like that illustrated in Figure 4.5, can be used. In this system, an operator still requires a production card and the necessary materials to begin processing. However, instead of removing a move card from the incoming materials, the worker simply removes the production card from the upstream process and sends it back upstream. If one looks closely, it is apparent that a two-card system is identical to a one-card system in which the move operations are treated as workstations. Hence, the choice of one over the other depends on the extent to which we wish to regulate the WIP involved in move operations. If these operations are fast and predictable, it is probably unnecessary. If they are slow and irregular, regulation of move WIP may be helpful.

The key controls in a kanban system (one- or two-card) are the card counts at each station. These govern the amount of WIP in the system and, by affecting the frequency with which machines are starved for parts, determine the throughput rate. We will examine the relationship between WIP and throughput in detail in Part II. For now, it is worthwhile to note the similarity between kanban and the reorder point methods

FIGURE 4.5

One-card kanban system



we discussed in Chapter 2. Consider the one-card kanban system with m production cards at a given station. Each time inventory in the downstream stockpoint falls below m, production cards are freed up, authorizing the station to replenish the buffer. The mechanics of this process are therefore precisely the same as those of the base stock model, with the downstream station acting as the demand and the card count m serving as the base stock level. The intuition we developed for this system in Chapter 2 carries over to the kanban system. However, the model does not directly apply because it assumes independent lead times for replenishing stock (i.e., the time to fill the nth and $(n + 1)$st orders are independent). Since the time to fill consecutive orders may well be correlated (i.e., if it takes a long time to fill the nth order, the $(n + 1)$st order may also have to wait a long time), a somewhat different model is required. We will develop models of this sort in Part II.

4.6 The Lessons of JIT

The range of issues touched on in this chapter makes it clear that JIT is not a simple procedure or technique. Nor can it be said to be a coherent, well-defined management strategy. Rather, it is an assortment of attitudes, philosophies, priorities, and methodologies that have been collectively labeled JIT. The real thread connecting them is that they have been practiced in recent times by a number of Japanese companies with notable success.

While JIT may not offer comprehensive policies for managing a manufacturing facility, its originators at Toyota and elsewhere have clearly demonstrated true genius in generating creative solutions to specific problems. Inherent in these solutions are some key insights that deserve a prominent place in the history of manufacturing management:

1. *The production environment itself is a control.* Strategies that involve reducing setups, changing product designs with manufacturing in mind, leveling production schedules, and so on, can have greater impact on the effectiveness of the production process than any decisions actually made on the factory floor.

2. *Operational details matter strategically.* Ohno and others reinforced the 100-year-old insight of Carnegie that the small details of the production process can confer a substantial competitive advantage. Like Carnegie, the JIT advocates concentrated on cost of manufacture and were willing to examine the most mundane aspects of the manufacturing process in their efforts to reduce waste.

3. *Controlling WIP is important.* The importance of the smooth and rapid flow of materials through the system was recognized by Ford in the 1910s and was echoed with emphasis by Ohno in the 1980s. Virtually all the benefits of JIT either are a direct consequence of low WIP levels (e.g., short cycle times) or are spurred by the pressure low WIP levels create (e.g., high quality levels).

4. *Flexibility is an asset.* JIT is inherently inflexible. In its essential form it calls for an absolutely steady rate and mix of production, virtually minute by minute. However, perhaps in reaction to this tendency toward inflexibility, the advocates of JIT have developed an acute appreciation for the value of flexibility in responding to a volatile marketplace. They have tempered JIT with a host of practices designed to promote flexibility, including short setup times, capacity cushions, worker cross-training, cellular plant layout, and many others.

5. *Quality can come first.* Although many of the basic quality concepts used by the Japanese in their JIT systems had long been championed by American quality experts,

Japanese firms were far more effective at putting these ideas into practice than were their American counterparts. They demonstrated to the world that a system in which quality takes precedence over throughput and is assured at the source not only works, but is profitable as well.

6. *Continual improvement is a condition for survival.* In sharp contrast to Henry Ford's belief in a perfectible product and process, the Japanese recognize that manufacturing is a continually changing game. Standards that sufficed yesterday will not be adequate tomorrow. Despite our terming JIT a "revolution," it took about 25 years (from the 1940s to the late 1960s) of constant attention for Toyota to reduce setups from three hours to three minutes. More than anything, the successful practitioners of JIT have been devoted to doing things better and better, a little bit at a time.

Discussion Point

1. Consider the following statement:

   Henry Ford practiced short-cycle manufacturing in the 1910s. The basic tools of Total Quality Management were developed and practiced at Western Electric in the 1920s. Kanban is equivalent to a base stock system, which was well known since the 1930s. Thus, just-in-time is nothing more than a repackaging of traditional American ideas, for which its Japanese proponents have been greatly overpraised.

   a. Comment on the accuracy of this statement.
   b. What aspects of JIT seem radically distinct from older techniques? Do these justify terming JIT a *revolution?*
   c. What aspects of JIT are particularly rooted in Japanese culture? What implications might this have for the transferability of JIT to America?

Study Questions

1. What are the seven zero goals of JIT? Of these, which are actually achievable? Which are completely outrageous if taken literally?
2. Discuss the fundamental difference between the zero defects goal in JIT and the acceptable quality level of former times. What does this have to do with the adage, "If you don't have time to do it right, when will you find time to do it over?"
3. Why is zero setup time desirable? Why is zero lead time?
4. Under the JIT philosophy, why is inventory often said to be evil?
5. What is meant by the common analogy of a stream, where WIP is represented by water and problems by rocks? What difficulties might arise from the perspective this analogy suggests?
6. What does Ohno mean by the "five whys"?
7. In what way does Ohno describe an American-style supermarket as an inspiration for JIT? What potential problems exist with using a supermarket as an analogy for a manufacturing system?
8. What role does total quality management (TQM) play in JIT? Does JIT depend on TQM, promote TQM, or both?
9. Describe autonomation.
10. Why is flexible labor important in a JIT system?
11. What are manufacturing cells? What role do they play in a JIT system?
12. What are the advantages of mixed model production?
13. Explain how two-card kanban works.

14. How is two-card kanban equivalent to one-card kanban? What is left out in the two-card case?
15. What is the "magic" of kanban? Is it the fact that stock is pulled from one station to the next, or is it something more fundamental?
16. Give at least two reasons that Toyota's kanban system has not been universally adopted by industry in America (or Japan).
17. Why are a relatively constant volume and relatively stable product mix essential to kanban?
18. List three ways in which the intrinsic rigidity of JIT is compensated for in practice.
19. What is the fundamental difference between a pull production system and a push production system?
20. In a serial production line, at which station (first, last, middle, etc.) would it be best to have the bottleneck in a push system? Where in a pull system? Explain your reasoning.
21. For each of the following situations, indicate whether kanban or MRP would be more effective.
    a. An auto plant producing three styles of vehicle
    b. A custom job shop
    c. A circuit board plant with 40,000 active part numbers
    d. A circuit board with 12 active part numbers
    e. A plant with one assembly line where all parts are purchased

CHAPTER

5 WHAT WENT WRONG

Look ma, the emperor has no clothes!
Hans Christian Andersen

Our task now is not to fix the blame for the past, but to fix the course for the future.
John F. Kennedy

5.1 Introduction

By the 1980s, there were signs that not all was well with American manufacturing. Slowdowns in productivity growth (see Dertouzous, Lester, and Solow (1989) and Baumol, Blackman, and Wolff (1989) for discussions), declines in American shares of various markets, widespread perception that many goods in America were inferior in quality to their foreign counterparts, persistently large trade deficits, and many other troubling trends reminded us daily that America's once-undisputed manufacturing supremacy was no more. The "decline" of American manufacturing served as a serious wakeup call that we had entered a new globally competitive era. From that point forward, long-term success would require world-class performance and continual improvement on a variety of fronts: product development, marketing, human resource management, finance, *and* operations management. One of the main lessons of the Japanese success in the 1980s was that operations management can be (must be?) part of an effective modern manufacturing business strategy.

Conventional American operations management practices employed between World War II and 1990 can be roughly grouped into three schools of thought:

1. *Scientific management* is characterized by a rational, deductive, quantitative, modeling-oriented view of manufacturing systems. The original scientific management movement of the early 20th century spawned the quantitative methods for inventory control, scheduling, forecasting, aggregate planning, and many other manufacturing functions.

2. *Material requirements planning* is characterized by a central computerized planning approach to production control and integration. As more functions were incorporated into the system, the original MRP evolved into manufacturing resources planning (MRP II) and then into enterprise resources planning (ERP).

3. *Just-in-time* is characterized by a low-inventory, flow-oriented focus on the manufacturing environment. The original emphasis on Japanese kanban methods expanded

into the broader view of lean manufacturing. JIT was also the impetus for total quality management, which both was part of JIT and evolved into a separate movement.

While each of these certainly offers good ideas, none has been consistently successful in elevating firms to the level required to thrive in the competitive environment of the next century. In this chapter, we trace the reasons that the above approaches have failed to offer a comprehensive solution to the competitiveness problem. We will build on these negative insights, as well as on the positive ones of the previous four chapters, to develop an integrated approach to operations management in Parts II and III of this book.

5.2 Trouble with Scientific Management

In Chapter 1, we discussed two cultural tendencies we feel have had a significant influence on the way operations management has developed and been viewed in America:

1. *Faith in the scientific method,* which runs deep in the American soul, has motivated academics and practitioners to emphasize methods that are precise, quantitative, and high-technology. Taylor, with his shoveling formulas, clearly took this approach, as did the developers of the inventory control methods discussed in Chapter 2.
2. *The frontier ethic,* which glorifies wide-open spaces, rugged individualism, and sweeping adventure, is fundamentally in conflict with an attitude of careful husbanding of resources. This, coupled with the almost total lack of serious global competition in the first half of the 20th century, led many of the best and brightest in America to shun operations for more exciting careers in marketing, finance, or other fields.

It is not that either a frontier mentality or a quantitative outlook is intrinsically bad. However, the unique American combination of the two proved deadly in the 1970s and 1980s. The emphasis on marketing and finance took top management out of the loop as far as operations were concerned. This caused responsibility to devolve to middle managers, who lacked the perspective to see operations management in a strategic context. As a result, middle managers and the academic research community that supported them approached operations from an extremely narrow, reductionist perspective. Given this, our scientific bent led us to devote tremendous energy to applying increasingly sophisticated techniques to increasingly irrelevant problems.

This technical but unrealistic approach to operations management was already evident in 1913 when Harris published his original EOQ paper. Writing at the height of the early scientific management movement, Harris placed great emphasis on precision and elegance. For this reason, he made a number of simplifying assumptions about the lot-sizing problem that allowed him to derive his appealing "square root formula," but which rendered his results highly questionable for many real-world production systems. As we discussed in Chapter 2, these unrealistic assumptions included

- A fixed, known setup cost.
- Constant, deterministic demand.
- Instantaneous delivery (infinite capacity).
- A single product or no product interactions.

Because of these assumptions, EOQ makes much more sense in purchasing environments than in the production environments for which Harris intended it. In a purchasing environment, setups (i.e., purchase orders) may adequately be characterized with a

constant cost. However, in manufacturing systems, setups cause all kinds of other problems (e.g., product mix implications, capacity effects, variability effects), as we will discuss in Part II. The assumptions of EOQ completely gloss over these important issues.

Even worse than the simplifying assumptions themselves was the myopic perspective toward lot sizing that the EOQ model promoted. By treating setups as exogenously specified constraints to be worked around, the EOQ model and its successors blinded operations management researchers and practitioners to the possibility of deliberately reducing the setups. It took the Japanese, approaching the problem from an entirely different perspective, to fully recognize the benefits of setup reduction.

In Chapter 2 we discussed similar aspects of unrealism in the assumptions behind the Wagner–Whitin, base stock, and (Q, r) models. In each case, the flaw of the model was not that it did not start with a real problem or a real insight. It did. As we have noted, the EOQ insight into the tradeoff between inventory and setups is fundamental to the behavior of a plant. So is the (Q, r) insight into the tradeoff between inventory (safety stock) and service. However, with our fascination for things scientific, these insights rapidly became secondary to the mathematics. Realism was sacrificed for precision and elegance. Instead of working to broaden and deepen the insights by studying the behavior of different types of real systems, we focused on faster computational procedures for solving the simplified problems. Instead of working to integrate disparate insights into a strategic framework, we concentrated on ever smaller pieces of the overall problem in order to achieve neat mathematical formulas.

Although the separation between models and reality existed right from the start of the operations management (OM) literature, it grew steadily worse. As OM became increasingly established as an academic discipline, fewer and fewer researchers drew directly on manufacturing facilities as a source of problems. Stylized standard problems became objects of volumes of research.

A classic example of this trend occurred in the field of flow shop scheduling, which was initiated by the publication of a paper by Johnson in 1954. Johnson's paper considered the problem of minimizing the total amount of time to process a fixed number of jobs (called **makespan**) on a two-machine production line. The processing times were assumed fixed and known, but not identical. The only issue, therefore, was the order in which to do the jobs on the machines. Johnson derived a simple and intuitive algorithm for computing an optimal schedule for this problem.

Unfortunately, the problem itself virtually never occurs in industry. Most manufacturing settings have jobs entering the system continually, so the issue of how to schedule a fixed number of jobs to minimize makespan is not relevant. However, the problem *is* of interest mathematically, because when the number of machines in the line is larger than three, it becomes very difficult (in a theoretical mathematical sense). Because researchers drew their inspiration from the literature and not from industry, Johnson's paper spawned an enormous number of follow-on papers addressing variations of his original problem. For the most part the variations were no more realistic than the original, and a recent survey of the flow shop scheduling research could find almost no evidence of influence on scheduling practice. Dudek, Panwalkar, and Smith (1992) summed up the history of this research area as follows:

> At this time, it appears that one research paper (that by Johnson) set a wave of research in motion that devoured scores of person-years of research time on an intractable problem of little practical consequence.

Similar stories can be told for other areas of the operations management literature, such as aggregate planning, inventory control, equipment replacement, and capacity planning. Throughout the OM field, far more was published than practiced.

The fact that most academic research had little impact on industry certainly did not help the competitiveness of American manufacturing, but it probably did not directly hurt it much either. A more insidious consequence of this research affected university teaching. By carrying the disjointed, models-oriented, unempirical approach of their research to the classroom, professors encouraged generations of engineering and business students to look at operations in a narrow, exclusively technical manner.

In engineering schools, operations management became operations research and focused almost exclusively on methodologies, such as linear programming and probability modeling. Even in courses aimed at production topics, methodologies often came first. Many scheduling classes, reflecting the scheduling literature, virtually became mathematics classes as they concentrated more on the complexity of the algorithms than on the issues involved in real scheduling situations. The contents of many "operations" texts emphasized operations research methodology over production applications.

In business schools, students were less patient and less interested in mathematics for its own sake. Therefore, as operations management courses became collections of quantitative methods applied to a host of loosely related problems (e.g., inventory control, scheduling, quality assurance, maintenance), they grew increasingly unpopular. In the 1970s and 1980s, some schools dropped OM from the curriculum! Others watered down the courses until the courses were mere compilations of anecdotal case studies.

The effect was that neither the engineering nor the business students were given much preparation for dealing with real-life operations problems. At best, this simply meant they were on their own to invent ad hoc solutions to problems as best they could. At worst, it meant they applied the mathematical methods they learned in school to situations for which the methods were ill adapted. (Our impression is that most industry practitioners have intelligently opted for the former and have largely ignored their academic training in production.)

By the late 1980s, stiff competition from the Japanese, Germans, and others made academics and practitioners alike realize that a change was necessary. Numerous distinguished voices called for a new emphasis on operations. For instance, professors from Harvard Business School stressed the strategic importance of operational details (Hayes, Wheelwright, and Clark 1988, 188):

> Even tactical decisions like the production lot size (the number of components or subassemblies produced in each batch) and department layout have a significant cumulative impact on performance characteristics. These seemingly small decisions combine to affect significantly a factory's ability to meet the key competitive priorities (cost, quality, delivery, flexibility, and innovativeness) that are established by its company's competitive strategy. Moreover, the fabric of policies, practices, and decisions that make up the manufacturing system cannot easily be acquired or copied. When well integrated with its hardware, a manufacturing system can thus become a source of sustainable competitive advantage.

Their counterparts across town at Massachusetts Institute of Technology agreed, calling for operations to play a larger role in the training of managers (Dertouzos, Lester, and Solow 1989, 161):

> For too long business schools have taken the position that a good manager could manage anything, regardless of its technological base.... Among the consequences was that courses on production or operations management became less and less central to business-school curricula. It is now clear that this view is wrong. While it is not necessary for every manager to have a science or engineering degree, every manager does need to understand how technology relates to the strategic positioning of the firm ...

But while there is now increasing agreement that operations management is important, there is not yet agreement on what should be taught or how to teach it. The old

approach of presenting operations solely as a series of mathematical models has been widely discredited. The pure case study approach is still in use at some business schools and may be superior because cases can provide insights into realistic production problems. However, covering hundreds of cases in a short time only serves to strengthen the notion that executive decisions can be made with little or no knowledge of the fundamental operational details. Moreover, the factory physics approach in Part II is our attempt to provide both the fundamentals and an integrating framework. In it we build upon past insights surveyed in the present section and make use of the precision of mathematics to clarify and generalize these insights. Better insight builds better intuition, and good intuition is necessary for good decision making. We are not alone in seeking a framework for building practical operations intuition via models (see Askin and Stanbridge 1993, Buzacott and Shantikumar 1993, and Suri 1998 for others). We take this as a hopeful sign that a new paradigm for operations education is emerging.

By the 1990s the mantle of scientific management had been picked up by business process reengineering (BPR). At its core, BPR was systems analysis applied to management.[1] But in keeping with the American proclivity for the big and the bold, the emphasis was heavily on *radical* change. Leading proponents of BPR defined it as "the fundamental rethinking and radical redesign of business processes to achieve dramatic improvements in critical, contemporary measures of performance, such as cost, quality, service, and speed" (Hammer and Champy 1993). Because most of the redesign efforts spawned by BPR involved eliminating jobs, it soon became synonymous with downsizing.

As a buzzword, BPR fell out of favor as quickly as it arose. By the late 1990s it had been banished from most corporate vocabularies. Still it left some lasting legacies. The layoffs of the 1990s, during bad times and good, certainly had a positive effect on labor productivity. But because the layoffs affected both labor and middle management to an unprecedented degree, they undermined worker loyalty.[2] Moreover, BPR represented an extreme backlash against the placid stability of the golden era of the 1960s; radical change was not only no longer feared, it was sought. This paved the way for more revolutions. For example, it is hard to imagine management embracing the ERP systems of the late 1990s, which required fundamental restructuring of processes to fit software as opposed to the other way around, without first having been conditioned by BPR to think in revolutionary terms. It is ironic that BPR, with its roots in the ultra-rational field of systems analysis, may actually have left American manufacturing more vulnerable to irrational buzzword fads than ever before.

The bottom line is that the scientific management school of thought contains valuable tools for addressing the problem of manufacturing competitiveness, but is not itself a comprehensive solution. The original insight of scientific management—that management is a discipline that can be studied—certainly remains valid. But Taylor's original efficiency-oriented framework of scientific management is too narrow to encompass the modern customer-oriented manufacturing environment. The quantitative models spawned by scientific management are useful for understanding and solving subproblems. But they can be dangerous if confused with the manufacturing system itself. Systems analysis is a powerful problem-solving tool that offers much promise as the basis for a balanced approach to continual improvement. But by pushing extreme solutions (radical change) or a narrow class of solutions (downsizing), it loses its balance and can

[1] Systems analysis is a rational means-ends approach to problem solving in which actions are evaluated in terms of a specific objective function. We discuss it in greater detail in Chapter 6.

[2] The enormous popularity of "Dilbert" cartoons, which poke liberal fun at BPR and other management fads, tapped into the growing sense of alienation of the workforce in corporate America. Ironically, some companies actually responded by banning them from office cubicles.

become the basis for personality-driven fads. The challenge, therefore, is to keep the essential components of the scientific management school, but to develop a framework in which they are applied in the right way to the problems of greatest strategic importance. This is precisely what we seek to do with factory physics in Part II.

5.3 Trouble with MRP

From at least one perspective, MRP was a stunning success. The number of MRP systems in use by American industry grew from a handful in the early 1960s, to 150 in 1971 (Orlicky 1975). The American Production and Inventory Control Society (APICS) launched its MRP Crusade to publicize and promote MRP in 1972. By 1981, claims were made that the number of MRP systems in America had risen as high as 8,000 (Wight 1981). In 1984 alone, 16 companies sold $400 million in MRP software (Zais 1986). In 1989, $1.2 billion worth of MRP software was sold to American industry, constituting just under one-third of the entire American market for computer services (*Industrial Engineering* 1989). By the late 1990s, ERP had grown to a $10 billion industry—ERP consulting was an even larger industry—and SAP, the largest ERP vendor, was the fourth-largest software company in the world (Edmondson and Reinhardt 1997). So, unlike many of the inventory models we discussed in Chapter 2, MRP *was*, and is, used in industry.

But has it worked? Were the companies who implemented MRP systems better off as a result? There is considerable evidence that suggests not.

First, from a macro perspective, American manufacturing inventory turns remained roughly constant during the 1970s and 1980s, during and after the MRP crusade (see Figure 5.1). (Note that inventory turns have increased in the 1990s, but this is almost certainly a consequence of the pressure to reduce inventory generated by the JIT movement and not directly related to MRP.) But of course many firms were not using MRP during this period. So while it appears that MRP did not revolutionize the efficiency of the entire manufacturing sector, these figures alone do not make a clear statement about MRP's effectiveness at the individual firm level.

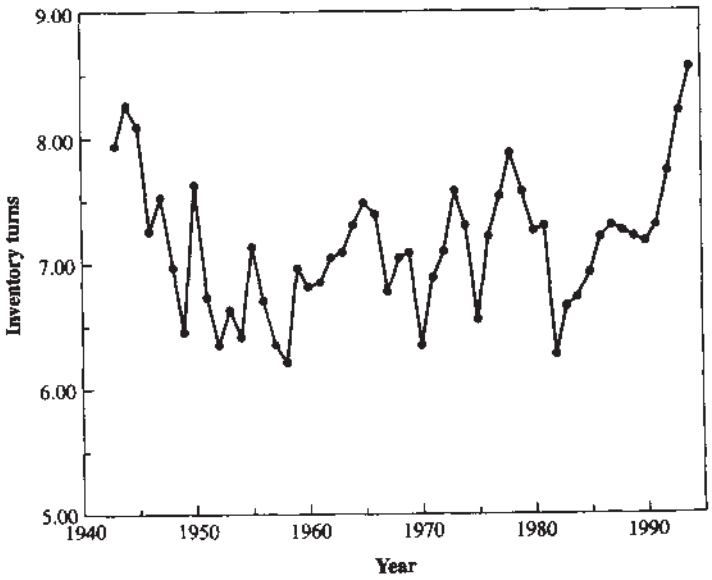


FIGURE 5.1
U.S. manufacturing inventory turns, 1943–1994

At the micro level, various surveys of MRP users did not paint a rosy picture either. Booz, Allen, and Hamilton, from a 1980 survey of more than 1,100 firms, reported that much less than 10 percent of American and European companies were able to recoup their investment in an MRP system within two years (Fox 1980). In a 1982 APICS-funded survey of 679 APICS members, only 9.5 percent regarded their companies as being class A users (Anderson et al. 1982).[3] Fully 60 percent reported their firms as being class C or class D users To appreciate the significance of these responses, we must note that the respondents in this survey were both APICS members and materials managers—people with strong incentive to see MRP in as good a light as possible! Hence, their pessimism is most revealing. A smaller survey of 33 MRP users in South Carolina arrived at similar numbers concerning system effectiveness; it also reported that the eventual total average investment in hardware, software, personnel, and training for an MRP system was $795,000, with a standard deviation of $1,191,000 (LaForge and Sturr 1986).

Such discouraging statistics and mounting anecdotal evidence of problems led many critics of MRP to make strong disparaging statements, such as MRP is a "$100 billion mistake," "90 percent of MRP users are unhappy," and "MRP perpetuates such plant inefficiencies as high inventories" (Whiteside and Arbose 1984).

This barrage of criticism prompted the proponents of MRP to defend it. While not denying that it was far less successful than they had hoped when the MRP crusade was launched, they did not attribute this lack of success to the system itself. The APICS literature (e.g., Orlicky as quoted by Latham 1981), cited a host of reasons for most MRP system failures but never questioned the system itself. John Kanet, a former materials manager for Black & Decker who wrote a glowing account of its MRP system in 1984, but had by 1988 turned sharply critical of MRP, summarized the excuses for MRP failures as follows.

> For at least ten years now, we have been hearing more and more reasons why the MRP-based approach has not reduced inventories or improved customer service of the U.S. manufacturing sector. First we were told that the reason MRP didn't work was because our computer records were not accurate. So we fixed them; MRP still didn't work. Then we were told that our master production schedules were not "realistic." So we started making them realistic, but that did not work. Next we were told that we did not have top management involvement; so top management got involved. Finally we were told that the problem was education. So we trained everyone and spawned the golden age of MRP-based consulting.

Because these efforts still did not make MRP effective, Kanet and many others concluded that there is something more fundamental wrong with MRP. The real reason for MRP's inability to perform plant performance is that *MRP is based on a flawed model*. As we discussed in Chapter 3, the key calculation underlying MRP is performed by using fixed lead times to "back out" releases from due dates. These lead times are functions only of the part number and are not affected by the status of the plant. In particular, lead times do not consider the loading of the plant. An MRP system assumes that the time for a part to travel through the plant is the same whether the plant is empty or overflowing with work. As the following quote from Orlicky's original book shows, this separation of lead times from capacity was deliberate and basic to MRP (Orlicky 1975, 152):

[3]The survey used four categories proposed by Oliver Wight (1981) to classify MRP systems, classes A, B, C, and D. Roughly, Class A users represent firms with fully implemented, effective systems. Class B users have fully implemented, but less than fully effective systems. Class C users have partially implemented, modestly effective systems. And class D users have marginal systems providing little benefit to the company.

> An MRP system is capacity-insensitive, and properly so, as its function is to determine what materials and components will be needed and when, in order to execute a given master production schedule. There can be only one correct answer to that, and it cannot therefore vary depending on what capacity does or does not exist.

But unless capacity is infinite, the time for a part to get through the plant *does* depend on the loading. Since all plants have finite capacity, the fixed-lead-time assumption is always an approximation of reality. Moreover, because releasing jobs too late can destroy the desired coordination of parts at assembly or cause finished products to come out too late, there is strong incentive to inflate the MRP lead times to provide a buffer against all the contingencies that a part may have to contend with (waiting behind other jobs, machine outages, etc.). But inflating lead times lets more work into the plant, increases congestion, and increases the flow time through the plant. Hence, the result is yet more pressure to increase lead times. The net effect is that MRP, touted as a tool to reduce inventories and improve customer service, can actually make them worse.

This flaw in MRP's underlying model is so simple, so obvious, that it may seem incredible that we came this far along the MRP path without noticing (or at least worrying about) it. To some extent, this is 20–20 hindsight. When viewed historically, MRP makes perfect sense and is, in some ways, the quintessential American production control system. When scientific management met the computer, MRP was the result. Unfortunately, the computer that scientific management met was a computer of the 1960s which had very limited power. Consequently, MRP is poorly suited to the environment and computers of the 1990s.

As we pointed out in Chapter 3, the original, laudable goal of MRP was to explicitly consider dependent demand, rather than to treat all demands as independent and use reorder point methods for lower-level inventories. This requires performing a bill-of-material explosion and netting demands against current inventories—both tedious data processing tasks in systems with complicated bills of material. Hence there was strong incentive to computerize.

The state of the art in computer technology in the mid-1960s, however, was an IBM 360 that used "core" memory with each *bit* represented by a magnetic doughnut about the size of the letter *o* on this page. When the IBM 370 was introduced in 1971, integrated circuits replaced the core memory. At that time a one-fourth inch square chip would typically hold less than 1,000 characters. As late as 1979, a mainframe computer with more than 1,000,000 bytes of RAM was a large machine. With such limited memory, performing all the MRP processing in RAM was out of the question. The only hope for realistically sized systems was to make MRP transaction-based. That is, individual part records would be brought in from a storage medium (probably tape), processed, and then written back to storage. As we pointed out in Chapter 3, the MRP logic is exquisitely adapted to a transaction-based system.

Thus, if one views the goal as explicitly addressing dependent demands in a transaction-based environment, MRP is not an unreasonable solution. The hope of the MRP proponents was that through careful attention to inputs, control, and special circumstances (e.g., expediting), the flaw of the underlying model could be overcome and MRP would represent a substantial improvement over older production control methods. This was exactly the intent of MRP II modules such as CRP and RCCP. Unfortunately, these were far from successful, and MRP II was roundly criticized in the 1980s while Japanese firms were strikingly successful by going back to methods that resemble the old reorder point approach. JIT advocates were quick to sound the death knell of MRP.

But MRP did not die, largely because MRP II handled important nonproduction data maintenance and transaction processing functions that were not replaced by JIT.

So MRP persisted into the 1990s, expanded in scope to include other business functions and multiple facilities, and was rechristened ERP. Simultaneously, computer technology advanced to the point where the transaction-based restriction of old MRP was no longer necessary. A host of independent companies emerged in the 1990s offering various types of finite-capacity schedulers to replace basic MRP calculations. However, because these were ad hoc and varied, many industrial users were reluctant to adopt them until they were offered as parts of comprehensive ERP packages. As a result, a host of alliances, licensing agreements, and other arrangements between ERP vendors and application software developers emerged.

There is much that is positive about the recent evolution of ERP systems. The integration and connectivity they provide make more data available to decision makers in a more timely fashion than ever before. Some finite-capacity scheduling modules are promising as replacements for old MRP logic in some environments. However, as we will discuss in Chapter 15, scheduling problems are notoriously difficult. It is not reasonable to expect a uniform solution for all environments. For this reason, ERP vendors are beginning to customize their offerings according to "best practices" in various industries. But the resulting systems are more monolithic than ever, often requiring firms to restructure their businesses to comply with the software. Although many firms, conditioned by the BPR movement to think in revolutionary terms, seem willing to do this, it may be a dangerous trend. The more firms conform to a uniform standard in the structure of their operations management, the less able they will be to use it as a strategic weapon, and the more vulnerable they will be to creative innovators in the future.

By the late 1900s, more cracks began to appear in the ERP landscape. In 1999, SAP AG, the largest ERP supplier in the world, was stung by two well-publicized implementation glitches at Whirlpool Corp., which resulted in the delay of appliance shipments to many customers and at Hershey Foods Corp., which left the shelves of candy retailers empty just before Halloween. Meanwhile, several companies decided to pull the plug on SAP installations costing between $100 and $250 million (Boudette 1999). Moreover, a survey by Meta Group of 63 companies revealed an average return on investment of a *negative* $1.5 million for an ERP installation (Stedman 1999).

Nonetheless, the original insight of MRP—that independent and dependent demands should be treated differently—remains fundamental. The hierarchical planning structure of MRP II and ERP provides coordination and a logical structure for maintaining and sharing data. However, to make effective use of the data processing power and scheduling sophistication promised by ERP systems of the future will require tailoring the operations system to a firm's business needs, not the other way around. This implies a sound understanding of core processes and the effects of specific planning and control decisions on them. *Factory Physics* provides a framework for understanding these core processes and the relationships between performance measures, as we show in Part II. In Part III, we use the insights of Part II to develop a planning hierarchy that parallels the MRP II hierarchy but incorporates advantages of pull production systems as well. We specifically focus on the scheduling problem, including approaches for working with MRP, in Chapter 15.

5.4 Trouble with JIT

As we noted in Chapter 4, the collection of ideas, priorities, and techniques that became collectively known as just-in-time (JIT) contains many creative and powerful insights.

The key question, however, is whether or not JIT represents a *system* and, if so, whether this system is transportable from Japan to America.

The early literature on JIT was somewhat contradictory on this point. On one hand, the first books on Japanese manufacturing techniques published in America suggested that these techniques are eminently transportable. In the first widely available book on JIT, Schonberger (1982, vii) said so directly: "I believe that the approaches travel easily to other countries ... Japanese production and quality management works in non-Japanese settings." Monden (1983, v) concurred, in his book describing the Toyota system: "The author firmly believes the Toyota production system can play a great role in the task for improving the constitutions of American and European companies..." Hall (1983), in his widely read JIT text, never even questioned whether JIT is a system, and proceeded to give detailed information on implementing it through such steps as flow balancing, quality improvements, and setup reduction.

In contrast to these optimistic viewpoints, other early observers of Japanese manufacturing practices were not sure that the Japanese had a system at all, let alone a transportable one. The Toyota kanban system was far from universal; in fact, it was almost exclusive to Toyota. Moreover, in a tour of six Japanese facilities, Robert Hayes (1981) did not find prevalent use of modern automation technology, quality circles, or uniform compensation systems. In short, he found "no exotic, strikingly different Japanese way of doing things."

Which view was correct? Were the Japanese practicing a well-defined JIT system that was responsible for their success, or were we simply observing a number of highly successful Japanese firms using a variety of disparate approaches?[4] The answer, perhaps, is that both views are partially correct. While Hayes did not see widely common procedures, he did observe two common effects:

1. Japanese plants were very clean and orderly.
2. Japanese plants exhibited much less work-in-process inventory than their American counterparts.[5]

Assuming that orderliness is an indicator of (or side effect from, or support to) a smoothly running system, these two effects are in fundamental agreement with the basic JIT tenets of *establishing a flow* and *eliminating waste* as described by Ohno (1988). While the Japanese firms may not have used the same methods, they do seem to have exhibited philosophical commonalities. But philosophy is trickier to transport than tools, leading Hayes (1981, 57) to be far less sanguine than Schonberger about the transferability of JIT:

> The modern Japanese factory is not, as many Americans believe, a prototype of the factory of the future. If it were, it might be, curiously, far less of a threat. We in the United States, with our technical ability and resources, ought then to be able to duplicate it. Instead, it is something much more difficult for us to copy; it is the factory of today running as it should.

Some of the controversy about its transportability was due to the fact that the term *JIT* did not mean the same thing to all people. Judging from the usage of the term in

[4] It is worthwhile to note here that the most successful Japanese firms are precisely the ones we saw most. Less-well-run Japanese companies simply could not survive the rigorous competitive task of marketing their goods halfway around the globe. As a result, we almost certainly overestimated the quality of overall Japanese manufacturing.

[5] A survey by Booz, Allen, and Hamilton of 1,000 firms in the United States, Europe, and Japan confirmed this observation, reporting that inventory turns were 50 percent higher in Japan than in the United States or Europe and that the gap was growing (Fox 1980).

the academic and practitioner literature, it appears that JIT represented both a *system* of beliefs and a *collection* of methods. This distinction was undoubtedly responsible for some of the considerable confusion over JIT in industry. We have heard of more frequent supplier deliveries, quality circles, smaller lot sizes, cellular layouts, material handling changes, worker participation programs, and so forth, all presented as JIT systems. The reality seems to be that whatever is systematic about them is a result of the company's own invention. Not surprisingly, some of these "systems" worked while others did not.

Zipkin (1991) aptly described the dichotomy of views on JIT expressed in the literature by separating *romantic JIT* from *pragmatic JIT*. By romantic JIT, he meant the stirring rhetoric that was built up around the idealized goals of zero inventories, zero defects, lot sizes of one, and so on, and was embodied in such lyrical slogans as "Simplify, and goods will flow like water" (Schonberger 1982). From this perspective, Zipkin (1991, 42) says, "JIT represents an aesthetic ideal, a natural state of simplicity. To implement JIT, what we need to do is to strip away needless layers of complexity." Although Schonberger generally acknowledges that working toward the JIT ideals requires grappling with myriad details, in passages of romantic fervor he has gone so far as to imply that JIT is easy, almost trivial, to implement (Schonberger 1990, 308): "Kanban is something that can be installed between any successive pair of processes in fifteen minutes, using a few containers and masking tape." While such statements are in fact true, there is a difference between *installing* kanban and *implementing* kanban.[6] Such statements invited readers, particularly those skimming through the literature rather than reading it carefully, to confuse simplicity of ideals with simplicity of implementation. As a result, many practitioners were led to believe that not only is JIT better than traditional American practices, but also it is easier.

While a senior manager who is far removed from the factory floor might be content with blithely contemplating the image of the ideal factory portrayed in romantic JIT, junior managers and operators on the shop floor who are charged with actually carrying out the revolution had no choice but to confront the other side of JIT—pragmatic JIT. Zipkin (1991, 41) describes pragmatic JIT as consisting of a host of nuts-and-bolts methods, including "engineering techniques to facilitate change-overs, cleaner plant layouts, quality-control training, scheduled maintenance, simpler product designs, and much more." The books of Hall (1983), Monden (1983), Shingo (1989), and Schonberger (1982, 1983) are replete with detailed descriptions of mechanical devices, plant layouts, and organizational structures with which to implement JIT. It is from this smorgasbord of techniques that practitioners were to achieve the environment of continuous improvement called for in romantic JIT.

Unfortunately, for all their detail, the methods described in the pragmatic JIT literature were far from being off-the-shelf technology. Indeed, they formed a much less complete system than MRP, which itself has been widely criticized as requiring an enormous amount of institutional attention to implement. To choose appropriate pragmatic JIT methods and construct a coherent set of operating policies requires a huge creative effort on the part of the practitioner.

Undoubtedly, the pioneers at Toyota *were* able to achieve a steady stream of improvements via the methods they described as JIT. But they were the creators of the methods, and they were very clever. Also, as the methods developed were specifically tailored to address *their* manufacturing environments, it is no wonder they were effective. Genius coupled with steadfast attention is a strong combination indeed.

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[6] We will find in Part II that such "easy" installations do reduce work in process but at the expense of lost throughput and revenue.

The vast majority of companies did not have the benefit of a genius such as Ohno or Shingo (or Taylor or Ford, for that matter). Leading a revolution is a very risky and tricky business. Everyone with a vested interest in the status quo will be against the revolutionary, while those who are interested in change will offer only lukewarm support (Machiavelli 1532). Ford owned the business. He could do whatever he wanted. Ohno and Shingo were in a unique situation of "do or die" and were therefore allowed a free rein.

Although less likely to result in a complete success, imitation is a far less risky practice to the manager. If it is successful—great! If not, who can be blamed for doing what the "best in the business" were doing? Imitation, euphemistically called "benchmarking," became a standard practice for American companies in the 1980s. Unfortunately, it was based on compartmentalized descriptions of pragmatic JIT that detailed individual techniques, but could not evoke the spark of creativity required to select, develop, and balance them in a particular manufacturing setting. The lack of systematic guidance on where and how to apply the pragmatic JIT methods, coupled with the deceptively alluring visions of simplicity conjured up by romantic JIT, led too many managers to adopt specific JIT techniques with little overall coordination or prioritization.

Although some Americans may have perceived them as such, Ohno and Shingo never intended their methods as any sort of quick-fix panacea for every manufacturing environment. As we noted earlier, the dramatic setup reductions at Toyota were actually achieved by 25 years of slow, incremental work. Shingo (1989) seemed somewhat amused by the thought that Americans could rapidly adopt JIT methods successfully and quipped

> Some people imagine that Toyota has put on a smart new set of clothes, the kanban system, so they go out and purchase the same outfit and try it on. They quickly discover that they are much too fat to wear it.

Moreover, it is clear that the early JIT pioneers considered their developments a competitive advantage. Ohno admits that the Japanese used deliberately confusing terms to describe JIT. He once stated, "If the U.S. had understood what Toyota was doing, it would have been no good for us" (Myers 1990). Terms such as *JIT, zero inventories,* and *stockless production* may have served to delude Americans into thinking that JIT is far simpler than it is.

The fundamental difficulty in combining the ideals of romantic JIT with the details of pragmatic JIT into a coherent system lies in the fact that the ideals stress multiple, sometimes conflicting objectives. Throughput, quality, regularity of flow, flexibility, worker involvement, and other objectives are often cited as central to JIT. But which of these should take precedence? How is one to evaluate a policy that promotes some objectives but impedes others? The romantic JIT literature tended to oversimplify and minimize the difficulty of balancing conflicting concerns. Schonberger (1990, viii) went so far as to ban the word *tradeoff* (he calls it the *t-word*) from civilized conversation![7] But refusing to talk about them does not make tradeoffs go away. The Japanese originators of JIT *did* balance these tradeoffs—but subtly, artfully, and in the context of their specific manufacturing environments. The subtlety of the Japanese system for making tradeoffs allowed it to be easily overlooked, and consequently this aspect of JIT was lost in popular American descriptions of it.

But the failure of the American JIT literature to develop the intuition and systematic framework needed for balancing competing objectives was a serious one. The balance

[7] Zipkin (42) relates a story of a company that took Schonberger's overzealous advice literally and found itself inventing euphemisms for the word *tradeoff* in order to have meaningful discussions of options.

struck by Toyota and the other JIT pioneers was probably more important than any particular methodology. Ignoring it was tantamount to throwing away the banana and keeping the peel.

As a specific example, consider the fondness of the JIT literature of referring to inventory as the "root of all evil." Without a perspective on tradeoffs, this simple slogan implies that removing inventory can only benefit the system. In fact, in the often-cited JIT analogy of a stream, with WIP as water and problems as rocks, lowering the water (i.e., removing WIP) is necessary for promoting improvement. Thus, many firms in the 1980s ambitiously pursued WIP reduction programs.

Without question, many firms ultimately benefited from such efforts because inventory levels were too high. But how many went too far?[8] How many caused themselves unnecessary disruption by removing WIP before eliminating the environmental flaws that necessitated the WIP? Inman (1993) has observed that inventory is better described as the "flower" rather than the "root" of all evil, since high levels of inventory are a consequence of other problems. To pursue the stream analogy a bit further, it would be better to use sonar to locate the rocks, remove them, and then lower the water, rather than to lower the water and smash into the rocks in order to find them. Unfortunately, JIT, as described in the American literature, offered neither sonar (i.e., models that predict the effects of system changes) nor a sense of the relative economics of level reduction versus rock removal—that is, procedures for evaluating the tradeoffs between the benefits of WIP reduction and the costs of eliminating problems.

Thus, American firms implementing JIT struck, explicitly or implicitly, their own balance among competing objectives. Those that did this with a basic understanding of their fundamental processes created effective systems. The rest were probably disappointed in their JIT experiences. In any case, because putting together a coherent JIT system is a daunting task, firms across the board have frequently relied on outside consultants to help them in JIT implementation. The expense of such consulting, plus the substantial training expenses that are required, can make JIT a costly option. Indeed, Inman and Mehra (1990) reported that such expenses can put JIT beyond the reach of many small companies. So despite some well-publicized success stories and a great deal of romantic JIT hyperbole, just-in-time has proved to be neither simple nor inexpensive.

In addition to the legacy of low-inventory, flow-oriented production, the JIT movement left another important mark on the manufacturing landscape, namely, total quality management (TQM). Originally an essential component of JIT—low inventory production cannot be implemented without good quality and good quality is impossible without low inventory levels and short cycle times—TQM soon spawned a movement of its own. TQM quickly eclipsed JIT and became the preeminent manufacturing buzzword of the 1980s. By the end of the 1980s virtually all American companies had some type of TQM program, whether or not they were making use of other JIT techniques. Quality was elevated from a low-level staff function to the executive suite through the appointment of vice-presidents of quality and proclamations by CEOs of the central role of quality (e.g., Bob Galvin of Motorola stated emphatically: "No company has ever hurt profits by improving quality"). Uniform quality "standards" (e.g., ISO 9000) became part of the business landscape. The 1980s were dubbed the "decade of quality."

But by the middle 1990s quality was passé. Companies discontinued programs and renamed positions. Business students objected to TQM courses as "out of date."

[8] We know of a furniture manufacturer that nearly put itself out of business via inventory reduction. The reason was that rising wood prices in recent years meant that competitors who carried more inventory were able to buy it earlier at lower prices and therefore had lower costs.

Depending on the commentator, the 1990s were the decade of "speed" or "agility" or anything but quality.

This was due in part to the success of the TQM movement. The quality of American manufactured goods really did improve during the 1980s. American firms in industries threatened by higher-quality imports (e.g., the auto industry) managed to close the gap through significant investments in facilities and procedures. But gaps still exist, and most companies are still nowhere near their "parts per million" or "six-sigma" targets. Moreover, customers, conditioned by competition to demand higher standards, are still far from ecstatic about most manufactured products. So opportunties still exist to gain competitive advantage through higher quality, although it is no longer fashionable to speak in these terms.

Another reason that TQM diminished as a thrust is that quality is not always the most promising competitive lever. Ford's concentration on quality (and cost) in the 1920s and 1930s almost destroyed the firm because it neglected the diversity factor so successfully introduced by General Motors. A similar dynamic was at play in the semiconductor industry in the 1980s and 1990s, where yield losses in microprocessor wafer fabs almost never reached one in 100 let alone one in one million, before the next generation of technology was introduced. The reason, of course, was that the benefits of rapid product development outweighed the benefits of extremely high levels of quality.

These factors may explain why quality fell out of fashion as a buzzword, but they do not diminish its importance as a competitive dimension. Just as quality did not eliminate cost as a concern—indeed, one of the main challenges of TQM was to elevate quality without increasing cost—the new dimensions of speed or flexibility do not replace quality. A key to competitiveness in the future will be the ability to elevate quality even while delivering products to customers faster and in greater variety than ever before. This will require steep learning curves, which precludes reliance on trial-and-error methods. The only way to do this is to have a sufficiently sophisticated body of theory to predict performance and "do it right the first time."

We can summarize the outcome of the JIT and TQM movements by noting that both have produced a number of important and useful insights about manufacturing management. At the same time, what was described in the American JIT literature as a *system* is really a loosely coordinated *collection* of techniques infused with an inspiring stream of romantic rhetoric. The well-publicized success of the Japanese in the 1980s, appealing JIT slogans, and the apparent simplicity of JIT techniques led us to expect far more than we received from the JIT "revolution." Similarly, the elevated awareness of quality and the specific statistical tools of TQM are unquestionably essential components of modern manufacturing management. But like JIT, TQM was sold in romantic terms with near religious fervor. As a result, it failed to develop a coordinated system with which to integrate the pieces, balance quality with other business objectives, and facilitate compression of the learning curve. In both TQM and JIT, what is lacking is a fundamental paradigm that connects practices with business performance. We propose one such paradigm with factory physics in Part II and specifically examine the science of pull in Chapter 10 and the relationships between quality and logistics in Chapter 12.

5.5 Where from Here?

In Part I of the book, and particularly this chapter, we have made the following points:

1. Scientific management, particularly the quantitative methods, has reduced the manufacturing management problem to analytically tractable subproblems, often with

unrealistic modeling assumptions, to the point where they provide little useful guidance from an overall perspective. The mathematical methods and some of the original insights can certainly still be useful, but we need a better framework for applying these in the context of an overall business strategy. Business process reengineering exhorts managers to rethink their processes, but does not provide a framework and became too closely identified with exclusively radical solutions and downsizing to provide a balanced alternative.

2. MRP is fundamentally flawed, not in the details, but in the basics, because it uses an infinite-capacity, fixed-lead-time approach to control work releases. "Patches," such as MRP II and CRP, may help but cannot rectify this basic problem. The original insight of MRP, that dependent demands are distinct from independent ones, is still valid; the planning hierarchy established by MRP II is useful; and the data maintenance and sharing functions of MRP systems are essential. Finite-capacity scheduling modules in ERP systems offer the potential for a fix. However, a single scheduling approach is unlikely to be effective in all types of systems. Moreover, by building "best practices" into the systems demanded by their software, ERP could have the undesirable effect of stifling creativity and preventing firms from crafting systems well suited to their needs. To design appropriate scheduling modules and apply them effectively in an effective planning hierarchy will require careful coordination with the principles governing production system behavior.

3. JIT and TQM are collections of methods and slogans, not systems. Because of this, it is simply not possible to imitate the Japanese successes of the 1980s in cookbook fashion. The many central and creative insights of the JIT and TQM founders need to be appreciated and built upon. However, only by establishing a framework for balancing competing objectives can we develop effective manufacturing management systems.

The real lesson in all this is that there is *no easy solution*. We Americans seem to have a resolute faith in a swift and permanent resolution of the manufacturing problem. Witness the famous economist John Kenneth Galbraith who stated years ago that we had "solved the problem of production" and could move on to other things (Galbraith 1958). Even though it quickly became apparent that the production problem was far from solved, our faith in the possibility of solving it remained unshaken. Each successive approach to manufacturing management—scientific management, operations research, MRP, JIT, TQM, BPR, ERP, etc.—has been sold as *the* solution. Each one has disappointed us, but we continue to look for the elusive "technological silver bullet" to save American manufacturing.

When will we learn? Manufacturing is complex, large scale, multiobjective, rapidly changing, and highly competitive. There *cannot* be a simple, uniform solution that will work well across a spectrum of manufacturing environments. Moreover, even if a firm can come up with a system that performs extremely well today, failure to continue improving is an invitation to be overtaken by the competition. Ultimately, each firm is on its own to develop an effective manufacturing strategy, support it with appropriate policies and procedures, and continue to improve these over time. As global competition intensifies, the extent to which a firm does this will become not just a matter of profitability, but one of survival.

Discussion Points

1. Consider the following quote referring to the two-machine minimize-makespan scheduling problem:

   > At this time, it appears that one research paper (that by Johnson) set a wave of research in motion that devoured scores of person-years of research time on an intractable problem of little practical consequence. (Dudek, Panwalkar, and Smith 1992)

   a. Why would academics work on such a problem?
   b. Why would academic journals publish such research?
   c. Why didn't industry practitioners either redirect academic research or develop effective scheduling tools on their own?

2. Consider the following quotes:

   > An MRP system is capacity-insensitive, and properly so, as its function is to determine what materials and components will be needed and when, in order to execute a given master production schedule. There can be only one correct answer to that, and it cannot therefore vary depending on what capacity does or does not exist. (Orlicky 1975)

   > For at least ten years now, we have been hearing more and more reasons why the MRP-based approach has not reduced inventories or improved customer service of the U.S. manufacturing sector. First we were told that the reason MRP didn't work was because our computer records were not accurate. So we fixed them; MRP still didn't work. Then we were told that our master production schedules were not "realistic." So we started making them realistic, but that did not work. Next we were told that we did not have top management involvement; so top management got involved. Finally we were told that the problem was education. So we trained everyone and spawned the golden age of MRP-based consulting. (Kanet 1988)

   a. Who is right? Is MRP fundamentally flawed, or can its basic paradigm be made to work?
   b. What types of environment are best suited to MRP?
   c. What approaches can you think of to make an MRP system account for finite capacity?
   d. Suggest opportunities for integrating JIT concepts into an MRP system.

Study Questions

1. Why have relatively few CEOs of American manufacturing firms come from the manufacturing function, as opposed to finance or accounting, in the past half century? What factors may be changing this situation now?
2. In what way did the American faith in the scientific method contribute to the failure to develop effective OM tools?
3. What was the role of the computer in the evolution of MRP?
4. In which of the following situations would you expect MRP to work well? To work poorly?
   a. A fabrication plant operating at less than 80 percent of capacity with relatively stable demand
   b. A fabrication plant operating at less than 80 percent of capacity with extremely lumpy demand
   c. A fabrication plant operating at more than 95 percent of capacity with relatively stable demand
   d. A fabrication plant operating at more than 95 percent of capacity with extremely lumpy demand

e. An assembly plant that uses all purchased parts and highly flexible labor (i.e., so that effective capacity can be adjusted over a wide range)

f. An assembly plant that uses all purchased parts and fixed labor (i.e., capacity) running at more than 95 percent of capacity

5. Could a breakthrough in scheduling technology make ERP the perfect production control system and render all JIT ideas unnecessary? Why or why not?
6. What is the difference between *romantic* and *pragmatic* JIT? How may this distinction have impeded the effectiveness of JIT in America?
7. Name some JIT terms that may have served to cause confusion in America. Why might such terms be perfectly understandable to the Japanese but confusing to Americans?
8. How long did it take Toyota to reduce setups from three hours to three minutes? How frequently have you observed this kind of diligence to a low-level operational detail in an American manufacturing organization?
9. How would history have been different if Taiichi Ohno had chosen to benchmark Toyota against the American auto companies of the 1960s instead of using other sources (e.g., Toyota Spinning and Weaving Company, American supermarkets, and the ideas of Henry Ford expressed in the 1920s)? What implications does this have for the value of benchmarking in the modern environment of global competition?

PART

II FACTORY PHYSICS

A theory should be as simple as possible, but no simpler.
Albert Einstein

CHAPTER

6 A SCIENCE OF MANUFACTURING

I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but have scarcely, in your thoughts, advanced to the stage of Science, whatever the matter may be.

Lord Kelvin

6.1 The Seeds of Science

These are confusing times for manufacturing managers. The barrage of books, short courses, software packages, videotapes, Web sites, and other sources pushing competing manufacturing philosophies and tools is enough to overwhelm even the most experienced professional. Moreover, as we saw in Part I, major approaches to manufacturing management (e.g., classical inventory control, MRP, and JIT) are not fully compatible with one another and suffer individually from serious flaws.

Many in manufacturing have come to view their discipline in terms of a blizzard of management buzzwords (for example, MRP, MRP II, ERP, JIT, CIM, FMS, OPT, TQM, BPR) and a succession of gurus. Micklethwait and Woolridge (1996) describe this trend in their revealingly titled book *The Witchdoctors*.

While they frequently offer kernels of truth, the very nature of buzzword approaches is to sell a single solution for all situations. Hence, they provide little balanced perspective on what works well and when. This has often led to a "management by bandwagon" mentality with unfortunate results. Employees, battered by one "revolution" after another, settle into a cynical attitude that "this too will pass." But undaunted, many managers keep to the faith, believing that someone, somewhere has a silver bullet that will solve all their operations problems. As a result, buzzword books and consultants prosper, but little real progress is made.

Certainly part of the confusion stems from the excessive hyperbole used by vendors and consultants to market their wares. Glitzy promotional materials built around vague,

sweeping claims make it difficult for managers to accurately compare systems. However, we suspect the roots of the problem are deeper than this. We believe that a large measure of the confusion is a direct consequence of our lack of an underlying **science of manufacturing.**

6.1.1 Why Science?

In a field such as physics, where the objective is to understand the physical universe, the need for science is obvious. But manufacturing management is an applied field, where the objective is financial performance, not discovery of knowledge. So why does it need science?

The simplest response is that many applied fields rely on science. Medicine is based on biology, chemistry, and other sciences. Civil engineering is premised on statics, dynamics, and other branches of physics. Electrical engineering depends on the sciences of electricity and magnetism. In each case, the scientific foundation provides a powerful set of tools, but is not in itself the complete applied discipline. For example, the practice of medicine involves much more than simply applying the principles of biology.

More specifically, science offers a number of uses in the context of manufacturing management.

First, science offers precision. As the quote at the beginning of this chapter attests, "when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind." So one reason to develop a science of manufacturing is to provide more precise characterization of how systems will work. Relations that provide predictions are the **basics** of science. For example, $F = ma$ is a basic relation of physics. Probability tools, like those we used to model demand uncertainty in inventory systems in Chapter 2, are examples of important basics of factory physics.

Science also offers **intuition.** The formula $F = ma$ is intuitive. Double the force and, for the same mass, acceleration doubles. Elementary school students are required to take science courses, not so they can calculate the outcome of an experiment, but so they can better understand the world around them. Knowing that water expands when it freezes and that expanding ice can crack an engine block convinces one of the need for antifreeze (whether or not one can compute the molality of a solution). Similarly, a manager frequently does not have time to conduct a detailed analysis of a decision. In such cases, the real value of models is to sharpen intuition. Good intuition enables managers to focus their energies on issues of maximum leverage.

Finally, science facilitates **synthesis** of disparate perspectives by providing a unified framework. For instance, for many years, electricity and magnetism and optics were thought to be different fields. James Clerk Maxwell unified them with four equations. In manufacturing, key performance measures, such as work in process and cycle time, are often treated as if they are independent. But as we will see in Chapter 7, there are well-defined and useful relationships between these measures. Moreover, manufacturing enterprises are complex systems involving people, equipment, and money. As such, they can be reasonably viewed in a variety of ways: as a collection of people with shared values, as a creative community for developing new products, as a set of interrelated physical processes, as a network of material flows, or as a set of cost centers. By providing a consistent framework, a science of manufacturing offers a means to synthesize these disparate views. Bringing the different parts of a system into an effective whole is close to the core of the management function.

To further highlight the need for a science of manufacturing, we consider two examples.

Example: A Product Design

First, suppose the marketing department of an automotive company has proposed a concept for a new car that entails

- A mass of 1,000 kilograms (about 2,200 pounds), for safety and luxury.
- Acceleration from 0 to 60 in 10 seconds (approximately 2.7 meters per second squared), for sportiness.
- An engine that generates no more than 200 newtons of force (approximately 45 lb), for fuel efficiency.

Can it be done?

When framed in such simple terms, we can give a simple answer to this question—*no way!* The elementary relationship from physics

$$F = ma$$

or, in this case,

$$200\ \text{N} \ll (1{,}000\ \text{kg})(2.7\ \text{m/s}^2) = 2{,}700\ \text{N}$$

clearly shows that the above requirements are inconsistent. Additionally, this physics analysis indicates where changes can be made to come up with a feasible design. Assuming that the acceleration requirement is fixed, we must either reduce the mass or increase the force of the engine. Hence, we need to consult more sophisticated aspects of the theory behind automotive engineering to find ways to decrease the mass while maintaining stability and safety and/or increase the force of the engine while maintaining fuel economy.

Readers with physics and engineering backgrounds will be quick to point out that this example is oversimplified—that the size of the engine should be rated in units of power and torque, not force, and that the torque generated by the engine will vary with speed. But while these considerations would complicate the analysis, they would not change the fundamental point: that there *is* a theory that enables us to determine the feasibility of a given set of requirements.

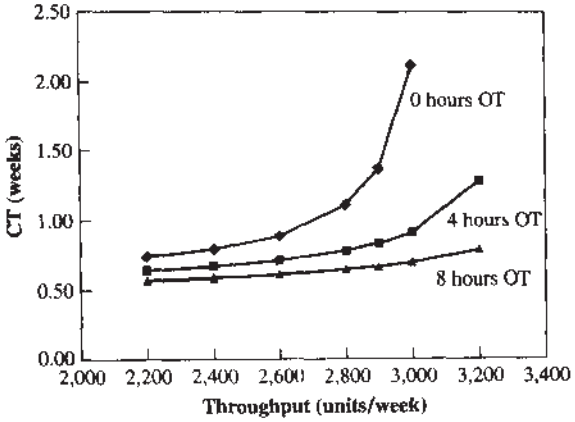
Many design decisions, for products ranging from semiconductors to bridges, are made on the basis of well-developed theoretical sciences. Although the sciences differ from one another, they all have the following features in common:

1. They offer *quantitative* relationships describing system behavior (e.g., $F = ma$).
2. They are founded on theories for *simple systems*, around which theories for more complex, real-world systems are built (e.g., the classical mechanics relationships are all stated for systems without air resistance or friction).
3. They contain *intuitive* key relationships. For example, $F = ma$ clearly indicates that doubling the mass halves the acceleration under a constant force. For a given set of observations, a much more complex formula than $F = ma$ might actually fit the data better, but would not provide the same clear insights and hence would be less powerful.

Example: A Factory

Next, suppose we are given specifications for a factory instead of for a product. Specifically, the vice president of manufacturing has demanded that a printed-circuit board (PCB) plant produce

FIGURE 6.1
Relation between cycle time and demand



- 3,000 PCBs per week to meet demand.
- With an average cycle time (delay between job release and completion) of not more than one week, to maintain responsiveness,
- No overtime (workweek of 40 hours), to keep costs low.

Can it be done?

This time, the answer is not so clear. The equivalent of $F = ma$ of factory design is not widely known;[1] and the factory analogs to the more sophisticated elements of automotive engineering have not even been developed.

If it did exist, what might a theory of factory design show us? One possibility would be the relationships necessary to generate the graph in Figure 6.1 for the PCB plant. The x axis indicates the throughput with the y axis, showing the resulting average cycle time. The three curves show the relationship for the cases of no overtime, four hours of overtime, and eight hours of overtime per week.

From this graph, we see that the immediate answer to the vice president's question is no. If we insist on no more than one week of average cycle time and no overtime, the best we can do is 2,600 units per week. If we insist on an average cycle time of less than one week and 3,000 units per week, we will need an additional four hours per week of overtime. As long as the characteristics of the plant yield this set of curves, there is no way to comply with the vice president's demand. This does not mean it is impossible, only that it cannot be done with the current plant configuration. Presumably, therefore, the next thing we want from our theory is an indication of how to change the plant in a cost-effective manner so as to alter Figure 6.1 to meet the vice president's requirements.

Notice that the relationships in Figure 6.1 satisfy the previously cited properties of design sciences: they are *quantitative, simple* (we will see how they are derived for simple systems later on as we develop the results upon which these curves are based), and *intuitive*. Thus, even if they were not used to answer numerical questions, such as that posed by the vice president of manufacturing, relationships like these contain valuable management insights. They indicate that efforts to increase release rate (and

[1] A plausible analog to $F = ma$ for factory design does exist, as we will see in Chapter 7, but it is not sufficient by itself to answer the question posed here.

hence throughput) may result in a sharp increase in cycle time. They also show that adding capacity (in this case overtime) makes cycle time less sensitive to release rate. We will conjecture laws that govern this and other behavior in the remainder of Part II.

6.1.2 Defining a Manufacturing System

Before we can develop a science of manufacturing, we must define precisely what we mean by a manufacturing system. We use the following definition, a modification of one suggested by Deuermeyer (1994):

> A **manufacturing system** is an *objective*-oriented *network* of *processes* through which *entities flow.*

The key words of the definition are emphasized in italic. First, a manufacturing system has an **objective.** This generally relates to making money, but as we discuss below, specifying the fundamental objective requires some care. A manufacturing system contains **processes.** These may include the usual physical processes (cutting, grinding, welding, etc.), but can also include other steps that support the direct manufacturing processes (order entry, kitting, shipping, maintenance, etc.). **Entities** include not only the parts being manufactured, but also the information that is used to control the system. The **flow** of the entities through the system describes how materials and information are processed. Management of this flow is a major part of a manufacturing manager's job. Finally, it is important to recognize that a manufacturing system is a **network** of interacting parts. Managing the interactions is as important as managing individual processes and entities, if not more so.

This definition of a manufacturing system serves to highlight the roles of the different disciplines that deal with manufacturing. For instance, mechanical and electrical engineering deal principally with manufacturing processes and the design of the entities (products), while industrial engineering (and chemical engineering in continuous flow systems, such as oil refineries and chemical plants) focuses on the flows and the network. Management is concerned with ensuring compliance with the objective and measuring progress toward that goal.

6.1.3 Prescriptive and Descriptive Models

In the previous examples we used **descriptive models** to determine whether our system would meet the desired specifications. In manufacturing management teaching and research, most of the models used are **prescriptive models.** That is, they seek to *prescribe* or *optimize* design or control of a production system. Clearly prescriptive models are needed, but it is essential to understand the basic relationships governing a system *before* attempting to optimize it.

Prescriptive models are typically derived from a set of *mathematical* assumptions. As such, they differ from models used in the sciences such as physics and chemistry which are statements about nature. They are not derived from mathematics, but instead are essentially independent conjectures. For example, the overarching goal of physics is to explain the most phenomena with the fewest elementary conjectures. The resulting descriptive models provide the foundation for prescriptive models used by practitioners in applied fields such as mechanical and chemical engineering for guidance in designing and controlling complex systems (such as chemical plants).

As an example, consider the problem faced by a civil engineer in selecting a bridge design. Each available design strategy represents a prescriptive solution based on both

experience and models. For instance, over a long span, a suspension bridge is often a good option. Suspension bridges are supported by cables made of steel, which can accommodate enormous *tensile* stresses but are almost worthless when faced with *compression* stresses. In contrast, a shorter span is often better served with a reinforced-concrete bridge, where the supporting members curve upward slightly, producing compression stresses in the load-bearing members. Concrete can support large compression stresses but does not work well under tension.

How do civil engineers know these things? Early in their education, before taking a course on building large structures, they take a set of engineering science courses. One of these, statics and dynamics, covers compression and tension forces. Here one learns how an arch transmits load from its top to its base. Another early course describes the strength of materials such as steel and concrete. In our parlance, these are descriptive courses. Only after these basic concepts are understood, does the prospective engineer begin to take design or prescriptive courses.

One could argue that the models traditionally taught in operations management courses represent the descriptive model foundation of manufacturing management. Like the models taught in engineering science courses, they are elementary and are used as building blocks for more complex systems. However, there is a fundamental difference. As Little (1992) pointed out, most of the mathematical models used in operations management and industrial engineering (IE) are *tautologies*. That is, given a particular set of assumptions, the system can be proved to behave in a particular manner. The emphasis is on proper derivation from the assumptions to the conclusions and not on whether the model is a realistic representation of an actual system. In essence, the *truth* of the model is self-contained. Little even demonstrated that a "law" named for himself (and one that we will explore in Chapter 7) is not a law at all but is a tautology. Since it can be shown to hold mathematically, there is no point in checking Little's law with empirical data.

Unlike mathematical tautologies, the models taught in engineering science courses *do* make conjectures about the outside world. They invite the student to check particular statements against empirical evidence (and students do exactly this in laboratory sections). The formula $F = ma$ is one such conjecture. This law is certainly not a mathematical tautology; indeed it isn't even strictly true (it is only correct for speeds that are slow compared to the speed of light). Nonetheless, it is enormously useful and is at the heart of many complex engineering models. Important results in physics, such as $F = ma$ and other Newtonian laws are also remarkable for their simplicity. However, as any sophomore engineering student can attest, the field of statics and dynamics is anything but simple, even though it is based solely on a small set of extremely simple statements about nature.

It is important to note that no scientific law can ever be proven. Derivation from first principles is not a proof since the first principles are themselves conjectured laws. Since we can never observe all possible situations (unlike mathematical induction), we can never know if our current explanation of observed phenomena is the right one or whether some other better explanation will come along. If history is any guide, it is a good bet that all the laws of physics will eventually be challenged and overthrown.

However, the practice of science is not as hopeless as it might seem. An unproved or even refuted law (such as $F = ma$) can be quite useful. The key is to understand where it does and *does not* apply. This is why it is important not to seek to verify our hypotheses but instead to try our best to refute them. The more we refute, the more we learn about the system and the better the surviving law will be (Polya 1954). We call this process **conjecture and refutation** (Popper 1963). In many ways, conjecture

and refutation is to science what "ask why five times" is to JIT implementation. Both represent procedures for getting beyond the obvious and down to root causes.

While there is yet no universally accepted basic science of operations management, a number of researchers and teachers are beginning to address this gap (see Askin and Standridge 1993, Buzacott and Shantikumar 1993, and Schwarz 1996). This book represents our attempt to structure a science of manufacturing. Admittedly it is far from complete. The factory physics relationships we can offer at this time are a combination of insights from historical practices, recent developments by researchers and practitioners, equations from queueing theory, and a few results from our own research. However, factory physics is no buzzword. It is not easy nor does it pretend to offer a solution for all situations. Factory physics simply provides the basic relationships among fundamental manufacturing quantities such as inventory, cycle time, throughput, capacity, variability, customer service, and so on. It is our hope that understanding these relationships in the context of a science of manufacturing, even an incomplete one, will better equip the reader to design and control effective manufacturing enterprises.

6.2 Objectives, Measures, and Controls

Developing a science of manufacturing is not a trivial task. Just as hard is applying this science to solve manufacturing problems. A process that is helpful in both regards is the systems approach.

6.2.1 The Systems Approach

The notion of conjecture and refutation is not only a vehicle for scientific research. It is also the foundation for an extremely useful problem-solving methodology, known as the **systems approach,** or **systems analysis.** Systems analysis (SA) has been studied formally for at least 30 years (see, e.g., Ackoff 1956, Churchman 1968, and Miser and Quade 1985, 1988 for discussions), but has been part of management thought, in spirit if not name, since at least as far back as the work of Chester Barnard (1938).

Briefly, systems analysis is a structured problem-solving approach characterized by

1. *A systems view.* The problem is viewed in the context of a system of interacting subsystems (e.g., a factory is a system composed of product flows supported by various subsystems consisting of different departments, shifts, lines, etc.). The emphasis is on taking a broad, holistic view of the problem, rather than a narrow, reductionist perspective.

2. *Means-ends analysis.* The objective is always specified first, and then alternatives are sought and evaluated in terms of this objective. Note that this is in sharp contrast with the "means first" approach frequently used in the political arena, in which alternatives are posed first and objectives are only introduced as expedient to the consensus-forming process.[2] For instance, a systems analysis might use the objective "to deliver finished goods swiftly and conveniently to customers," but would *not* use the objective "to improve the efficiency of processing purchase orders." The latter is a means-first approach, which could rule out potentially attractive options (e.g., doing away with purchase orders under an entirely new procedure).

[2]Lindblom (1959) terms the means-first approach *disjointed incrementalism* and argues that it may be better suited to the political process than the systems approach.

3. *Creative alternative generation.* With the objective in mind, the systems approach seeks as broad a range of alternative policies as possible. Many formalized brainstorming techniques have been developed to aid in this process. Regardless of the method used, however, the intent is to find nonobvious ways to improve the system. For instance, to reduce manufacturing cycle time (the time it takes to make a product), we should go beyond simply considering how to speed up the individual steps and think about ways to eliminate entire portions of the production process.

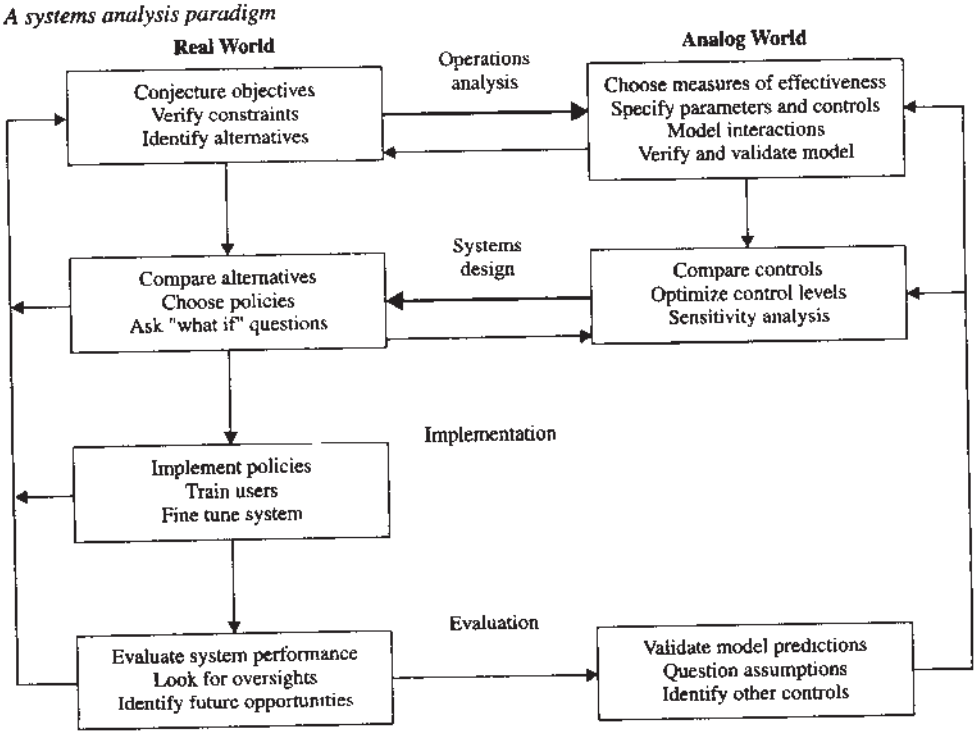
4. *Modeling and optimization.* To compare alternatives in terms of the objective, some kind of quantification is required. The modeling/optimization step for doing this may be as simple as computing costs for each alternative and choosing the cheapest one, or it may require analysis of a sophisticated mathematical model. The appropriate level of detail will vary depending on the complexity of the system under study and the magnitude of the potential impact of the actions (e.g., it makes no sense to do $50,000 worth of analysis to save $52,000).

5. *Iteration.* In almost every systems analysis, the objective, alternatives, and model are revised repeatedly. This is not because we are dumb; it is because real-world systems are complex. Catching errors and oversights is a natural part of the conjecture and refutation process.

Figure 6.2 depicts a schematic of the systems analysis process in four basic phases: operations analysis, systems design, implementation, and evaluation. As indicated by the feedback arrows, these phases are not sequential. Iteration can and should take place both within and between phases. Furthermore, the focus of the study generally shifts

FIGURE 6.2

A systems analysis paradigm



back and forth between the real world and the analog (or modeling) world as the analysis proceeds.

The process begins with the **operations analysis** phase, which focuses on the essentially *scientific* task of observing the actual system and developing an appropriate and useful model. To do this, we attempt to define objectives, constraints, and alternatives for the project. Although initially they might seem obvious, they usually prove to be more elusive than expected. Hence, we must conjecture them tentatively and then look for refutations. As the project proceeds, new objectives, constraints, and alternatives may arise, and the relative importance among them may change.

Another issue that frequently arises in reference to the iterative consideration of objectives and constraints is the choice of how to represent them. Often a particular preference may be sensibly stated as either an objective or a constraint. For example, minimizing the number of customer orders that are filled after their due dates could be an objective. Alternatively, requiring that less than two percent of orders be filled late could be a constraint that addresses the same concern. This technique of converting objectives to constraints is known as **satisficing** and is widely used in systems analysis (Majone 1985).

Iteratively considering objectives, constraints, and alternatives for the actual system is only the starting point for the operations analysis phase. In truly complex systems we cannot obtain a thorough understanding of the system and evaluate alternatives by looking at the actual system alone. There are two reasons for this. First, high-level objectives (e.g., maximize customer satisfaction) are generally not measurable. And second, real-world systems are typically too complex to allow direct description of the interaction between the various system components and the effects on the system of specific alternatives. To deepen our understanding of the system and its control alternatives, we develop an analog or model of the essential aspects of the system.

Modeling a real-world system begins with specification of low-level, quantifiable measures of effectiveness to serve as proxies for the true system objectives (e.g., fraction of jobs that are late to represent customer satisfaction). We then specify descriptive parameters, controllable variables, and their interactions to represent the system in some form of model. The art of developing a model that is sophisticated enough to capture the essential features of the system and yet simple enough to allow practical analysis is a complex task requiring a staff with skills in creative problem solving and mathematical methods. Models require both **verification** (i.e., checking the logic of the model) and **validation** (i.e., comparing the model results to reality). Model validation involves repeated iteration between the modeling and observation aspects of the analysis, and should take place throughout the study.

The **systems design** phase is the beginning of the predominantly *engineering* portion of the systems analysis paradigm. While in the operations analysis phase we work primarily from the real world to the analog world through modeling; in the systems design phase we work primarily from the analog world to the real world by translating results from the model to implementable policies. We do this by "optimizing" the model with respect to the chosen measures of effectiveness and then examining the robustness of the solution via sensitivity analysis. We then translate these mathematical or symbolic solutions to actual policies and examine the practicality of these policies in the actual environment. It is important to remember that no matter how good a mathematical model is, it is still a simplification of reality. Like developing appropriate models, interpreting the results to develop sensible courses of actions is an art that can never be fully mechanized.

A good systems analysis does not end with the presentation of the proposed policies. The **implementation** phase of the paradigm offers us the opportunity to see that they

are adopted properly and to identify unanticipated problems while there is still time to deal with them effectively.

Finally, in the **evaluation** phase, we review the system after the policies have been implemented and assess the results in terms of the original objectives. This is an extremely important phase because it offers the best opportunity to validate the usefulness of the model in improving the actual system, as opposed to simply describing the behavior of the system. Since systems analyses are *applied* problem solving exercises, the degree to which the desired objectives were met must always be the bottom line of the study. However, since most real-world systems are complex and constantly changing, the end of a particular study should not mark the end of analysis. Opportunities for future improvements in both the model and the actual system should be identified as input to further cycling of the systems analysis procedure.

6.2.2 The Fundamental Objective

Since, for our purposes, we have defined a manufacturing system as an *objective*-oriented network, the obvious starting point for a science of manufacturing is the **fundamental objective.** This is a broad goal that all parties can agree on. It is generally vague since it describes a long-term aspiration that may or may not be completely quantifiable. In some companies the fundamental objective is formalized into a *mission statement.* However, this deceptively complex exercise frequently becomes an exercise in rhetoric in which scores of person-hours are wasted in "workshops" that do little more than generate a new (and often cumbersome) slogan. It is important to recognize, therefore, that systems analysis only begins with identifying the fundamental objective. By itself, a fundamental objective (or mission statement) is of little tangible value.

"Use money to make more money" is an obvious choice as a fundamental objective. However, it presents problems when we consider that there are many ways to make money, including selling off the firm's assets (possibly good in the short term, but terrible for the future) and dealing in illicit drugs (profitable, but illegal and immoral). Other popular slogans such as "Give the customers what they want" are similarly incomplete—customers would be very satisfied if we provided them with better products for free! To gain widespread support, a fundamental objective must balance the concerns of all parties involved in the organization. The following statement is vague enough to serve as the fundamental objective for almost any manufacturing firm:

> Increase the well-being of the stakeholders (stockholders, employees, and customers) over the long term.

We realize that this is a "Mom and apple pie" statement, which is too vague to yield much concrete guidance. But it does provide a point of common ground for all the stakeholders and stresses that many parties at interest may be affected by changes to a manufacturing system.

6.2.3 Hierarchical Objectives

As soon as we specify a fundamental objective, conflicts arise, since what is good for one stakeholder is not always good for another. Cost reduction through lower wages is good for profitability and hence stockholders, but is not good for employees. To strike a balance, we need to narrow our fundamental objective slightly, perhaps to something like

> Make a "good" return on investment (ROI) over the long term.

This statement will satisfy stockholders because ROI supports stock price. It will also satisfy employees in one regard since they will continue to be employed and in a position to receive better wages. Finally, customers will be satisfied, because if they are not, good returns will be impossible over the long run. Thus, this statement, while still very high level, relates to the concerns of the primary parties at interest and is directly measurable.

But we cannot simply inform the workers of the firm's high-level objectives. No amount of encouraging slogans plastered about the plant exhorting workers to achieve a good return on investment will stimulate manufacturing excellence. People have to know how *their* jobs affect the fundamental objective in order to be able to influence it in a positive fashion. For this, we need to identify measures more directly related to production.

First, note that profit and return on investment (ROI) are computed from three financial quantities—(1) **revenue,** (2) **assets,** and (3) **costs**—as follows:

$$\text{Profit} = \text{Revenue} - \text{Costs}$$

$$\text{ROI} = \frac{\text{Profit}}{\text{Assets}}$$

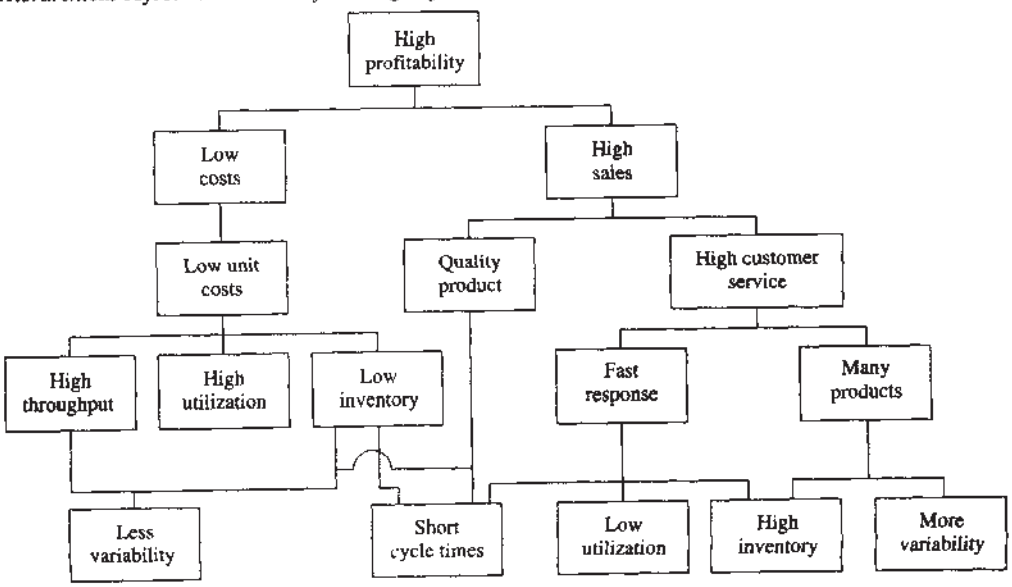
But even these measures are too high-level for day-to-day plant operation.

The plant-level equivalents of revenue, assets, and costs are (1) **throughput,** the amount of product *sold* per unit time (it does no good to make it and not sell it); (2) **assets,** particularly *controllable* assets such as inventory; and (3) **costs,** consisting of operating expenditures of the plant, particularly cost variances such as overtime, subcontracting, and scrap. These three basic measures provide the link between the high-level financial measures (say, ROI), and the lower-level measures (e.g., machine availability) that are more directly related to manufacturing activities.

Figure 6.3 illustrates a sample hierarchy of objectives from the fundamental objective to various supporting **subordinate objectives.** From the formula for profit, we see

FIGURE 6.3

Hierarchical objectives in a manufacturing organization



that high profitability requires low costs and high throughput (sales). Low costs imply low unit costs, which require high throughput, high utilization, and low inventory. As we will see later in Part II, less variability in production is required to achieve low inventory and high throughput. On the other side of the hierarchy, to increase sales requires a high-quality product that people want to buy, plus good customer service. High customer service requires fast response and many products (anything the customer wants). Fast response requires short cycle times, low equipment utilization, and/or high inventory levels. To keep many products available requires high inventory levels and more variability (in product). However, to obtain high quality, we need less variability (in processing) and short cycle times (to catch defects when they occur).

Note that this hierarchy contains some conflicts. For instance, we want high inventory for fast response, but low inventory for keeping total assets low so that the return on assets will be high. We want high utilization to keep unit costs down, but low utilization for good responsiveness. We want more variability for greater product variety, but less variability to keep inventory low and throughput high. Despite the reluctance of some JIT advocates to use the "t word," we have no choice but to make **tradeoffs** to resolve these conflicts.

Finally, it is useful to observe from Figure 6.3 that short cycle times support both lower costs and higher sales. This is the motivation behind the emphasis during the 1990s on speed, embodied in slogans such as **quick response manufacturing** and **time-based competition.** We will take up the important topic of cycle time reduction in Part III, after establishing basic relationships involving variability later on in Part II.

6.2.4 Control and Information Systems

Manufacturing managers face a wide array of controls with which to try to achieve their objectives. Product design, facility design, equipment maintenance, work scheduling, personnel policies, and many other areas present opportunities for controlling a manufacturing system. Despite our focus on flows in factory physics, it is important not to concentrate too narrowly on controls directly related to movement of material (e.g., scheduling). Other controls may seem less closely related to generating throughput, but may be just as important in attaining the fundamental objective of the system.

To provide a structure for thinking about the range of alternatives, Schwartz (1998) compared the practice of an operations manager to that of a financial portfolio manager. A portfolio manager mixes securities in order to get a good and stable return on investment. An operations manager has three basic assets to manage to generate return on investment: information, control, and buffers. Information involves what is known about the system (e.g., inventory status data from the ERP system). Control involves operating policies that affect system behavior (e.g., inventory stocking rules). And buffers involve protection against variability (e.g., safety stocks). The three components must be managed together to obtain effective overall performance. If any one component is lacking (e.g., imperfect information regarding demand), it must be compensated by some combination of the other two (e.g., more control by assigning due dates or more buffers by carrying safety stock).

As an example, consider a make-to-stock operation controlled by an MRP system. The information system collects data on current inventory, scheduled receipts, and capacity and forecasts demand. The control system uses MRP to translate this information to actual jobs released to the floor and then tracks them as they are completed. The control system might also involve expediting as demands change. Buffers include safety stock, safety lead time, and excess capacity in the system. These are needed because the forecast is never exactly right and because MRP is an imperfect model of the production process.

Any of the three parts of the portfolio offers opportunities for improvement, as does adjusting their mix to improve overall system performance. For instance, better (earlier) information about demand would reduce the need for inventory buffers by allowing more of the demand to be satisfied in make-to-order fashion. A completely flexible workforce (more control) would reduce the need for excess capacity (less buffer). Better prediction of the flows through the system (better information than is offered by the MRP model) would reduce the need for excessive safety lead times and WIP levels (buffers). These are the types of policies espoused in lean manufacturing, which is fundamentally about reducing the need for buffers through better use of information and control. However, we will find in Chapter 9 that no matter how perfect the information and how powerful the controls, there will always be a need for buffers.

6.3 Models and Performance Measures

A hierarchy of objectives like that in Figure 6.3 presents two practical questions. First, how do we resolve the conflicts it identifies? Second, how do we translate these high-level objectives to detailed operational policies?

The answer to the first question is the use of *models* to quantify tradeoffs. The challenge is to develop models that are accurate enough to represent these tradeoffs appropriately, but simple enough to give us good intuition. Much of the remainder of Part II is devoted to several such models that will underlie our discussion of operating procedures in Part III.

6.3.1 The Danger of Simple Models

As we discussed in Chapter 5, the use of fixed lead times in MRP leads to systems that are unresponsive to customers and bloated with inventory. The main reason is the underlying model of cycle time. MRP assumes that cycle time (CT) will be the same regardless of the work-in-process (WIP) level of the line. That is, no matter how much we load into the system, jobs will take the same amount of time to be completed. Mathematically, the MRP model is simply $CT = T_{MRP}$.

A more sophisticated model of cycle time can be constructed by separating the approximation into two cases, one for when the line is relatively empty and one for when it is saturated. When the line is relatively empty, we use an MRP-like model of cycle time $CT = T_{approx}$, where T_{approx} represents the time for a job to go through an uncongested line. When it is saturated, we assume that the line can produce at most C_{approx}, which is the capacity of the line. Hence, the time to process a quantity of WIP will be $CT = WIP/C_{approx}$. Since the cycle time cannot be less than T_{approx}, the complete model of cycle time is

$$CT = \max\left\{T_{approx}, \frac{WIP}{C_{approx}}\right\}$$

We call this the **conveyor model** of a production line because it behaves as a conveyor. The time to go down a conveyor is constant until the conveyor is full. Once it is full, the time to get down the conveyor is computed by dividing the amount of work on and in front of the conveyor by the rate of the conveyor.

In practice, it makes sense to set T_{approx} slightly higher than the actual time to go through an empty line (to account for some congestion) and $CT = T_{approx}$ slightly below the maximum capacity of the line (to account for some inefficiency in the line).

The conveyor model is only slightly more complex than the MRP model, since it requires estimation of two parameters instead of one. However, it is much more accurate. Figure 6.4 shows a sample of the actual relationship between WIP and cycle time, along with the MRP and conveyor models. While the MRP model fits very poorly, particularly for high WIP levels, the conveyor model tracks the basic relationship much more closely.

6.3.2 Building Better Prescriptive Models

Better descriptive models provide the basis for better prescriptive models. For instance, we can use the conveyor model to solve capacitated scheduling problems, as we illustrate in the following example.

Consider a line with a capacity of 100 units per day that requires three days to process jobs (when there is no congestion). Currently there are 50 units in finished goods inventory (FGI), 95 units that have been in the line for three days (i.e., that will start coming out immediately), 95 units that have been in for two days, and 100 units that have been in one day. Since less than 100 units were started two and three days ago, output for those days is limited by available WIP. Thus, the maximum output from this point forward is 50 (immediately from FGI), 95 today, 95 tomorrow, and 100 from that point on. Demands for the next 10 days are as follows: 100, 120, 100, 0, 200, 0, 200, 120, 0, 80. The 340 units in finished goods and currently in WIP cover demand for periods 1, 2, and 3 as well as 20 units of the 200 due on day four. Thus, the netted demand is 0, 0, 0, 0, 180, 0, 200, 120, 0, 80. If we offset these by three days to find out how much should be started, we obtain starting demands of 0, 180, 0, 200, 120, 0, and 80. Our task at hand is to find a start schedule that minimizes inventory and is capacity-feasible.

We first solve the problem by using the MRP model with a lead time of four days. Since cycle time is assumed constant, releases are simply netted demand offset by four days, or 80, 0, 200, 120, 0, 80, for the next six days. But given the capacity restriction, we will finish 100 on the first day, the remaining 80 on the second day, and 100 on all subsequent days. This results in a shortage of 20 units on the eighth day. If instead we use a lead time of five days in an attempt to eliminate the shortage, the problem becomes infeasible (from an MRP standpoint because we would need to start product before the first day).

As a more sophisticated procedure based on the MRP model of cycle time, we could use the Wagner–Whitin algorithm from Chapter 2 and use the setup cost as a surrogate for a capacity constraint. The schedule becomes feasible only when we make the setup

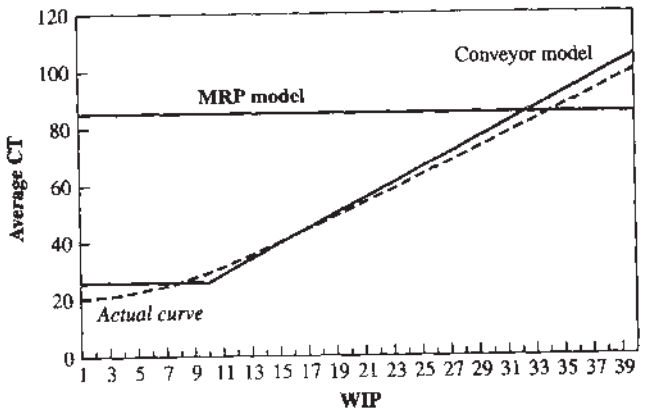


FIGURE 6.4
Relation between cycle time and WIP

cost so high that all production is started in the first period. Specifically, the ratio of setup to holding cost must be at least 1,200. This results in an inventory carrying cost of $340h$, where h is the cost to carry one unit of inventory for one period. If we reduce the setup to holding cost ratio to 1,199, the schedule becomes infeasible with a shortage of 120 units in period 9. Further reductions of the ratio only make the infeasibility worse. Hence, Wagner–Whitin generates either high-inventory or infeasible solutions.

Alternatively, we can formulate a simple solution procedure based on the conveyor model (we develop this further in Chapter 15). To do this, let

D_t = demand on day t

X_t = production on day t, decision variable

I_t = ending inventory on day t

C_t = capacity available on day t

We compute production quantities backward from day 10. First, we set the desired ending inventory for day 10 to zero: $I_{10} = 0$. Then for each day t the production quantity is given by

$$X_t = \min\{C_t,\ D_t + I_t\} \tag{6.1}$$

Notice that unlike in the MRP model, the conveyor model assumes that production on a day is limited by capacity. Therefore, cycle time is a function of inventory (and backlogged demand). To proceed to day $t - 1$, we compute

$$I_{t-1} = I_t + D_t - X_t \tag{6.2}$$

and continue with Equation (6.1). If the ending inventory for period 0 is greater than zero, then demands *cannot* be met using the current capacity no matter what the schedule.

In our example, we can apply this procedure to the netted demands to obtain a start schedule of 100, 100, 100, 100, 100, 0, 80. The inventory holding cost of this schedule is $260h$. Unlike the MRP schedule, this schedule meets all the demands. It also carries 24 percent less inventory than the "optimal" Wagner–Whitin algorithm. This procedure can be extended to multistage production systems, as we show in Chapter 15. The conveyor model can also be applied to throughput tracking and due date quoting, as we discuss in Chapters 13 and 15.

6.3.3 Accounting Models

The mathematical models one normally studies in a course on operations management (EOQ, MRP, forecasting models, linear programming models, etc.) are by no means the only models for measuring performance and evaluating management policies in manufacturing systems. Indeed, some of the most common models used by manufacturing managers are those related to accounting methods. Although accounting is sometimes viewed as mere bookkeeping or cost tracking, it is actually based on models and is therefore subject to the same pitfalls concerning assumptions that face any modeling exercise.

One of the key functions of cost accounting is to estimate how much individual products cost to make. Such estimates are widely used to make both long-term decisions (Should we continue to make this product in house?) and short-term decisions (What price should we quote to this customer?). But because many costs in manufacturing systems are not directly attributable to individual products, they can only be estimated by means of a model.

Direct costs, such as raw materials, are simple to assign. If castings are purchased and machined into switch housings, then the price of the castings must be included in the

unit cost of the switches. Direct labor can be slightly more difficult to assign if workers produce more than one type of product. For instance, if a machinist makes two types of switch housings, then we must decide what fraction of her time she spends on each, in order to allocate the cost of her time accordingly. But this is still a relatively simple computation.

The difficulty, and hence the need for a model, arises in the allocation of **overhead** costs. Overhead (also called **fixed costs** or **burden**) refers to costs that are not directly associated with products. Mortgage payments on the factory, the salary of the chief executive officer, the cost of a research and development laboratory, and the cost of the company mail room are examples of costs that do not vary directly with the production of individual products. But since they are part of the cost of doing business, they are indirectly part of the cost of producing products. The challenge is to apportion the overhead cost among the different products in a reasonable manner.

The traditional approach (model) for allocating overhead costs was to use labor hours. That is, if a particular product used two percent of the hours spent by workers producing products, then it would be assigned two percent of the overhead cost. The rationale for this was that at the turn of the century, when "modern" accounting techniques were developed, direct labor and material typically represented up to 90 percent of the total cost of a product (see Johnson and Kaplan 1987 for an excellent history of accounting methods). Today, direct labor constitutes less than 15 percent of the cost of most products, and hence the traditional methods have been increasingly challenged as inappropriate. The title of the book by Johnson and Kaplan is *Relevance Lost.*

The leading contender to replace traditional cost accounting techniques is known as **activity-based costing (ABC).** ABC differs from traditional methods in that it seeks to link overhead costs to *activities* instead of directly to products. For instance, purchasing might be an activity that is responsible for overhead costs. By measuring the amount of purchasing activity in units of purchase orders and then allocating the purchasing overhead costs to each product on the basis of the fraction of purchase orders it generates, the ABC approach tries to accurately apportion this part of the overhead cost. Similar allocations are done for any other portions of the overhead cost that can be assigned to specific activities. Appendix 6A gives an example illustrating the mechanics of ABC and contrasting it with the traditional labor-hour approach.

Because ABC divides overhead costs into categories, it can promote better understanding, and eventually reduction, of these costs. As such, it is a positive step in the area of cost modeling. However, it is by no means a panacea. Cost-based models, however detailed, can sometimes be misleading.

First, there are cases when the *allocation* of costs is simply a poor modeling focus from a systems point of view. One of the authors worked in a chemical plant in which considerable debate and analysis were devoted to determining the price that should be exchanged for a commodity that was a by-product of one product and a raw material for another. The users of the commodity argued that the price should be zero since it would be wasted if they were not using it. The producers of the commodity argued that the users should pay what it would cost if they had to produce the product themselves. In actuality, neither of the processes would have been profitable as a stand-alone operation, but they were quite profitable when taken together. A better focus for the analysis and debate would have been on how and where to improve yields (how much product was produced) of the two processes.

Second, no matter how detailed the model, it is extremely difficult to accurately represent the value of limited resources by using a cost-based approach common to all accounting methodologies. This applies to both the **full costing** or **absorption costing** method described above and **variable costing** where overhead is not considered.

Full absorption costing is appropriate if we are building a new plant and so are concerned with all the costs of the plant. Variable costing is suited to operating an existing plant, where we should only concern ourselves with costs that can be controlled within a short time frame. For instance, in a new plant, machine and labor costs should all be considered. If one plan requires more setups and those setups take labor to perform, then that plan will truly cost more than a plan requiring fewer setups. On the other hand, in an existing plant we should completely ignore the cost of machines since they have already been purchased. It is a **sunk cost.** Managers are sometimes tempted to run more product on a more expensive machine in order to "recover its cost." But from an overall perspective this may not make sense, especially if the more expensive machine is less suited to running some products than a cheaper one is.

Most product costing (ABC included) is based on fully absorbed and not variable costs. This can lead to bad decisions. For instance, if a customer is asking for a part that requires a long time at the process center that currently has the most work, the cost is great. But if there is demand for an item that flows only through processes that currently have little work to do, the cost is essentially raw materials cost. In essence, the machines and labor are both free since they are there with little else to do. The following example illustrates the danger of using fully absorbed costs to make production decisions.

Example: Production Planning

Consider a plant consisting of three machines that make two products, A and B, as illustrated in Figure 6.5. Product A costs \$50 in raw material and requires two hours on machine 1 and two hours on machine 3. Product B costs \$100 in raw material and requires two and one-half hours on machine 2 and one and one-half hours on machine 3. Thus, both products require four hours of machine and four hours of labor time. Labor cost is \$20 per hour (including benefits, etc.). The plant runs an average of 21 days per month with two shifts or 16 hours per day (workers relieve one another for breaks, etc.), for a total of 336 hours per month. Nonmaterial expenditures to run the plant (i.e., labor, supervision, administration, etc.) are \$100,000 per month. Both products sell for \$600 per unit and make use of exactly the same amount of overhead activities. Marketing estimates a demand of no more than 140 units per month for both products. Also, to maintain market position, the company needs to produce at least 75 units of product A per month. Table 6.1 summarizes the data for this example.

Suppose we cost the products by using an absorption method and then use these costs to help plan how much of each product to make. Since both products require the same number of labor hours and activities, they will receive the same overhead charge regardless of how we allocate overhead. Since these would not affect the *relative* costs of the two products, we can ignore them when choosing between products to produce. The profit per unit of A sold (neglecting overhead and labor costs) is \$600 − \$50 = \$550, while the profit per unit of B sold is \$600 − \$100 = \$500. Since A is more profitable, it would seem that our production plan should favor production of A.

There are 21 × 16 = 336 hours available in a month. Since each unit of B requires two hours of time on machines 1 and 3 to produce, maximum monthly production of

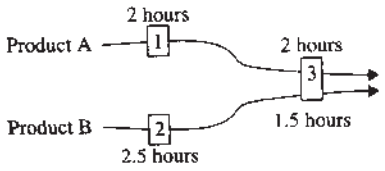


FIGURE 6.5
Layout of two-product plant

TABLE 6.1 Data for Two-Product Plant Example

Product Name	Price ($)	Raw Material Cost ($)	Total Labor Hours	Unit Cost ($)	Minimum; Maximum Demand per Month
A	600	50	4	130	75; 140
B	600	100	4	180	0; 140

either is $336/2 = 168$ units. Since potential demand is only 140, it seems reasonable to set production to maximum demand level for A (140 units per month) which, of course, meets our minimum demand requirement of 75. This uses up 280 hours per month on machine 3, leaving $336 - 280 = 56$ hours on machine 3 for the production of B. Hence, we can produce $56/1.5 = 37$ units of B per month (actually 37.33, but we round to the largest integer quantity).[3]

The monthly profit from this plan can be computed by multiplying the production quantities of A and B by their unit profits and subtracting the nonmaterial costs:

$$\text{Profit} = 140(\$550) + 37(\$500) - \$100{,}000 = -\$4{,}500$$

This plan loses money!

Instead of relying on an accounting model, we could have used an optimization model based on **linear programming.** The idea behind linear programming is to formulate a model to maximize profit subject to the demand and capacity constraints. For this example, we can state our problem as follows:

Maximize Profit

Subject to: Time used on M1 $\leq$ 336 hours

Time used on M2 $\leq$ 336 hours

Time used on M3 $\leq$ 336 hours

75 $\leq$ amount of A $\leq$ 140

0 $\leq$ amount of B $\leq$ 140

Defining X_A and X_B to represent the monthly production quantities of products A and B, we can formalize our model as

$$\begin{aligned} \text{Maximize} \quad & 550X_A + 500X_B - 100{,}000 \\ \text{Subject to:} \quad & 2X_A \leq 336 \\ & 2.5X_B \leq 336 \\ & 2X_A + 1.5X_B \leq 336 \\ & 75 \leq X_A \leq 140 \\ & 0 \leq X_B \leq 140 \end{aligned}$$

This model is an example of a **linear program.**[4] We will go into detail on how to formulate and solve linear programs in Chapter 16. For now, we simply note that there

[3]Note that we did not have to worry about machine 2, since it is only used by product B. The entire 336 hours per month are available for production of B, which is enough to produce $336/2.5 = 134$ units. Hence, it is capacity on machine 3 that determines how much B we can produce.

[4]It is *linear* because the objective function and constraints involve the decision variables X_A and X_B in linear expressions (i.e., multiplied by constants and summed). The term *program* comes from the historical fact that the technique was devised to find optimal programs (i.e., schedules) of resource use; it has nothing to do with the fact that these kinds of problems are generally solved by means of a computer program.

are very efficient techniques for solving this type of optimization model, and we report that for this example the solution results in a plan calling for 75 units of A and 124 units of B per month. Notice that this plan is completely counterintuitive when we consider the "cost" of the products; we are making more of the lower-profit product! However, the profit from this plan is

$$\text{Profit} = 75(\$550) + 124(\$500) - \$100{,}000 = \$3{,}250$$

which is profitable!

The moral of this example is that *the value of limited resources depends on how they are used.* A static cost-based model, no matter how detailed, cannot accurately assign costs to limited resources, such as machines subject to capacity constraints, and therefore may produce misleading results. Only a more sophisticated optimization model, which dynamically determines the costs of such resources as it computes the optimal plan, can be guaranteed to avoid this.

In addition to offering an alternative to the cost accounting perspective, constrained optimization models are useful in a wide variety of operations management problems. In Part III, we will specifically address problems related to scheduling, long-range production planning, and workforce planning with such models. Methods for analyzing constrained optimization models, such as linear programming, are therefore key tools for the manufacturing manager.

6.3.4 Tactical and Strategic Modeling

As useful as models are, it is important to remember that they are only tools, not reality. The appropriate formulation of a model depends on the decision it is intended to assist. Parameters that are reasonably considered constrained for the purposes of tactical decision making are often subject to control at the strategic level. Thus, while one model may be effective in planning production quantities over the intermediate term, another (possibly still a constrained optimization model) is needed for planning over the long term. Chapter 13 explains the hierarchical relationship between production planning and control models in greater detail. Here we will highlight the distinctions between tactical and strategic planning by means of the previous example.

Because the above example focused on the tactical problem of planning production over the next few months, it made perfect sense to treat capacity and demand as constrained. Over the longer strategic term, however, both capacity and demand are subject to influence. Capacity could be increased by adding a third shift or decreased by reducing the second shift. Price discounts could boost demand, while an announcement of a competing (e.g., next-generation) product could reduce demand.

Models can clarify the relationships between tactical and strategic decisions and help ensure consistency between them. For instance, by using the sensitivity analysis capabilities of linear programming (Chapter 16), we can determine that the constraint to produce at least 75 units per month of product A is detrimental to profit. In fact, if we eliminate this constraint and re-solve the model, it generates a plan to produce 68 units of A and 133 units of B, which yields a monthly profit of $3,900, an increase of $650 per month.

This suggests that we should consider the strategic reasons for the constraint to produce at least 75 units per month of A. If the reason is a firm commitment to a specific customer, it may be necessary. But if it is only an approximation of the number needed

to meet our commitments, then using a lower limit of 68 might be just as reasonable, and more profitable.

Another piece of information provided by the sensitivity analysis function of linear programming is that for every additional hour of time available at machine 3 (up to seven extra hours per day), profits increase by $275. Since overtime does not cost nearly $275 per hour, we should probably consider adding some to the short-term plan. But in the longer term, the tactical decision of whether to use overtime relates to the strategic decisions of whether to increase the size of the workforce, add equipment, subcontract production, and so on. Thus, the model also suggests that these be considered as potential future options.

Effective planning calls for the use of different models for different problems and coordination between models. A tactical model, such as the constrained optimization model used earlier to generate a production plan for the next few months, can provide intuition (i.e., what variables are important), sensitivity information (i.e., where there is leverage), and data (e.g., identification of the current bottleneck resource) for use in strategic planning. Conversely, a strategic model, such as a long-term capacity planning model, can provide data (e.g., capacity constraints) and suggest alternatives (e.g., dynamic subcontracting) for use at the tactical level. We will discuss coordination in Chapter 13 and specific models for various levels throughout Part III.

6.3.5 Considering Risk

There are many sources of uncertainty in manufacturing management situations, including demand fluctuations, disruptions in materials procurement, variable yield loss, machine breakdowns, labor unrest, actions by competitors, and so on. In some cases, uncertainty should be explicitly represented in models. In other cases, as we will see in Part III, uncertainty can be safely ignored in the modeling process. But in all cases related to both modeling and management, the existence of uncertainty makes it essential to consider in some fashion what will happen if an assumption fails to hold.

As a high-level strategic example, consider the experience of a major American automobile manufacturer. In the late 1970s and early 1980s, many people in the corporation recognized a need to invest in improved product quality and proposed product and process changes to achieve this. However, funding for many of these projects was denied as not financially justified. The implicit assumption on the part of the corporate staff was that the competitive position of the company's products relative to the competition would remain unchanged. Hence, the cost of such products could not be justified by the promise of greater sales revenues. But when the competition upgraded the quality of its products at a faster pace than anticipated, the corporation experienced a disastrous loss of market share, and only in the 1990s, after a decade of huge losses and widespread plant closings, did the company return to profitability (but nowhere near its former market share).

The flaw in the firm's analysis was fundamental. The quality improvement projects were evaluated on the basis of their potential to improve profits instead of their need to avoid lost profits. Thus, management failed to consider adequately what would happen if the competition outpaced the company by offering better products. Product and process improvement should not have been viewed as an option for increased profitability but rather as a constraint to stay in business.

The procedure of evaluating the potential negative consequences in an uncertain situation is known as **risk analysis** and has been widely used in riskier industries such as petroleum exploration. Using a model, the analyst conjectures several possible scenarios

and assigns a probability of each occurring.[5] Since the scenarios often involve strategic moves on the part of the competition, such analyses are generally undertaken by a senior manager working with a technical expert and a model. One approach for evaluating potential decisions is to weight the various outcomes with the probabilities and to compute an expected value of some performance measure (e.g., profit). An alternative, and sometimes more realistic, approach is to examine the various scenarios and choose a course of action that prevents really bad things from happening. This is the **minimax** (i.e., minimize the maximum damage) strategy that is often used by the military.

Had the previously mentioned automotive company employed a minimax strategy, most likely it would have approved many more product and process improvement projects than it did, as a hedge against improvements by the competition. Of course, since hindsight is 20/20, it is easy for us to say this in retrospect. The best policy is generally not obvious in advance. Indeed, the primary job of upper-level management is to chart reasonable long-term strategies in the face of considerable uncertainty about the future. These executives are highly paid in large part because their task is so difficult. (The question of whether they are smart or just lucky is moot so long as the company is successful.)

At the plant level, operations managers must perform an analogous function to that of upper management, only with a shorter time horizon and on a smaller scale. For example, consider the commonly faced operations problem of selecting machines for a new line.

Example: Risk Analysis

Suppose all the machines for a planned line, except one particular machine, the 3C 273, are capable of switching to any conceivable new product that the firm may choose to produce in the near future. A different machine, the 4C 273, could be substituted for the 3C 273 at a cost of an additional $100,000. The 4C 273 has all the same process characteristics (speed, availability, quality, etc.) as the 3C 273, but is also flexible enough to process any of the new products that might be introduced in the future. The problem is to choose between the 3C 273 and the 4C 273.

First, we articulate the possible scenarios. Either the firm will decide to produce a new product in the near future, or it will not. If it does not, then either the 3C 273 or the 4C 273 will suffice. If it does, then the 4C 273 will be required. If we install the 4C 273 now, it will cost an additional $100,000. But if we install the 3C 273 and the firm chooses to produce a new product, then we will need to replace it with the 4C 273. Suppose that this will cost $375,000 for the new machine plus $200,000 in lost revenue during the installation period and that the old machine can be sold for $50,000. Hence, the net cost incurred if we install the 3C 273 and then decide to produce a new product will be[6]

$$375 + 200 - 50 = \$525{,}000$$

Table 6.2 summarizes the costs of the four possible decision-scenario pairs.

[5] One can also perform scenario analysis without the use of probabilities for contingency planning. See, e.g., Wack (1985).

[6] Note that we are not considering the fact that this cost will actually be incurred in the future. To more accurately compare it to the $100,000 cost of installing the 4C 273 now, we should really multiply it by an appropriate discount factor to represent the time value of money. But for the sake of simplicity we will omit this.

TABLE 6.2 Costs of Decision-Scenario Pairs for Machine Installation Example

Scenario	Decision	
	3C 273	4C 273
Don't introduce new product	0	100
Introduce new product	525	100

Next we apply a decision criterion to the data. If we use the mini-max approach, we select the decision that minimizes the maximum cost. In this case, the maximum cost for the 3C 273 decision is \$525,000, while the maximum cost for the 4C 273 decision is \$100,000, so the mini-max criterion recommends installing the 4C 273.

However, the mini-max criterion may be overly conservative. If it is very unlikely that the firm will decide to produce a new product, then it may make more sense to install the 3C 273 and take our chances. To incorporate the likelihood of the different scenarios into our analysis, we might choose to use an expected value approach. Letting p represent the probability that the firm will introduce a new product, we see the expected cost of installing the 3C 273 is

$$0 \times (1 - p) + 525 \times p = \$525p$$

The expected cost of installing the 4C 273 is \$100,000 (since we incur this cost regardless of the scenario that occurs). Hence, the expected cost of the two scenarios is equal when

$$525p = 100$$
$$p = \frac{100}{525} = 0.19$$

Thus, if p is more than 0.19, the expected cost of installing the 4C 273 is smaller than that of installing the 3C 273. If p is less than 0.19, then the expected cost of the 3C 273 is smaller. To use the expected-value criterion to make a decision, therefore, we need only decide which region p lies in.

There are two important things to note about the above analysis:

1. Instead of guessing a value for p and using it to compute the expected costs of the two options, we worked backward to find the cutoff point for p that makes one option preferred to the other. The reason for this is that it is sometimes difficult to choose a value for something as intangible as the likelihood of a new product being introduced. Decision makers are generally more comfortable making the rough decision of whether a parameter lies in one range or another than trying to pin it down precisely. Since this decision does not necessarily require a highly accurate estimate of p to resolve, we set up the analysis so as not to ask for it.
2. We treated the need to meet demand for a new product as a constraint. In actuality, of course, this is a decision that will be addressed in the future. However, in order to consider the uncertainty concerning this decision when making the current equipment selection, we simply treat it as a scenario that may or may not unfold.

Modeling decision problems under uncertainty is a broad subject treated in the field of **decision analysis.** The books by Raiffa (1968), Brown (1974), and French (1986) provide good introductions to this vast discipline.

6.4 Conclusions

This chapter lays the foundation for our factory physics approach to developing the basics, intuition, and synthesis skills needed by the modern manufacturing manager. The main observations about the scientific, systems analysis, and modeling paradigm represented by this approach are as follows:

1. *Manufacturing management needs a science.* Although considerable folk wisdom exists about manufacturing, there is still only a small body of empirically verified, generalizable knowledge for supporting the design, control, and management of manufacturing facilities. If we are to move beyond fads and slogans, researchers and practitioners need to join forces to evolve a true science of manufacturing.
2. *The systems approach is a valuable manufacturing management tool.* By encouraging a holistic view of manufacturing enterprises and promoting a clear link between policies and objectives, systems analysis is the logical foundation for almost all manufacturing problem solving.
3. *Good descriptive models lead to good prescriptive models.* Trying to optimize a system we do not understand is futile. We need descriptive models to sharpen our intuition and focus our attention on the parameters with maximum leverage. Furthermore, policies based on accurate descriptions of system behavior are more likely to work with, rather than against, the system's natural tendencies. Such policies are apt to be more robust than those that try to force the system to behave unnaturally.
4. *Models are a necessary, but not complete, part of a manufacturing manager's skill set.* Because systems analysis demands that alternatives be evaluated with respect to objectives, some form of model is needed to make tradeoffs for virtually all manufacturing decision problems. Models can range from simple quantification procedures to sophisticated optimization and analysis methodologies. The *art* of modeling is in the selection of the proper model for a given situation and the coordination of the many models used to assist the decision-making process.
5. *Cost accounting typically provides poor models of manufacturing operations.* The purpose of accounting is to tell where the money went, not where to spend new money. Operations decisions require good characterization of *marginal,* not fully absorbed, costs and appropriate consideration of resource constraints.

From this base, we will now turn to developing specific models that describe the behavior of manufacturing systems.

Appendix 6A
Activity-Based Costing

There are four basic steps to ABC cost allocation (Baker 1994):

1. Determine the relevant activities.
2. Allocate overhead costs to these activities.

TABLE 6.3 Calculations for ABC Example

Category	Requisition	Engineering	Shipping	Sales	Sum
Total cost	$50,000	$65,000	$35,000	$100,000	$250,000
Units used, hot	600	2,500	6,000	400	—
Units used, mild	300	2,500	3,000	200	—
Unit cost	$55.56	$13.00	$3.89	$166.67	—
Total OH, hot	$33,336	$32,500	$23,333	$66,667	$155,836
Total OH, mild	$16,664	$32,500	$11,667	$33,333	$94,164

3. Select an allocation *base* appropriate for each activity.
4. Allocate cost to products using the base.

To illustrate the mechanics of ABC and contrast it with the traditional labor-hour approach, let us consider an example. Suppose a production facility makes two different products, hot and mild, and sells 6,000 units per month of hot and 3,000 units per month of mild. Total overhead costs are $250,000 per month. The plant runs 5,000 hours per month, of which 2,500 hours are devoted to hot and 2,500 to mild.

Traditional accounting would allocate the overhead equally among the two products because the number of labor hours devoted to each is the same. Hence, we would add $125,000 to the total cost of each product. This implies a unit charge of $125,000/6,000 = $20.83 for hot and $125,000/3,000 = $41.67 for mild. The unit cost of each product would then be computed by adding these unit overhead charges to the direct material and labor costs per unit. Notice that because fewer units of mild are produced, this procedure serves to inflate its unit cost more than that of hot.

Now reconsider the overhead allocation problem using the ABC approach. Suppose that we determine that the principal activities that account for the overhead (OH) cost are (1) requisition of material, (2) engineering support, (3) shipping, and (4) sales. Furthermore, suppose we can allocate the overhead cost to each activity as follows: $50,000 for requisition, $65,000 for engineering, $35,000 for shipping, and $100,000 for sales. The base (i.e., unit of measure) for requisition is the number of purchase orders (a total of 900); for engineering, the number of machine hours (5,000 hours); for shipping, the number of units shipped (9,000); and for sales, the number of sales calls made (600). Using these, a cost per base unit can be computed. The overhead allocation for a given product is then determined by the number of the base units used by that product times the cost per base unit. Finally, the unit overhead allocation is computed by dividing the total overhead allocation by the number of units. Table 6.3 summarizes the data and calculations for this example.

The unit overhead charge for hot is the sum of the "Total OH, Hot" entries divided by the number of units sold, that is, $155,833/6,000 = $25.97. Similarly, the unit overhead charge for mild is $94,164/3,000 = $31.38. Notice that while mild still receives a higher unit overhead charge than hot (due to its smaller volume), the difference is not as great as that resulting from the traditional labor-hour approach. The reason is that ABC recognizes that because of its higher volumes, greater effort, and hence cost, in the activities of requisition, engineering, and sales is devoted to hot. The net effect is to make mild look relatively more profitable than it would under traditional accounting methods.

Study Questions

1. What relevance does something as abstract as a "science of manufacturing" have to manufacturing management?

2. How many consistent observations does it take to prove a conjecture? How many inconsistent observations does it take to disprove a conjecture?
3. How can the concept of "conjectures and refutations" be used in a practical problem-solving environment?
4. List some dimensions along which manufacturing environments differ. How might these affect the "laws" governing their behavior? Do you think that a single science of manufacturing is possible for every manufacturing environment?
5. Indicate how each of the following might promote and impede the objective to maximize long-run profitability:
   a. Decrease average cycle time
   b. Decrease WIP
   c. Increase product diversity
   d. Improve product quality
   e. Improve machine reliability
   f. Reduce setup times
   g. Enhance worker cross-training
   h. Increase machine utilization
6. Why do you think that many writers in the JIT and TQM literature are loath to acknowledge the existence of tradeoffs? Do you think this has had positive, negative, or both impacts?
7. Why might the objective to maximize profits be difficult to use at the plant level? What advantages, or disadvantages, are there to using "minimize unit cost" instead?
8. We have suggested net profit and return on investment as firm-level measures. Do these capture the essence of a healthy firm? What characteristics are not adequately reflected in these measures? Can you suggest alternatives?
9. We have suggested
   - revenue (total quantity of good product sold per unit time)
   - operating expenses (operating budget of the plant)
   - assets (money tied up in plant, including inventories)

   as plant-level measures. How do these translate to the firm-level measures of total profit and ROI? Are there plant-level activities that are not reflected in the plant-level measures that affect the firm-level objectives? How might these be addressed?
10. Why does the distinction between objectives and constraints tend to blur in actual decision-making practice?
11. Give a specific example where "gaming behavior" (i.e., considering the other guy) is important in a manufacturing environment.

Problems

1. Consider a two-station production line in which no inventory is allowed (i.e., the stations are tightly coupled). Station 1 consists of a single machine that has potential daily production of one, two, three, four, five, or six units, each outcome being equally likely (i.e., potential production is determined by the roll of a single die). Station 2 consists of a single machine that has potential daily production of either three or four units, both of which are equally likely (i.e., it produces three units if a fair coin comes up heads and four units if it comes up tails).
   a. Compute the capacity of each station (i.e., in units per day). Is the line balanced (i.e., do both stations have the same capacity)?
   b. Compute the expected daily throughput of the line. Why does this differ from your answer to *a*?

c. Suppose a second identical machine is added to station 1. How much does this increase average throughput? What implications might this result have concerning the desirability of a balanced line?

d. Suppose a second identical machine is added to station 2 (but not station 1). How much does this increase average throughput? Is the impact the same from adding a machine at stations 1 and 2? Explain why or why not.

2. A manufacturer of vacuum cleaners produces three models of canister-style vacuum cleaners—the X-100, X-200, and X-300—on a production line with three stations—motor assembly, final assembly, and test. The line is highly automated and is run by three operators, one for each station. Data on production times, material cost, sales price, and bounds on demand are given in the following tables:

Product	Material Cost ($/Unit)	Price ($/Unit)	Minimum Demand (Units per Month)	Maximum Demand (Units per Month)
X-100	80	350	750	1500
X-200	150	500	0	500
X-300	160	620	0	300

Product	Motor Assembly (Minimum per Unit)	Final Assembly (Minimum per Unit)	Test (Minimum per Unit)
X-100	8	9	12
X-200	14	12	7
X-300	20	16	14

Labor costs $20 per hour (including benefits), and overhead for the line is $460,000 per month. The current production plan calls for production of X-100, X-200, and X-300 to be 625, 500, and 300 units per month, respectively.

a. What is the monthly profit that results from the current production plan (i.e., sales revenue minus labor cost minus material cost minus overhead)?

b. Estimate the profit per unit of each model, using direct labor hours to allocate the overhead cost per month. Which product appears most profitable? Is the current production plan consistent with these estimates? If not, propose an alternate production plan and compute its monthly profit.

c. Suppose overhead costs are categorized into plant and equipment, management, purchasing, and sales and shipping. Plant and equipment costs use square footage as a base, where floor space dedicated to specific products (e.g., product-specific inventory sites) is assigned to individual products, while shared space is allocated equally. Management costs use labor hours as the base (i.e., as used in part *b* for all overhead costs). Purchasing uses purchase orders, where parts ordered for a specific product are counted toward that product and common parts are divided equally. Sales and shipping costs are allocated according to customer orders, where, again, orders for unique products are counted by product and orders for multiple products are split equally. The breakdown of overhead costs and the allocation of base units by product are given as follows:

Category	Plant and Equipment	Management	Purchasing	Sales and Shipping
Total cost	$250,000	$100,000	$60,000	$50,000
Base	Square feet	Labor hours	Purchase orders	Customer orders
Total units used	120,000	49,625	2,000	150
Units X-100	40,000	18,125	500	100
Units X-200	50,000	16,500	600	30
Units X-300	30,000	15,000	900	20

i. Compute the unit profit for each product, using an ABC allocation of overhead cost based on the above breakdowns. Compare these with the estimates of unit profits obtained by using a labor-hours allocation scheme.
ii. Do the ABC unit profits suggest a different production plan? If not, suggest one and compute its monthly profit and compare to that of the current plan and that suggested by the labor-hours cost allocation.
iii. What is wrong with using the approach of computing unit profits for each product and then using them to produce as much as possible of the most profitable products?

C H A P T E R

7 BASIC FACTORY DYNAMICS

I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

Isaac Newton

7.1 Introduction

In the previous chapter, we argued that manufacturing management needs a science of manufacturing. In this chapter, we begin the process of fleshing out such a science by examining some basic behavior of production lines.

To motivate the measures and mechanics on which we will focus, we begin with a realistic example. HAL, a computer company, manufactures printed-circuit boards (PCBs), which are sold to other plants, where the boards are populated with components ("stuffed") and then sent to be used in the assembly of personal computers. The basic processes used to manufacture PCBs are as follows:

1. *Lamination.* Layers of copper and prepreg (woven fiberglass cloth impregnated with epoxy) are pressed together to form cores (blank boards).
2. *Machining.* The cores are trimmed to size.
3. *Circuitize.* Through a photographic exposing and subsequent etching process, circuitry is produced in the copper layers of the blanks, giving the cores "personality" (i.e., a unique product character). They are now called *panels*.
4. *Optical test and repair.* The circuitry is scanned optically for defects, which are repaired if not too severe.
5. *Drilling.* Holes are drilled in the panels to connect circuitry on different planes of multilayer boards. Note that multilayer panels must return to lamination after being circuitized to build up the layers. Single-layer panels go through lamination only once and do not require drilling or copper plating.
6. *Copper plate.* Multilayer panels are run through a copper plating bath, which deposits copper inside the drilled holes, thereby connecting the circuits on different planes.

7. *Procoat.* A protective plastic coating is applied to the panels.
8. *Sizing.* Panels are cut to final size. In most cases, multiple PCBs are manufactured on a single panel and are cut into individual boards at the sizing step. Depending on the size of the board, there could be as few as two boards made from a panel, or as many as 20.
9. *End-of-line test.* An electrical test of each board's functionality is performed.

HAL engineers monitor the capacity and performance of the PCB line. Their best estimates of capacity are summarized in Table 7.1, which gives the average process rate (number of panels per hour) and average process time (hours) at each station. (Note that because panels are often processed in batches and because many processes have parallel machines, the rate of a process is not the inverse of the time.) These values are averages, which account for the different types of PCBs manufactured by HAL and also the different routings (e.g., some panels may visit lamination twice). They also account for "detractors," such as machine failures, setup times, and operator efficiency. As such, the process rate gives an approximation of how many panels each process could produce per hour if it had unlimited inputs. The process time represents the average time a typical panel spends being worked on at a process, which includes time waiting for detractors but *does not* include time waiting in queue to be worked on.

The main performance measures emphasized by HAL are throughput (how many PCBs are produced), cycle time (the time it takes to produce a typical PCB), work in process (inventory in the line), and customer service (fraction of orders delivered to customers on time). Over the past several months, throughput has averaged about 1,100 panels per day, or about 45.8 panels per hour (HAL works a 24-hours a day). WIP in the line has averaged about 37,000 panels, and manufacturing cycle time has been roughly 34 days, or 816 hours. Customer service has averaged about 75 percent.

The question is, how is HAL doing?

We can answer part of this question immediately. HAL management is not happy with 75 percent customer service because it has a corporate goal of 90 percent. So this aspect of performance is not good. However, perhaps the reason for this is that overzealous salespersons are promising unrealistic due dates to customers. It may not be an indication of anything wrong with the line.

The other measures—throughput, WIP and cycle time—are more difficult to deal with. We need to establish some sort of baseline against which to compare them. One

TABLE 7.1 Capacity Data for HAL Printed-Circuit Board Line

Process	Rate (parts per hour)	Time (hour)
Lamination	191.5	1.2
Machining	186.2	5.9
Circuitize	150.5	6.9
Optical test/repair	157.8	5.6
Drilling	185.9	10.0
Copper plate	136.4	1.5
Procoat	146.2	2.2
Sizing	126.5	2.4
EOL test	169.5	1.8

way to do this would be to benchmark against a competitor's operation. But even if HAL could get such data, there would still be the question of how comparable they really were. After all, every facility is unique. To be better or worse than a different type of facility does not necessarily mean much. A better baseline would be one that compares actual performance against what is theoretically possible for this facility.

In this chapter, we examine the extremes of behavior that are possible for simple idealized production lines, and we use the resulting models to develop a scale with which to rate actual facilities. We will return to the HAL example and use this scale to evaluate the performance of its PCB line. But first we must define our terms.

7.2 Definitions and Parameters

The scientific method absolutely requires precise terminology. Unfortunately, use of manufacturing terms in industry and the OM literature is far from standardized. This can make it extremely difficult for managers and engineers from different companies (and even the same company) to communicate and learn from one another. What it means for us is that the best we can do is to define our terms carefully and warn the reader that other sources will use the same terms differently or use different terms in place of ours.

7.2.1 Definitions

In Part II, we focus on the behavior of production *lines*, because these are the links between individual processes and the overall plant. Therefore, the following terms are defined in a manner that allows us to describe lines with precision. Some of these terms also have broader meanings when applied to the plant, as we note in our definitions and will occasionally adopt in Part III. However, to develop sharp intuition about production lines, we will maintain these rather narrow definitions for the remainder of Part II.

Workstation: A **workstation** is a collection of one or more machines or manual stations that perform (essentially) identical functions. Examples include a turning station made up of several vertical lathes, an inspection station made up of several benches staffed by quality inspectors, and a burn-in station consisting of a single room where components are heated for testing purposes. In **process-oriented layouts,** workstations are physically organized according to the operations they perform (e.g., all grinding machines located in the grinding department). Alternatively, in **product-oriented layouts** they are organized in lines making specific products (e.g., a single grinding machine dedicated to an individual line). The terms **station, workcenter,** and **process center** are synonymous with *workstation*.

Part: A **part** is a piece of raw material, a component, a subassembly, or an assembly that is worked on at the workstations in a plant. **Raw material** refers to parts purchased from outside the plant (e.g., bar stock). **Components** are individual pieces that are assembled into more complex products (e.g., gears). **Subassemblies** are assembled units that are further assembled into more complex products (e.g., transmissions). **Assemblies** (or final assemblies) are fully assembled products or end items (e.g., automobiles). Note that one plant's final assemblies may be another's raw material. For instance, transmissions are the final assemblies of a transmission plant, but are raw materials or purchased components to the automotive assembly plant.

End item: A part that is sold directly to a customer, whether or not it is an assembly, is called an **end item.** The relationship between end items and their constituent parts

(raw materials, components, and subassemblies) is maintained in the **bill of material (BOM),** which Chapter 3 presented in detail.

Consumable: For the most part, **consumables** are materials such as bits, chemicals, gases, and lubricants that are used at workstations but do not become part of a product that is sold. More formally, we distinguish between parts and consumables in that parts are listed on the bill of material, while consumables are not. This means that some items that do become part of the product, such as solder, glue, and wire, can be considered either parts if they are recorded on the bill of material or consumables if they are not. Since different purchasing schemes are typically used for parts and consumables (e.g., parts might be ordered according to an MRP system, while consumables are purchased through a reorder point system), this choice may influence how such items are managed.

Routing: A **routing** describes the sequence of workstations passed through by a part. Routings begin at a raw material, component, or subassembly stock point and end at either an intermediate stock point or finished-goods inventory. For instance, a routing for gears may start at a stock point of raw bar stock; pass through cutting, hobbing, and deburring; and end at a stock point of finished gears. This stock of gears might in turn feed another routing that builds gear subassemblies. The bill of material and the associated routings contain the basic information needed to make an end item.

Order: A **customer order** is a request from a customer for a particular part number, in a particular quantity, to be delivered on a particular date. The paper or electronic **purchase order** sent by the customer might contain several customer orders. Henceforth, we will refer to a customer order as simply an **order.** Inside the plant, an order can also be an indication that certain inventories (e.g., safety stocks) need to be replenished. While timing may be more critical for orders originating with customers, both types of orders represent demand.

Job: A **job** refers to a set of physical materials that traverses a routing, along with the associated logical information (e.g., drawings, BOM). Although every job is triggered by either an actual customer order or the anticipation of a customer order (e.g., forecasted demand), there is frequently not a one-to-one correspondence between jobs and orders. This is because (1) jobs are measured in terms of specific parts (uniquely identified by a part number), not the collection of parts that may make up the assembly required to satisfy an order, and (2) the number of parts in a job may depend on manufacturing efficiency considerations (e.g., batch size considerations) and thus may not match the quantities ordered by customers.

Throughput (TH): The average output of a production process (machine, workstation, line, plant) per unit time (e.g., parts per hour) is defined as the system's **throughput,** or sometimes **throughput rate.** At the firm level, throughput is defined as the production per unit time that is *sold.* However, managers of production lines generally control what is made rather than what is sold. Therefore, for a plant, line, or workstation, we define throughput to be the average quantity of *good* (nondefective) parts (the manager does have control over quality) produced per unit time. In a line made up of workstations in tandem dedicated to a single family of products and where all products pass through each station exactly once, the throughput at every station will be the same (provided there is no yield loss). In a more complex plant, where workstations service multiple routings (e.g., a job shop), the throughput of an individual station will be the sum of the throughputs of the routings passing through it.

Capacity: An upper limit on the throughput of a production process is its **capacity.** In most cases, releasing work into the system at or above the capacity causes the system to become unstable (i.e., build up WIP without bound). Only very special systems can operate stably at capacity. Because this concept is subtle and important, we will inves-

tigate it more thoroughly later in this chapter, once we have introduced the appropriate notation and concepts.

Raw material inventory (RMI): As noted, the physical inputs at the start of a production process are typically called **raw material inventory.** This could represent bar stock that is cut up and then milled into gears, sheets of copper and fiberglass that are laminated together to make circuit boards, wood chips that become pulp and then paper stock, or rolls of sheet steel that are pressed into automobile fenders. Typically, the stock point at the beginning of a routing is termed raw material inventory even though the material may have already undergone some processing.

"Crib" and finished goods inventory (FGI): The stock point at the end of a routing is either a **crib inventory location** (i.e., an intermediate inventory location) or **finished goods inventory.** Crib inventories are used to gather different parts within the plant before further processing or assembly. For instance, a routing to produce gear assemblies may be fed by several crib inventories containing gears, housings, crankshafts, and so on. Finished goods inventory is where end items are held prior to shipping to the customer.

Work in process (WIP): The inventory between the start and end points of a product routing is called **work in process (WIP).** Since routings begin and end at stock points, WIP is all the product between, but not including, the ending stock points. Although in colloquial use WIP often includes crib inventories, we make a distinction between crib inventory and WIP to help clarify the discussion.

Inventory turns: A commonly used measure of the efficiency with which inventory is used is **inventory turns,** or the **turnover ratio,** which is defined as the ratio of throughput to average inventory. Typically, throughput is stated in yearly terms, so that this ratio represents the average number of times the inventory stock is replenished or turned over. Exactly which inventory is included depends on what is being measured. For instance, in a warehouse, all inventory is FGI, so turns are given by TH/FGI. In a plant, we generally consider both WIP (inventory still in the line) and FGI (inventory waiting to ship), so turns are given by TH/(WIP + FGI). In any case, it is essential to make sure that throughput and inventory are measured in the same units. Since inventory is usually measured in cost dollars (i.e., rather than price or sales dollars), throughput should also be measured in cost dollars.

Cycle time (CT): The **cycle time** (also called variously **average cycle time, flow time, throughput time,** and **sojourn time**) of a given routing is the average time from release of a job at the beginning of the routing until it reaches an inventory point at the end of the routing (i.e., the time the part spends as WIP).[1] Although this is a precise definition of cycle time, it is also narrow, allowing us to define cycle time only for individual routings. It is common for people to refer to the cycle time of a product that is composed of many complex subassemblies (e.g., automobiles). However, it is not clear exactly what is meant by this. When does the clock start for an automobile? When the chassis starts down the assembly line? When the engine begins production? Or, as in Henry Ford's terms, when the ore is mined from the ground? We will discuss measuring cycle time for such assembled parts later, but for now we restrict our definition to single routings.

Lead time, service level, and fill rate: The **lead time** of a given routing or line is the time allotted for production of a part on that routing or line. As such, it is a management constant.[2] In contrast, cycle times are generally random. Therefore, in a line functioning

[1] Cycle time also has another meaning in assembly lines as the time allotted for each station to complete its task. It can also refer to the processing time of an individual machine (e.g., the time for a punch press to cycle). We will avoid these other uses of the term *cycle time* to prevent confusion.

[2] Recall that the time phasing function of MRP is critically dependent on the choice of such lead times.

in a *make-to-order* environment (i.e., it produces parts to satisfy orders with specific due dates), an important measure of line performance is **service level,** which is defined as

$$\text{Service level} = P\{\text{cycle time} \leq \text{lead time}\}$$

Notice that this definition implies that for a given distribution of cycle time, service level can be influenced by manipulating lead time (i.e., the higher the lead time, the higher the service level).

If the line is functioning in a *make-to-stock* environment (i.e., it fills a buffer from which customers or other lines expect to be able to obtain parts without delay), then a different performance measure may be more appropriate than service level. A logical choice is **fill rate,** which is defined as the fraction of orders that are filled from stock and was discussed in Chapter 2. Since fill rate and many other performance measures are often referred to as "service levels," the reader is cautioned to look for a precise definition whenever this term is encountered. We will consistently use the former definition of service level throughout Part II, but will return to the fill rate measure in Chapter 17.

Utilization: The **utilization** of a workstation is the fraction of time it is not idle for lack of parts. This includes the fraction of time the workstation is working on parts or has parts waiting and is unable to work on them due to a machine failure, setup, or other detractor. We can compute utilization as

$$\text{Utilization} = \frac{\text{Arrival rate}}{\text{Effective production rate}}$$

where the effective production rate is defined as the maximum average rate at which the workstation can process parts, considering the effects of failures, setups, and all other detractors that are relevant over the planning period of interest.

7.2.2 Parameters

Parameters are numerical descriptors of manufacturing processes and therefore vary in value from plant to plant. Two key parameters for describing an individual line (routing) are the bottleneck rate and the raw process time. We define these below, along with a third parameter, the *critical* WIP level, that can be computed from them.

Bottleneck rate (r_b): The **bottleneck rate** of the line, r_b, is the rate (parts per unit time or jobs per unit time) of the workstation having the highest long-term utilization. By long term we mean that outages due to machine failures, operator breaks, quality problems, etc., are averaged out over the time horizon under consideration. This implies that the proper treatment of outages will differ depending on the planning frequency. For example, for daily replanning, outages typically experienced during a day should be included; but unplanned long outages, such as those resulting from a major upset, should not. In contrast, for planning over a year-long horizon, time lost to major upsets should be included, if such occurrences are not unlikely over the course of a year.

In lines consisting of a single routing in which each station is visited exactly once and there is no yield loss, the arrival rate to every workstation is the same. Hence, the workstation with the highest utilization will be that with the least long-term capacity (i.e., slowest effective rate). However, in lines with more complicated routings or yield loss, the bottleneck may not be at the slowest workstation. A faster workstation that experiences a higher arrival rate may have higher utilization. For this reason, it is important to define the bottleneck in terms of utilization as we have done here.

Raw process time (T_0): The **raw process time** of the line, T_0, is the sum of the *long-term average* process times of each workstation in the line. Alternatively, we can

define raw process time as the average time it takes a single job to traverse the empty line (i.e., so it does not have to wait behind other jobs). Again, we must be concerned about the length of the planning horizon when deciding what to include in the "average" process times. Over the long term, T_0 should include infrequent random and planned outages, while over a shorter term it should include only the more frequent delays.

Critical WIP (W_0): The **critical WIP** of the line, W_0, is the WIP level for which a line with given values of r_b and T_0 but having no variability achieves maximum throughput (that is, r_b) with minimum cycle time (that is, T_0). We show below that critical WIP is defined by the bottleneck rate and raw process time by the following relationship:

$$W_0 = r_b T_0$$

7.2.3 Examples

We now illustrate these definitions by means of two simple examples.

Penny Fab One. Penny Fab One consists of a simple production line that makes giant one-cent pieces used exclusively in Fourth of July parades. The line consists of four machines in sequence that use well-known, stable processes. The first machine is a punch press that cuts penny blanks, the second stamps Lincoln's face on one side and the Memorial on the back, the third places a rim on the penny, and the fourth cleans away any burrs. Each machine takes exactly two hours to perform its operation. (We will relax this requirement that process times be deterministic later.) After each penny is processed, it is moved immediately to the next machine. The line runs 24 hours per day, with breaks, lunches, etc., covered by spare operators. For our purposes, the market for giant pennies can be assumed to be unlimited, so that all product made is sold; thus, more throughput is unambiguously better for this system.

Since this is a tandem line with no yield loss, the bottleneck is defined as the slowest workstation. However, the *capacity* of each machine is the same and equals one penny every two hours, or one-half part per hour. Hence, any of the four machines can be regarded as the bottleneck and

$$r_b = 0.5 \text{ penny per hour}$$

or 12 pennies per day. Such a line is said to be **balanced,** since all stations have equal capacity.

Next, note that the raw process time is simply the sum of the processing times at the four stations, so

$$T_0 = 8 \text{ hours}$$

The critical WIP level is given by

$$W_0 = r_b T_0 = 0.5 \times 8 = 4 \text{ pennies}$$

We will illustrate that this is indeed the level of WIP that causes the line to achieve throughput of $r_b = 0.5$ penny per hour and cycle time of $T_0 = 8$ hours. Notice that W_0 is equal to the number of machines in the line. This is *always* the case for balanced lines, since having one job per machine is just enough to keep all machines busy at all times. However, as we will see, it is not true for unbalanced lines.

Penny Fab Two. Now consider a somewhat more complex Penny Fab Two, which represents an unbalanced line with multimachine stations. Penny Fab Two still produces giant pennies in four steps: punching, stamping, rimming, and deburring; but the

workstations now have different numbers of machines and processing times, as shown in Table 7.2.

The presence of multimachine stations complicates the capacity calculations somewhat. For a single machine, the capacity is simply the reciprocal of the process time (e.g., if it takes one-half hour to do one job, the machine can do two jobs per hour). The capacity of a station consisting of several identical machines in parallel must be calculated as the individual machine capacity times the number of machines. For example, in Penny Fab Two, the capacity per machine at station 3 is

$$\frac{1}{10} \text{ penny per hour}$$

so the capacity of the station is

$$6 \times \frac{1}{10} = 0.6 \text{ penny per hour}$$

Notice that the station capacity can be computed directly by dividing the number of machines by the process time. This is done for each station in Table 7.2.

The capacity of the line with multimachine stations is still defined by the rate of the bottleneck, or slowest station in the line. In Penny Fab Two, the bottleneck is station 2, so

$$r_b = 0.4 \text{ penny per hour}$$

Notice that the bottleneck is neither the station that contains the slowest machines (station 3) nor the one with the fewest machines (station 1).

The raw process time of the line is still the sum of the process times. Notice that adding machines at a station does not decrease T_0, since a penny can be worked on by only one machine at a time. Hence, the raw process time for Penny Fab Two is

$$T_0 = 20 \text{ hours}$$

Regardless of whether the line has single- or multiple machine stations, the critical WIP level is always defined as

$$W_0 = r_b T_0 = 0.4 \times 20 = 8 \text{ pennies}$$

In Penny Fab Two, as in Penny Fab One, W_0 is a whole number. This, of course, need not be the case. If W_0 comes out to a fraction, it means that there is no constant WIP level that will achieve throughput of exactly r_b jobs per hour and cycle time of T_0 hours. Furthermore, notice that the critical WIP level in Penny Fab Two (eight pennies) is less than the number of machines (11). This is because the system is not balanced (i.e., stations have different amounts of capacity), and therefore some stations will not be fully utilized.

TABLE 7.2 Penny Fab Two: An Unbalanced Line

Station Number	Number of Machines	Process Time (hour)	Station Capacity (Jobs per Hour)
1	1	2	0.50
2	2	5	0.40
3	6	10	0.60
4	2	3	0.67

7.3 Simple Relationships

Now, in the pursuit of a science of manufacturing, we ask the fundamental question, What are the relationships among WIP, throughput, and cycle time in a single production line? Of course, the answer will depend on the assumptions we make about the line. In this section, we will give a precise (i.e., quantitative) description of the range of possible behavior. This will serve to sharpen our intuition about how lines perform and will give us a scale on which to rate (benchmark) actual systems.

A problem with characterizing the relationship between measures such as WIP and throughput is that in real systems they tend to vary simultaneously. For instance, in an MRP system, the line may be flooded with work one month (due to a heavy master production schedule) and very lightly loaded the next. Hence, both WIP and throughput are apt to be high during the first month and low during the second. For clarity of presentation, we will eliminate this problem by controlling the WIP level in the line so as to hold it constant over time. For instance, in the Penny Fabs, we will start the lines with a specified number of pennies (jobs) and then release a new penny blank into the line each time a finished penny exits the line.[3]

7.3.1 Best-Case Performance

To analyze and understand the behavior of a line under the best possible circumstances, namely, when process times are absolutely regular, we will *simulate* Penny Fab One. This is easily done by using a piece of paper and several pennies, as shown in Figure 7.1.

We begin by simulating the system when only one job is allowed in the line. The first penny spends two hours successively at stations 1, 2, 3, and 4, for a total cycle time of eight hours. Then a second penny is released into the line, and the same sequence is repeated.

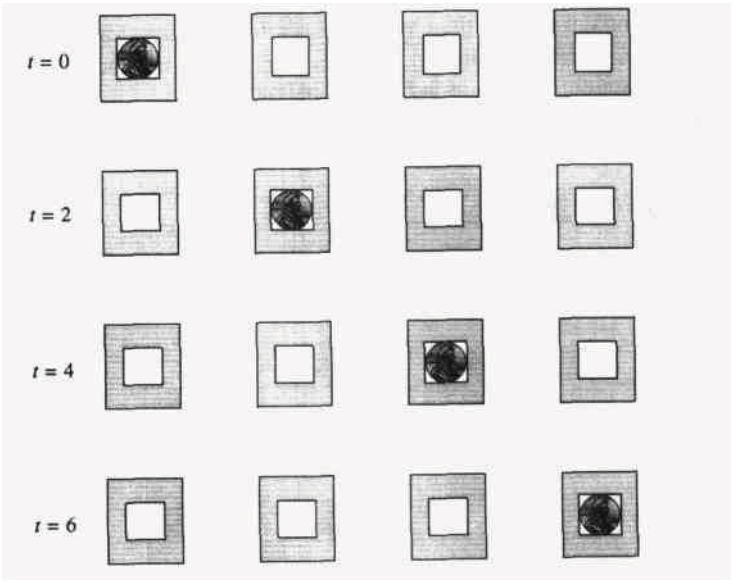


FIGURE 7.1
Penny Fab One with WIP = 1

[3] We say that such a line is operating under a CONWIP (*con*stant *WIP*) protocol, which is treated more thoroughly in Chapters 10 and 14

Since this results in one penny coming out of the line every eight hours, the throughput is one-eighth penny per hour. Notice that the cycle time is equal to the raw process time $T_0 = 8$, while the throughput is one-fourth of the bottleneck rate $r_b = 0.5$.

Now we add a second penny to the line (starting both at the front of the line). After two hours, the first penny completes processing at station 1 and starts on station 2. Simultaneously, the second penny starts processing on station 1. Thereafter, the second penny will follow the first, switching stations every two hours, as shown in Figure 7.2. After the initial wait experienced by the second penny, it never waits again. Hence, once the system is running in steady state, every penny released into the line still has a cycle time of exactly eight hours. Moreover, since two pennies exit the line every eight hours, the throughput increases to two-eighths penny per hour, double that when the WIP level was 1 and 50 percent of line capacity ($r_b = 0.5$).

We add a third penny. Again, after an initial transient period in which pennies wait at the first station, there is no waiting, as shown in Figure 7.3. Hence, cycle time stays at 8 h, while throughput increases to three-eighths part per hour, or 75 percent of r_b.

When we add a fourth penny, we see that all the stations stay busy all the time once steady state has been reached. Because there is no waiting at the stations, cycle time is still $T_0 = 8$ h. Since the last station is busy all the time, it outputs a penny every other hour, so throughput becomes one-half penny per hour, which equals the line capacity r_b. This very special behavior, in which cycle time T_0 (its minimum value) and throughput r_b (its maximum value) are only achieved when the WIP level is set at the critical WIP level, which we recall for Penny Fab One is

$$W_0 = r_b T_0 = 0.5 \times 8 = 4 \text{ pennies}$$

Now we add a fifth penny to the line. Because there are only four machines, a penny will wait at the first station, even after the system has settled into steady state. Since we measure cycle time as the time from when a job is released (the time it enters the queue at the first station) to when it exits the line, it now becomes 10 hours, due to the extra two hours of waiting time in front of station 1. Hence, for the first time, cycle time becomes larger than its minimal value $T_0 = 8$. However, since all stations are always busy, the throughput remains at $r_b = 0.5$ penny per hour.

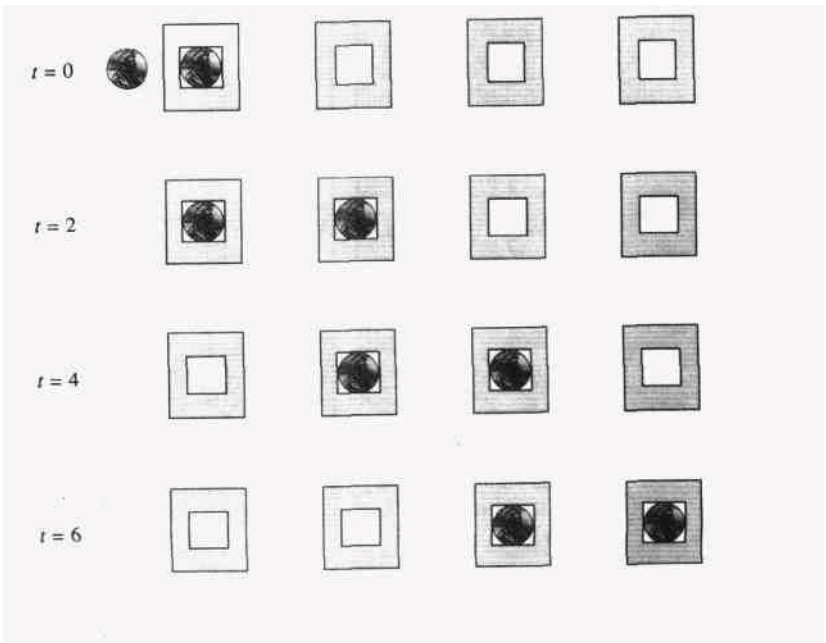


FIGURE 7.2
Penny Fab One with WIP = 2

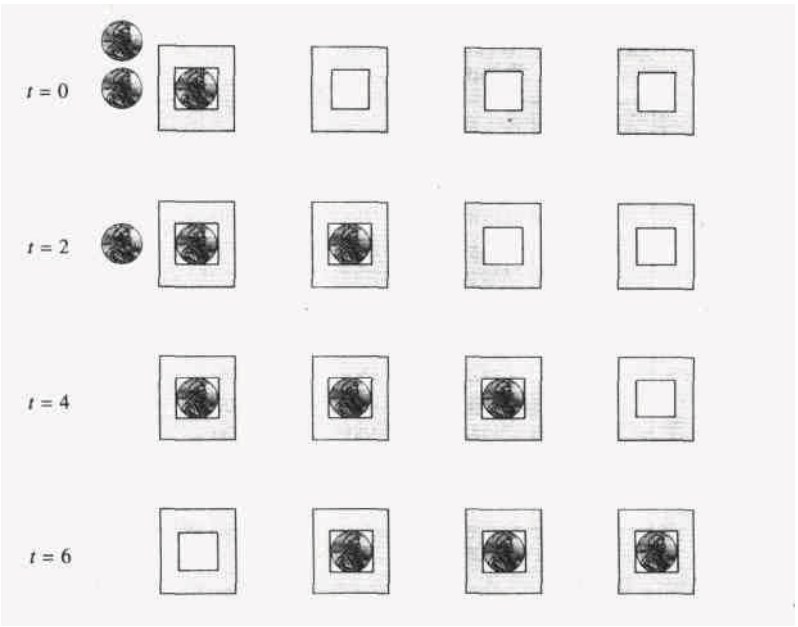


FIGURE 7.3
Penny Fab One with WIP = 3

Finally, consider what happens when we allow 10 pennies in the line. In steady state, a queue of six pennies persists in front of the first station, meaning that an individual penny spends 12 hours from the time it is released to the line until it begins processing at station 1. Hence, the cycle time is 20 hours. As before, all machines remain busy all the time, so throughput is still $r_b = 0.5$ penny per hour. It should be clear at this point that each penny we add increases cycle time by two hours with no increase in throughput.

We summarize the behavior of Penny Fab One with no variability for various WIP levels in Table 7.3, and we present the results graphically in Figure 7.4. From a performance standpoint, it is clear that Penny Fab One runs best when there are four pennies in WIP. Only this WIP level results in minimum cycle time T_0 and maximum throughput r_b—any less and we lose throughput with no decrease in cycle time; any more and we increase cycle time with no increase in throughput. This special WIP level is the critical WIP (W_0) that was defined previously.

In this particular example, the critical WIP is equal to the number of machines. This is always the case when the line consists of stations with equal capacity (i.e., a balanced line). For unbalanced lines, W_0 will be less than the number of machines, but still has the property of being the WIP level that achieves maximum throughput with minimum cycle time, and is still defined by $W_0 = r_b T_0$.

It is important to note that while the critical WIP is optimal in the case with zero variability, it will *not* be optimal in other cases. Indeed, the concept of an optimal WIP level is not even well defined in the presence of variability because, in general, increasing WIP will increase both throughput (good) and cycle time (bad).

Little's Law. Close examination of Table 7.3 reveals an interesting, and fundamental, relationship among WIP, cycle time, and throughput. At every WIP level, WIP is equal to the product of throughput and cycle time. This relation is known as *Little's law* (named for John D. C. Little, who provided the mathematical proof) and represents our first *factory physics* relationship:

Law (Little's Law):

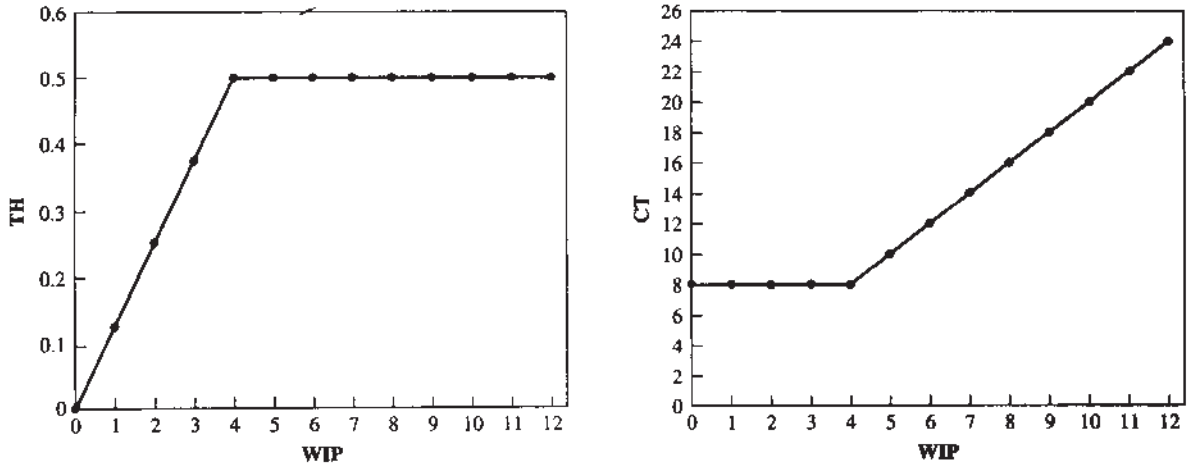
$$\text{WIP} = \text{TH} \times \text{CT}$$

TABLE 7.3 WIP, Cycle Time, and Throughput of Penny Fab One

WIP	CT	% T_0	TH	% r_b
1	8	100	0.125	25
2	8	100	0.250	50
3	8	100	0.375	75
4	8	100	0.500	100
5	10	125	0.500	100
6	12	150	0.500	100
7	14	175	0.500	100
8	16	200	0.500	100
9	18	225	0.500	100
10	20	250	0.500	100

FIGURE 7.4

Cycle time and throughput versus WIP for Penny Fab One



It turns out that Little's law holds for *all* production lines, not just those with zero variability. As we discussed in Chapter 6, Little's law is not a *law* at all but a *tautology*. For special cases (e.g., the case of observing the system for a time that goes to infinity), the relationship can be proved mathematically. However, it does not entirely hold in the less-than-infinite case (which, of course, involves all real cases) except for other special cases. Nonetheless, we will use it as a conjecture about the nature of manufacturing systems and use it as an approximation when it is not exact.

Little's law is quite useful in that it can be applied to a single station, a line, or an entire plant. As long as the three quantities are measured in consistent units, the above relationship will hold over the long term. This makes it immensely applicable to practical situations. Some straightforward uses of Little's law include these:

1. *Queue length calculations.* Since Little's law applies to individual stations, we can use it to calculate the expected queue length and utilization (fraction of time busy) at each station in a line. For instance, consider Penny Fab Two, which was summarized in Table 7.2, and suppose it is running at the bottleneck rate (that is, 0.4 job per hour). From Little's law, the expected WIP at the first station will be

$$\text{WIP} = \text{TH} \times \text{CT} = 0.4 \text{ job per hour} \times 2 \text{ hour} = 0.8 \text{ job}$$

Since there is only one machine at station 1, this means that it will be utilized 80 percent of the time. Similarly, at station 3, Little's law predicts an average WIP of four jobs. Since there are six machines, the average utilization will be $4/6 = 66.7$ percent. Notice that this is equal to the ratio of the rate of the bottleneck to the rate of station 3 (that is, 0.4/0.6), as we would expect.

2. *Cycle time reduction.* Since Little's law can be written as

$$\text{CT} = \frac{\text{WIP}}{\text{TH}}$$

it is clear that reducing cycle time implies reducing WIP, provided throughput remains constant. Hence, large queues are an indication of opportunities for reducing cycle time, as well as WIP. We will discuss specific measures for WIP and cycle time reduction in Chapter 17.

3. *Measure of cycle time.* Measuring cycle time directly can sometimes be difficult, since it entails registering the entry and exit times of each part in the system. Since throughput and WIP are routinely tracked, it might be easier to use the ratio WIP/TH as a perfectly reasonable indirect measure of cycle time.

4. *Planned inventory.* In many systems, jobs are scheduled to finish ahead of their due dates in order to ensure a high level of customer service. Because, in our era of inventory consciousness, customers often refuse to accept early deliveries, this type of "safety lead time" causes jobs to wait in finished goods inventory prior to shipping. If the **planned inventory** time is n days, then according to Little's law, the amount of inventory in FGI will be given by nTH (where TH is measured in units per day).

5. *Inventory turns.* Recall that inventory turns are given by the ratio of throughput to average inventory. If we have a plant in which all inventory is WIP (i.e., product is shipped directly from the line so there is no finished goods inventory), then turns are given by TH/WIP, which by Little's law is simply 1/CT. If we include finished goods, then turns are TH/(WIP + FGI). But Little's law still applies, so this ratio represents the inverse of the total average time for a job to traverse the line plus the finished goods crib. Hence, intuitively, inventory turns are one divided by the average residence time of inventory in the system.

In a sense, Little's law is the "$F = ma$" of factory physics. It is a broadly applicable equation that relates three fundamental quantities. At the same time, Little's law can be viewed as a truism about units. It merely indicates the obvious fact that we can measure WIP level in a station, line, or system in units of jobs or time. For instance, a line that produces 100 crankcases per day and has a WIP level of 500 crankcases has five days of WIP in it. Little's law is a statement that this unit's conversion is valid for average WIP, cycle time, and throughput, or

$$\text{CT} = \frac{\text{WIP}}{\text{TH}}$$

or

$$5 \text{ days} = \frac{500 \text{ crankcases}}{100 \text{ crankcases per day}}$$

We can now generalize the results shown in Table 7.3 and Figure 7.4 to achieve our original objective of giving a precise summary of the relationship between WIP and throughput for a "best-case" (i.e., zero-variability) line. We can then apply Little's law to extend this to describe the relationship between WIP and cycle time. Since these relationships were derived for perfect lines with no variability, the following expressions indicate the *maximum throughput* and *minimum cycle time* for a given WIP level for any system having parameters r_b and T_0. The resulting equations are our next *Factory Physics* law.

Law (Best-Case Performance): *The minimum cycle time for a given WIP level w is given by*

$$\mathrm{CT}_{\mathrm{best}} = \begin{cases} T_0 & \text{if } w \leq W_0 \\ \dfrac{w}{r_b} & \text{otherwise} \end{cases}$$

The maximum throughput for a given WIP level w is given by

$$\mathrm{TH}_{\mathrm{best}} = \begin{cases} \dfrac{w}{T_0} & \text{if } w \leq W_0 \\ r_b & \text{otherwise} \end{cases}$$

One conclusion we can draw from this is that, contrary to the popular slogan, zero inventory is *not* a realistic goal. Even under perfect deterministic conditions, zero inventory yields zero throughput and therefore zero revenue. A more realistic "ideal" WIP is the critical WIP W_0.

Penny Fab One represents an ideal (zero-variability) situation, in which it is optimal to maintain a WIP level equal to the number of machines. Of course, in the real world there are not many factories that run with such low WIP levels. Indeed, in many production lines the WIP-to-machines ratio is closer to 20:1 (Bradt 1983). If this ratio were to hold for Penny Fab One, the cycle time would be almost seven days with 80 jobs in WIP. Obviously, this is much worse than a cycle time of eight hours at a WIP level of four jobs (i.e., the "optimal" level). Why, then, do actual plants operate so far from the ideal of the critical WIP level?

Unfortunately, Little's law offers little help. Since TH = WIP/CT, we can have the same throughput with large WIP levels and long cycle times, or with low WIP levels and short cycle times. The problem is that Little's law is only one relation among three quantities. We need a second relation if we are to uniquely determine two quantities, given the third (e.g., predict both WIP and cycle time from throughput). Sadly, there is no universally applicable second relationship among WIP, cycle time, and throughput. The best we can do is to characterize the behavior of a line under specific assumptions. In addition to the best case, which we considered above, we will treat two other scenarios, which we term the **worst case** and the **practical worst case.**

7.3.2 Worst-Case Performance

Instead of imagining the best possible behavior of a line, we consider the worst. Specifically, we seek the *maximum cycle time* and *minimum throughput* possible for a line with bottleneck rate r_b and raw process time T_0. This will enable us to bracket the behavior and gauge the performance of real lines. If a line is closer to the worst case than to the best case, then there are some real problems (or opportunities, depending on your perspective).

To facilitate our discussion of the worst case, recall that we are assuming a constant amount of work is maintained in the line at all times. Whenever a job finishes, another is started. One way that this could be achieved in practice would be to transport jobs through the line on *pallets*. Whenever a job is finished, it is removed from its pallet and the pallet immediately returns to the front of the line to carry a new job. The WIP level, therefore, is equal to the (fixed) number of pallets.

Now, imagine yourself sitting on a pallet riding around and around a best-case line with WIP equal to the critical WIP (e.g., Penny Fab One with four jobs). Each time you arrive at a station, a machine is available to begin work on the job immediately. It is precisely because there is no waiting (queueing) that this line achieves the minimum possible cycle time of T_0.

To get the longest possible cycle times for this system, we must somehow increase the waiting time without changing the *average* processing times (otherwise we would change r_b and T_0). The very worst we could possibly make waiting time would be that every time our pallet reached a station, we found ourselves waiting behind *every* other job in the line. How could this possibly occur?

Consider the following. Suppose that you are riding on pallet number 4 in a modified Penny Fab One with four pallets. However, instead of all jobs requiring exactly two hours at each station, suppose that jobs on pallet 1 require eight hours, while jobs on pallets 2, 3, and 4 require zero hours. The average processing time at each station is

$$\frac{8+0+0+0}{4} = 2 \text{ hours}$$

as before, and hence we still have $r_b = 0.5$ job per hour and $T_0 = 8$ hours. However, every time your pallet reaches a station, you find pallets 1, 2, and 3 ahead of you (see Figure 7.5). The slow job on pallet 1 causes all the other jobs to pile up behind it at all times. This is the absolute maximum amount of waiting time it is possible to introduce, and hence this represents the worst case.

The cycle time for this system is

$$8 + 8 + 8 + 8 = 32 \text{ hours}$$

or $4T_0$, and since four jobs are output each time pallet 1 finishes on station 4, the throughput is

$$\frac{4}{32} = \frac{1}{8} \text{ job per hour}$$

or $1/T_0$ jobs per hour. Notice that the product of throughput and cycle time is $\frac{1}{8} \times 32 = 4$, which is the WIP level, so, as always, Little's law holds.

Let us summarize these results for a general line as our next factory physics law.

Law (Worst-Case Performance): *The worst-case cycle time for a given WIP level w is given by*

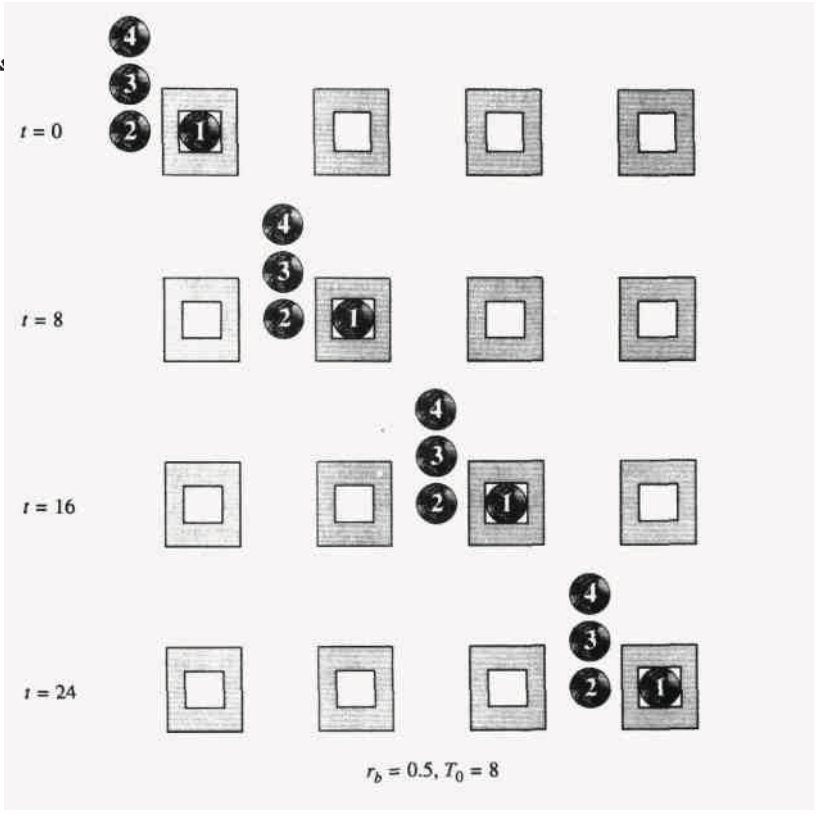
$$\text{CT}_{\text{worst}} = wT_0$$

The worst-case throughput for a given WIP level w is given by

$$\text{TH}_{\text{worst}} = \frac{1}{T_0}$$

It is interesting to note that both the best-case and worst-case performances occur in systems with no randomness. There is *variability* in the worst-case system, since jobs have different process times; but there is no *randomness*, since all process times are completely predictable. The literature on quality management stresses the need

FIGURE 7.5
Evolution of worst-ca line



for variability reduction, but sometimes implies that variability and randomness are synonymous. The above *Factory Physics* results show that this is not the case; variability can be the result of randomness or *bad control* (or both). We will examine this distinction in greater depth after we have developed the tools for treating variability in Chapters 8 and 9.

Finally, the reader may be justifiably skeptical about the realism of the worst case. After all, we arrived at this case by forcing the maximum amount of waiting time (in order to make cycle times as long as possible) by making the processing times as variable as possible. To do this, we assumed jobs on one of the pallets had long processing times, while all the others had zero processing times. Surely this could never happen in real life.

But it can and (at least to some extent) does happen. To see how, suppose that the four pallets used to carry jobs in Penny Fab One (when WIP equals four jobs) are themselves moved between stations with a forklift. Further, suppose that because the forklift has other obligations, it cannot afford to make the number of trips necessary to move each pallet individually. Instead, it waits until all four jobs are finished on a station and then moves them as a group to the next station. Similarly, it waits until all four pallets are empty at the end of the line to bring them back to the front to receive new jobs. Assuming that processing times of each job at each station are two hours (as in the original Penny Fab One), and that move times on the forklift are sufficiently short as to be reasonably treated as zero, the progress of the system will be *exactly* the same as that shown in Figure 7.5. Hence, worst-case behavior can result from **batch moves.**

Of course, it is rare to find real plants in which batch moves are so extreme as to cause every job in the line to travel together. More commonly, the WIP in a line will

be transported in several batches, possibly of varying size. While this kind of more modest batching will not produce worst-case behavior, it is one factor that can push the performance of a line closer to that of the worst case than the best case. Consequently, batching is a genuine problem (opportunity) in many production systems.

7.3.3 Practical Worst-Case Performance

Virtually no real-world line behaves literally according to either the best case or the worst case. Therefore, to better understand the behavior between these two extreme cases, it is instructive to consider an intermediate case. We do this by means of a case that, unlike the previous two, involves randomness. In fact, in a sense, it represents the "maximum randomness" case. We term this the **practical worst case** to express our belief that virtually any system with worse behavior is a target for improvement.

To describe the practical worst case and show why it can be regarded as the maximum randomness case, we must first define the concept of a system *state*. The state of the system is a complete description of the jobs at all the stations: how many there are and how long they have been in process. Under special conditions, which we assume here and describe below, the only information needed is the number of jobs at each station. Hence, we can give a concise summary of a state by using a vector with as many elements as there are stations in the line.

For instance, in a line with four stations and three jobs, the vector (3, 0, 0, 0) represents the state in which all three jobs are at the first station, while the vector (1, 1, 1, 0) represents the state in which there is one job each at stations 1, 2, and 3. There are 20 possible states for a system consisting of four machines and three jobs, which are enumerated in Table 7.4.

Depending on the specific assumptions about the line, not all states will necessarily occur. For instance, if all processing times in the four-station, three-job system are one hour and it behaves according to the best case, then only four states—(1, 1, 1, 0), (0, 1, 1, 1), (1, 0, 1, 1), and (1, 1, 0, 1)—will be repeated as illustrated in Figure 7.6. Similarly, if it behaves according to the worst case, then four different states—(3, 0, 0, 0), (0, 3, 0, 0), (0, 0, 3, 0), and (0, 0, 0, 3)—will be repeated, as illustrated in Figure 7.7. Because both of these systems have no randomness, other states are never reached.

TABLE 7.4 Possible States for a System with Four Machines and Three Jobs

State	Vector	State	Vector
1	(3, 0, 0, 0)	11	(1, 0, 2, 0)
2	(0, 3, 0, 0)	12	(0, 1, 2, 0)
3	(0, 0, 3, 0)	13	(0, 0, 2, 1)
4	(0, 0, 0, 3)	14	(1, 0, 0, 2)
5	(2, 1, 0, 0)	15	(0, 1, 0, 2)
6	(2, 0, 1, 0)	16	(0, 0, 1, 2)
7	(2, 0, 0, 1)	17	(1, 1, 1, 0)
8	(1, 2, 0, 0)	18	(1, 1, 0, 1)
9	(0, 2, 1, 0)	19	(1, 0, 1, 1)
10	(0, 2, 0, 1)	20	(0, 1, 1, 1)

FIGURE 7.6
States in best-case, four-machine, three-j line

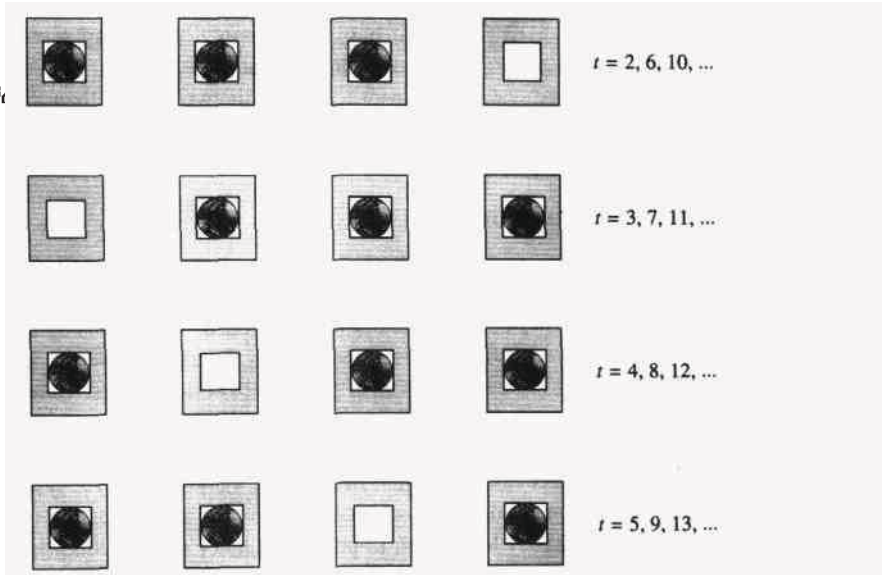
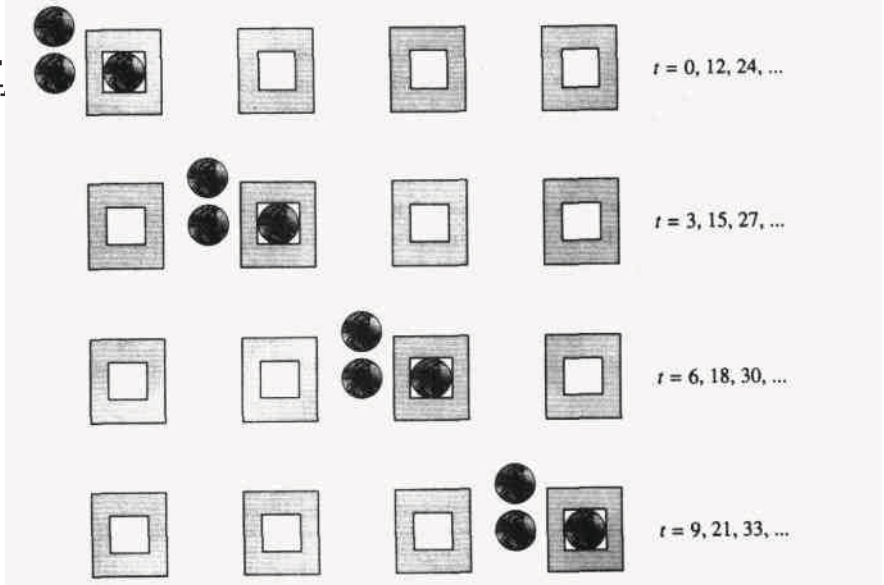


FIGURE 7.7
States in worst-case, four-machine, three- line



When randomness is introduced into a line, more states become possible. For instance, suppose the processing times are deterministic, but every once in a while a machine may break down for several hours. Then most of the time we will observe "spread out" states, like those in Figure 7.6, but occasionally we will see "clumped up" states, like those in Figure 7.7. If there is only a little randomness (e.g., machine failures are very rare), then the frequency of the spread-out states will be very high, whereas if there is a lot of randomness (e.g., machines are failing right and left), then *all* the states shown in Table 7.4 may occur quite often. Hence, we define the ***maximum randomness*** scenario to be that which causes every possible state to occur with equal frequency.

In order for all states to be equally likely, three special conditions are required:

1. The line must be balanced (i.e., all stations must have the same average process times).
2. All stations must consist of single machines. (This assumption also allows us to avoid the complexities of parallel processing and jobs passing one another.)
3. Process times must be random and occur according to a specific probability distribution known as the **exponential distribution.** The exponential is the *only* continuous distribution that has a special property known as the **memoryless property** (see Appendix 2A). What this means is that if the processing time on a machine is exponentially distributed, then knowledge of how long a part has been in process offers no information about when it will be finished. For instance, if process times on a machine are exponential with mean one hour and the current job has been in process for five seconds, then the expected remaining process time is one hour. If the current job has been in process for one hour, the remaining process time is one hour. If the current job has been in process for 942 hours, the expected remaining process time is one hour.[4] It is as if the machine forgets its past work when predicting the future—hence the term *memoryless*. Thus, if process times are exponentially distributed, there is no need to know about how long a job has been in process to completely define the system state.

To understand how the practical worst case (PWC) works, return to the thought experiment in which you envisioned yourself riding around on a pallet that cycles through the line again and again. Suppose there are N (single machine) stations, each with average processing times of t, and a constant level of w jobs in the line. Thus, the raw process time is $T_0 = Nt$, and the bottleneck rate is $r_b = 1/t$ for this line.

Since the above three conditions guarantee that all states are equally likely, then, from your vantage point on a pallet, you would expect to see on average the $w - 1$ other jobs equally distributed among the N stations each time you arrive at a station. So the expected number of jobs ahead of you upon arrival is $(w - 1)/N$. Since the average time you spend at the station will be the time for the other jobs to complete processing plus the time for your job to be processed, we can write

$$\begin{aligned}\text{Average time at a station} &= \text{Time for other jobs} + \text{Time for your job}\\ &= \frac{w-1}{N}t + t\\ &= \left(1 + \frac{w-1}{N}\right)t\end{aligned}$$

By assuming that the $(w-1)/N$ jobs ahead of you require an average of $[(w-1)/N]t$ time to complete, we are ignoring the fact that the job in process at the station was partially finished when you arrived. It is the memoryless property of the exponential distribution that enables us to do this.

Finally, since all stations are assumed identical, we can compute the average cycle time by simply multiplying the average time at each station by the number of stations N, to get

[4] Although it may be a stretch to imagine processing times behaving in this way, there certainly seem to be examples of this type of behavior in daily life, for instance, times until departure of delayed flights, times until the arrival of trains on certain railways, times until some contractors finish home improvement jobs, etc.

$$\begin{aligned} \mathrm{CT} &= N\left(1 + \frac{w-1}{N}\right)t \\ &= Nt + (w-1)t \\ &= T_0 + \frac{w-1}{r_b} \end{aligned}$$

To get the corresponding throughput, we simply apply Little's law:

$$\begin{aligned} \mathrm{TH} &= \frac{\mathrm{WIP}}{\mathrm{CT}} \\ &= \frac{w}{T_0 + (w-1)/r_b} \\ &= \frac{w}{W_0/r_b + (w-1)/r_b} \\ &= \frac{w}{W_0 + w - 1} r_b \end{aligned}$$

This provides our definition of practical worst-case performance.

Definition (Practical Worst-Case Performance): *The practical worst-case (PWC) cycle time for a given WIP level w is given by*

$$\mathrm{CT}_{\mathrm{PWC}} = T_0 + \frac{w-1}{r_b}$$

The PWC throughput for a given WIP level w is given by

$$\mathrm{TH}_{\mathrm{PWC}} = \frac{w}{W_0 + w - 1} r_b$$

Notice that the behavior of this case is reasonable for both extremely low and extremely high WIP levels. At one extreme, when there is only one job in the system ($w = 1$), cycle time becomes raw process time T_0, as we would expect. At the other extreme, as the WIP level grows very large (that is, $w \to \infty$), throughput approaches capacity r_b, while cycle time increases without bound. The intuition behind this latter result is that achieving throughput close to capacity in systems with high variability requires high WIP levels, in order to ensure high utilization of machines. But this also ensures a great deal of waiting and hence high cycle times.

The throughput and cycle time of the practical worst case are always between those of the best case and the worst case. As such, the PWC provides a useful midpoint that approximates the behavior of many real systems. By collecting data on average WIP, throughput, and cycle time (actually, because of Little's law, any two of these will suffice) for a real production line, we can determine whether it lies in the region between the best and practical worst cases, or between the practical worst and worst cases. Systems with better performance than the PWC (i.e., that have larger throughput and smaller cycle time for a given WIP level) are "good," and systems with worse performance are "bad." It makes sense to focus our improvement efforts on the bad lines because they are the ones with room for improvement. Thus, our three cases offer a sort of **internal benchmarking** methodology (i.e., as opposed to **external benchmarking** in which comparisons are made against outside systems).

For further guidance on *how* to improve a bad line, we can look to the three assumptions under which the PWC was derived:

1. Balanced line.
2. Single machine stations.
3. Exponential (memoryless) processing times.

Since these three conditions were chosen to maximize randomness in the line, improving any of them will tend to improve the performance of the line.

First, we could unbalance the line by adding capacity at a station. This could be accomplished by adding physical equipment, reducing downtime due to worker breaks or equipment failures, speeding up the process through more efficient work methods, and so on. Obviously, if we increase capacity at all stations, throughput will increase. But even if we increase capacity at only some stations, so that r_b does not change, this serves to reduce randomness (i.e., the states in Table 7.4 are no longer equally likely) and therefore causes the throughput-versus-WIP curve to increase more rapidly (i.e., less WIP in the system achieves the same throughput). We realize that line *unbalancing* is somewhat counter to the traditional industrial engineering emphasis on line balancing. However, as we will see in Chapter 18, line balancing is primarily applicable to *paced* assembly lines, not a line of independent workstations like those we are considering here.

Second, we could make use of parallel machines in place of single machines at workstations. If this is accomplished by adding extra machines, then it serves to increase capacity and therefore has essentially the same effects as those discussed above. But even replacing single machines with parallel ones with the same capacity can improve performance in some cases. For instance, reconsider Penny Fab One under the assumption that process times are exponential instead of deterministic with *average* process times still two hours at each station. Suppose stations 3 and 4 (rimming and deburring) are collapsed into a single station with two parallel machines, where the machines perform both rimming and deburring in a single step and take twice as long as before (i.e., an average of four hours per penny). Since the capacity of the station is one-half penny per hour, the bottleneck rate of the line is still $r_b = 0.5$. Also, the raw process time remains $T_0 = 8$ hours. But in the former arrangement, two pennies could have wound up at either rimming or deburring, with the consequence that one has to wait. In the revised line, anytime there are two pennies in rimming or deburring, we are guaranteed that both are being worked on. The result will be less waiting, and hence shorter cycle times, for a given WIP level in the revised system with parallel machines.

Finally, we could reduce the variability of the processing times to less than that implied by the exponential distribution. Reducing the likelihood of jobs clumping up behind stations, and hence waiting, will improve throughput and cycle time for a given WIP level. We will examine what is meant by variability reduction relative to the exponential in Chapter 8, and we will discuss practical methods for achieving it in Part III.

Figures 7.8 and 7.9 illustrate some of these concepts by plotting cycle time and throughput as a function of WIP level for Penny Fab Two under the assumption of exponentially distributed process times at all stations. For comparison, we have also plotted the best, worst, and practical worst cases for the same bottleneck rate and raw process time (i.e., for $r_b = 0.4$ and $T_0 = 20$). Even though processing times are exponential, because Penny Fab Two has an unbalanced line and parallel machine stations, it outperforms the practical worst case. If we were to reduce the variability of the processing times, this would improve it even more.

7.3.4 Bottleneck Rates and Cycle Time

Since the 1980s, a great deal of attention has been focused on the importance of bottlenecks in production systems (see, e.g., Goldratt and Cox 1984). Our discussion here

FIGURE 7.8
Cycle time versus WIP in Penny Fab Two

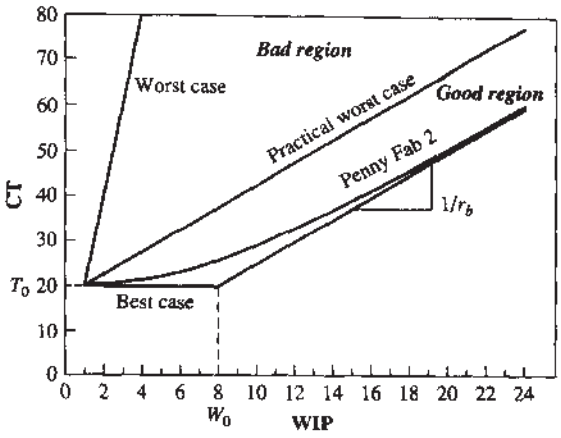
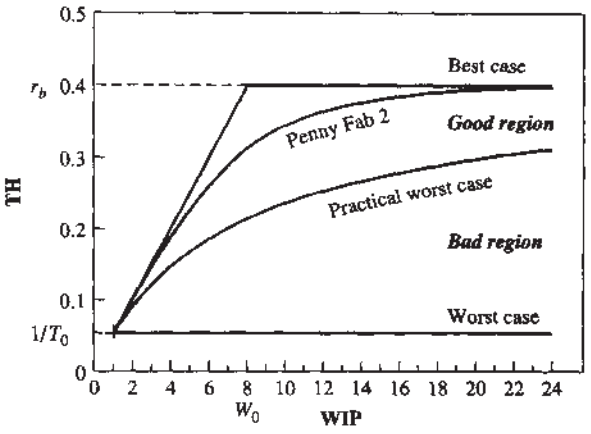


FIGURE 7.9
Throughput time versus WIP in Penny Fab Two



certainly concurs that the bottleneck rate r_b is important, since it establishes the capacity of the line. But the factory physics laws also give us insights into the role of bottlenecks beyond this obvious conclusion.

First, if we are operating a "good" line (i.e., throughput greater than the practical worst case for any WIP level), then at typical WIP levels (e.g., between 5 and 10 times W_0) the cycle time will be very close to w/r_b, where w is the WIP level. (This can be observed in Figures 7.8 and 7.9.) Hence, increasing the bottleneck rate r_b will reduce cycle time for any given WIP level.

Unfortunately, there are times when it is physically or economically impractical to speed up the bottleneck. For example, suppose the copper plater is the bottleneck in the HAL plant we described at the beginning of the chapter. The rate at which this machine runs is governed by the chemistry of the process. Therefore, if it is already running for the maximum number of hours per day (i.e., it does not suffer from staffing or maintenance problems that could be resolved to increase the effective capacity), then the only way to increase capacity is to add another plater. This is an extremely expensive option that would probably be overkill, since it would result in a 100 percent increase in capacity. In a situation like this, it may make economic sense to consider increasing capacity of nonbottleneck resources.

To see this, consider a system with four single machine stations. Each station takes 10 minutes to perform a job except the last station (the bottleneck) which takes 15 minutes. Thus, the bottleneck rate is four jobs per hour.

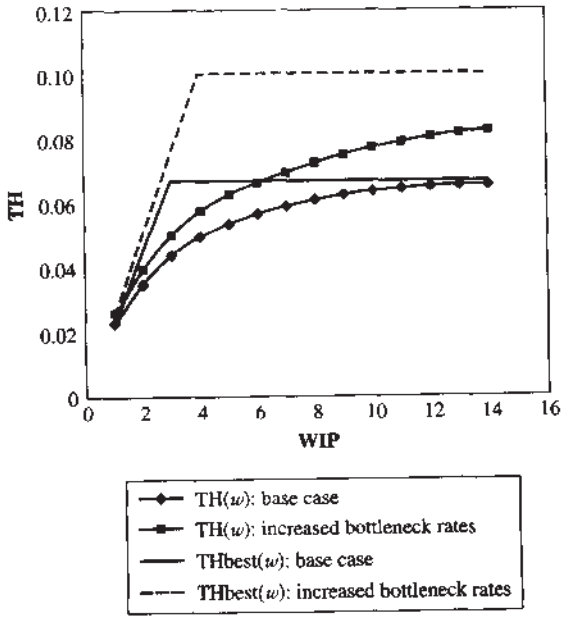
Now, suppose we speed up the bottleneck to 10 minutes per job (6 jobs per hour), thereby balancing the line. Figure 7.10 illustrates the impact on the throughput versus WIP curve for the line. Notice that the improved line has a higher limiting production rate (a new r_b), but the throughput curve stays further from it than the original system. The reason is that a balanced line tends to starve its bottleneck more frequently than an unbalanced line, and hence requires more WIP for throughput to approach capacity. Nonetheless, speeding up the bottleneck causes throughput to increase for any WIP level.

Alternatively, suppose we speed up all of the nonbottleneck processes so that they require only five minutes, but keep bottleneck time at 15 minutes. Figure 7.11 shows that this also increases throughput for any WIP level. Indeed, for small WIP levels, the increase in throughput is actually greater than that achieved by speeding up the bottleneck. However, for large WIP levels (six or above), increasing the bottleneck rate achieves a greater increase in throughput than does the increase in nonbottleneck rates. Also we note that we made a bigger change to the nonbottleneck stations than we did to the bottleneck station (i.e., we cut the process time in half at three machines as opposed to reducing the time at a single machine by 33 percent). If we had the freedom to reduce any process time by five minutes, the best place to do it would be the bottleneck, *always!* But since this is not always possible (economical), it is good to know that performance gains can be achieved by improving nonbottleneck resources.

7.3.5 Internal Benchmarking

We now have the tools to reconsider the HAL example from the beginning of the chapter. We can evaluate the PCB line by comparing actual performance to the best, worst, and practical worst cases. To do this, we must estimate the bottleneck rate r_b and raw process

FIGURE 7.10
Change in throughput curve due to increase in bottleneck rate



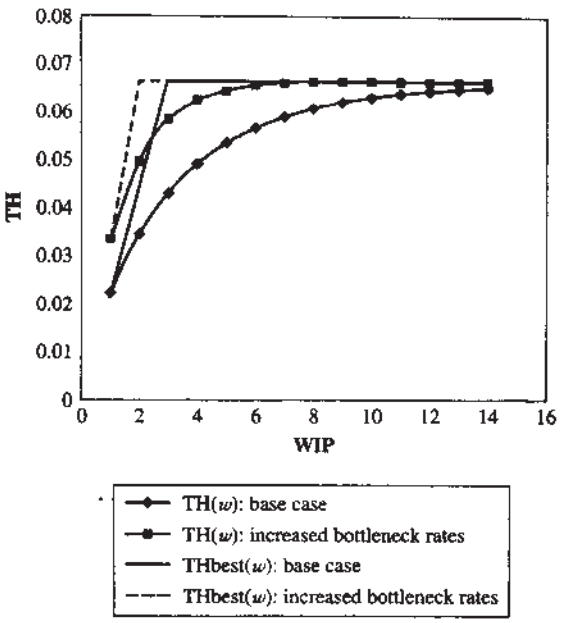


FIGURE 7.11
Change in throughput curve due to increase in rate of nonbottlenecks

time T_0. If we ignore multiple visits to some workstations (e.g., lamination), since this was considered in the rate and time data, the bottleneck is simply the process with the smallest capacity. This is sizing with $r_b = 126.5$ panels per hour. The raw process time is simply the sum of the process times in Table 7.1, which is $T_0 = 33.1$ hours. Hence, the critical WIP for the line is

$$W_0 = r_b \times T_0 = 126.5 \times 33.1 = 4,187 \text{ panels}$$

Recalling that the actual throughput was 45.8 panels per hour, actual cycle time was 816 hours, and actual WIP level was 37,000 panels, we can make some quick observations. First we make a quick Little's law check of the data:

$$\text{TH} \times \text{CT} = 1,100 \text{ panels/day} \times 34 \text{ days} = 37,400 \text{ panels} \approx 37,000 \text{ panels}$$

Since Little's law applies precisely only to long-term averages, we would not expect it to hold exactly. However, this is certainly well within the precision of the data and hence suggests no problems.

Second, we place these actual measures in context by noting that throughput is $45.8/126.5 = 36$ percent of the bottleneck rate, cycle time is $816/33.1 = 24.6$ times the raw process time, and WIP is $37,000/4,187 = 8.8$ times critical WIP. None of these look very good. However, we must be careful about drawing conclusions from any single measure. For instance, simply knowing that the WIP level is 8.8 times critical WIP does not by itself mean that the line is performing poorly. Even a very good line will require high WIP to attain a throughput level close to the bottleneck rate. But when WIP is high *and* throughput is low, this is a bad sign. Just how bad can be determined by comparing to the practical worst case.

There are two ways we can compare actual performance to the PWC. One way is to compute the throughput level that would be achieved by a PWC line with the same r_b, T_0, and WIP level as the HAL line and to compare to actual throughput. Using the

formula from the PWC definition, we get

$$\mathrm{TH_{PWC}} = \frac{w}{W_0 + w - 1} r_b = \frac{37{,}400}{4{,}187 + 37{,}400 - 1}(126.5) = 113.8 \text{ panels per hours}$$

Actual throughput of 45.8 panels per hour is less than one-half this level, indicating performance that is much worse than that in the practical worst case.

Alternatively, we can compute what WIP level would be required in a PWC line with the same r_b and T_0 as the HAL line, to achieve the observed level of throughput. That is,

$$\mathrm{TH_{PWC}} = \frac{w}{W_0 + w - 1} r_b = 45.8 = 0.36 r_b$$

which yields

$$\frac{w}{W_0 + w - 1} = 0.36$$

or

$$w = \frac{0.36}{0.64}(W_0 - 1) = 2{,}354 \text{ panels}$$

Actual WIP is more than 15 times this level, again indicating that the HAL line is far less efficient at converting WIP to throughput than the PWC.

We can put these calculations in graphical terms by plotting the best, worst, and practical worst throughput versus WIP curves and plotting the actual performance. This results in the graph in Figure 7.12. From this we can see dramatically that the WIP/throughput pair of (37,400, 45.8) is well into the "bad" region between the worst and practical worst cases. Clearly, lines that exhibit such behavior offer much more opportunity for improvement than lines in the "good" region between the practical worst and best cases.

This example shows that the models presented in this chapter can help diagnose a production line and determine whether it is operating efficiently or not. But they do not tell us why a line is operating poorly and therefore do not help us determine how to improve it. For this, we require a deeper investigation of what causes some lines to be

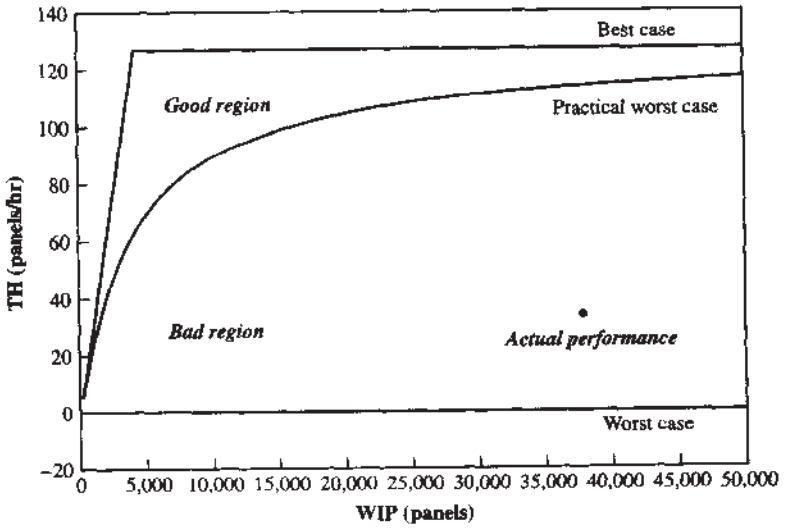


FIGURE 7.12
Throughput versus WIP in HAL example

very efficient at converting WIP to throughput and others to be very inefficient. This is the subject of the next two chapters.

7.4 Labor-Constrained Systems

Throughout this chapter, we have focused on lines in which machines are the primary constraint. We have implicitly assumed that if there are human operators, they are assigned to machines and can therefore be viewed as part of the workstations. However, in some systems, workers perform multiple tasks or tend more than one workstation. These types of systems exhibit more complex behavior than the simple lines considered so far, since the flow of work is affected by the number and characteristics of both machines and operators.

Although the subject of flexible labor is much too broad for us to treat comprehensively here, we can make some observations about how labor-constrained lines relate to the simple lines presented earlier. We do this by considering three situations below.

7.4.1 Ample Capacity Case

We begin with the case in which labor is the only constraint on output. That is, we assume sufficient equipment at each workstation to ensure that a worker is never blocked for lack of a machine. While one might think that such a situation would never arise in practice, there are realistic situations that approximate this behavior. An example the authors encountered was that of a prepress graphical production facility of catalogs and other marketing materials. This firm received content (text, photos, etc.) from its clients and converted these to electronic engraving data via a series of steps (e.g., scanning, color correction, page finishing), which it then sent to a printer to be made into paper products. Most of the prepress steps required a computer along with some peripheral equipment. Because computer equipment was inexpensive relative to the cost of delays, the firm installed enough duplicates of each station to ensure that technicians virtually never had to wait for equipment to perform the various tasks. The result was many more machines than people, which meant that labor was the key constraint in the system.

A primary reason the graphics company installed ample capacity at its stations was to facilitate its flexible labor policy. Instead of having specialists for each operation, the company had cross-trained the workforce so that almost everyone could do almost every operation. This allowed the company to assign workers to jobs instead of stations. A worker would follow a job through the system, performing each operation on the appropriate workstation, as shown in Figure 7.13. The extra computers made it very

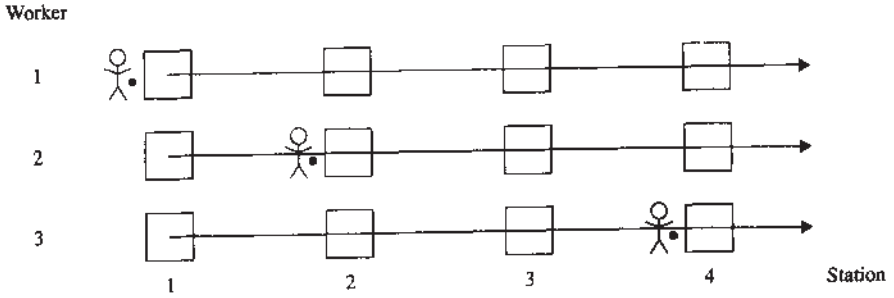


FIGURE 7.13
Ample capacity line with fully cross-trained workers

unlikely that someone would ever have to wait for equipment at a station. Having workers stay with a job all the way through the system meant that customers had a single person to contact and also made one person clearly responsible for quality.

In a system like this, capacity is defined by labor rather than equipment. To characterize capacity, we will continue to let T_0 represent the average time for one job to traverse the system, which we assume is independent of which worker is assigned to the job. Furthermore, we suppose that once a worker starts a job, he or she continues with it until it is done. Stopping work midway through a job cannot improve throughput and will only increase cycle time, so unless some customers have higher priority than others, there is no reason to do this. Under these assumptions, jobs are released into the system only when a worker becomes available, and since there is no blocking due to equipment, cycle time is always T_0. If there are n workers in the line, all working at the same rate, then each puts out a job every T_0 time units, which means that throughput is n/T_0.

Since the ample capacity case is an ideal situation, any changes to our assumptions can only decrease throughput. Examples of such changes include less-than-ample equipment so that blocking occurs, intermittent arrival of work that may cause starving, partial cross-training so that jobs may have to wait for a "specialist" at some stations, or any other change that prevents workers from being completely busy. Hence, we can state the following factory physics law.

Law (Labor Capacity): *The maximum capacity of a line staffed by n cross-trained operators with identical work rates is*

$$\mathrm{TH}_{\max} = \frac{n}{T_0}$$

This law provides a way to introduce labor into the capacity calculations. For instance, in a line that has more stations than workers, the bottleneck rate of the equipment r_b may be a poor estimate of the capacity of the line. Where throughput is constrained by labor, n/T_0 may be a more realistic and useful upper bound on capacity. This bound is applicable to a wide range of systems, including those with fully or partially cross-trained workers.

One class of systems to which it does not apply, however, is that in which a worker can process more than one job simultaneously. For instance, a manufacturing cell where a single operator can tend several automated machines at the same time may have throughput exceed n/T_0. Such systems are often appropriately viewed as equipment-constrained, where operator unavailability acts as a capacity detractor and variability inflator. We will examine detractors in Chapter 8.

7.4.2 Full Flexibility Case

To deepen our insight into how both equipment and labor affect capacity, we next consider the case in which workers are completely cross-trained (i.e., can operate every station in the line). Furthermore, we begin by assuming that workers are tied to jobs as in the ample capacity case. However, unlike in the ample capacity case, equipment is limited so workers may become blocked, as shown in Figure 7.14. Once a worker finishes a job at the end of the line, she goes back to the beginning and starts a new one.

If the workers in Figure 7.14 have identical work rates, then this line is logically identical to the CONWIP lines we considered previously, except that the WIP level is now the number of workers. Hence, the behavior of the line will lie somewhere between the best and worst cases, with the practical worst case defining the division between

FIGURE 7.14
Line with fully cross-trained workers tied to jobs

good and bad lines. Furthermore, all the improvement strategies we listed earlier—increasing capacity, reducing line balance, using parallel machine stations, and reducing variability—still apply to this case.

The assumption of fully cross-trained workers who walk jobs all the way through the line may not be realistic in many situations. For instance, if the workstations require very different skills, it may make sense to have workers pass jobs from one to another. One mechanism is the *bucket brigade* (see Bartholdi and Eisenstein 1996). In this system, whenever the worker farthest downstream in the line completes a job, he or she moves up the line and takes the job from the next worker upstream. That worker in turn moves upstream and takes the job from the next worker. And so on, until the worker farthest upstream takes a new job. If all workers work at the same speed and there is no delay due to the handing off of the jobs, then there is no logical difference in this system from the one depicted in Figure 7.14. The line still operates as a CONWIP line with the WIP level set by the number of workers. Only the identities of the workers assigned to each job are changed.

While the bucket brigade system may not differ logically from the system with workers tied to jobs, it does differ practically. Each worker will tend to operate machines in a zone. Indeed, in the case where all process times are perfectly deterministic (i.e., the best case), the line will settle into a repetitive cycle where each worker processes jobs through the same sequence of stations. The cross-training and job transfers allow the line to balance itself so that each worker spends the same amount of time with a job. This type of system has been used effectively in automobile seat construction (see Chapter 10 for a discussion of this system at Toyota), warehouse picking, and fast-food sandwich construction (Subway).

Notice that blocking is still possible in the bucket brigades. Whenever an upstream worker catches up with the next worker downstream, she or he will be blocked unless the station has extra equipment. Hence, it makes sense to organize the workers so as to minimize the frequency with which this happens, by placing the fastest workers downstream and the slowest workers upstream. Bartholdi and Eisenstein (1996) show that this arrangement from slowest to fastest can significantly improve throughput and observed that this tends to be the practice in industry where such systems are used.

7.4.3 CONWIP Lines with Flexible Labor

If workers stay tied to jobs (or hand off jobs directly to one another as in the bucket brigade system), then the number of jobs in the system always equals the number of workers and the system behaves logistically as a CONWIP line. But in many, if not most, systems, the number of jobs will typically exceed the number of workers. If workers can rove through the system and work at different stations, then the performance of the system will depend on how effectively labor is allocated to promote flow through the system. This can get complex, since there are countless ways that labor can be dynamically allocated in the system.

One approach, which is a natural extension of the bucket brigade system to the case with more jobs than workers, is to have any worker who becomes free take the

next job upstream, either from the upstream worker or from a buffer (see Figure 7.15 for an illustration of the mechanics). Whenever a worker becomes blocked because a downstream station is busy, the worker drops the job in the buffer in front of the station and moves upstream to get another job. This continues as long as the total number of jobs in the system does not exceed some preset limit (without such a limit, a fast worker at the front of the line would flood the line with WIP).

If all stations consist of single machines, so that no passing is possible, then at any time worker n (the last worker in the line) will be working on the job farthest downstream. Worker $n-1$ will be working on the next-farthest job downstream that is not blocked by worker n. And so on. If passing on multimachine stations is possible, then the workers can get out of order. But the basic intent is still to keep workers working whenever possible on the jobs farthest downstream. Keeping workers busy tends to maximize throughput; working on downstream jobs tends to minimize cycle times. Hence, we would expect this policy to work reasonably well.

Of course, other flexible labor policies are possible. Which is appropriate depends on a variety of factors, including the degree of worker cross-training, the relative speed of the workers at the different stations, and the efficiency with which jobs can be passed from one worker to another. If there is no difference in the speed of workers, then the throughput of the system depends entirely on how often unblocked jobs are idle for lack of a worker. If this never happens, then the system will operate like a regular CONWIP line. If it happens so frequently that the workers might just as well be tied to one job each, then the system will operate as a CONWIP line with only as many jobs as workers. Hence, we can bound the throughput of a CONWIP line with flexible workers as in the following factory physics law.

Law (CONWIP with Flexible Labor): *In a CONWIP line with n identical workers and w jobs, where $w \geq n$, any policy that never idles workers when unblocked jobs are available will achieve a throughput level* $\mathrm{TH}(w)$ *bounded by*

$$\mathrm{TH}_{\mathrm{CW}}(n) \leq \mathrm{TH}(w) \leq \mathrm{TH}_{\mathrm{CW}}(w)$$

where $\mathrm{TH}_{\mathrm{CW}}(x)$ *represents the throughput of a CONWIP line with all machines staffed by workers and x jobs in the system.*

This law can give us some insight into the value of cross-training in a system. For instance, in a line with fixed workers, where the number of workers is at least equal to critical WIP and performance is close to the best case, there is clearly little benefit to cross-training. The reason is that the throughput of a CONWIP line with any WIP above critical WIP will be close to the bottleneck rate, so $\mathrm{TH}_{\mathrm{CW}}(n)$ will be approximately equal to $\mathrm{TH}_{\mathrm{CW}}(w)$. The reason is that because there is little variability in the system, there will not be many occasions in which the capability of moving workers between stations will be of value.

On the other hand, if a line has significant variability, then the potential improvement from cross-training can be substantial. To see this, consider a practical worst-case line

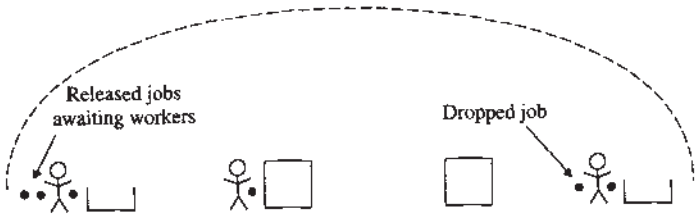
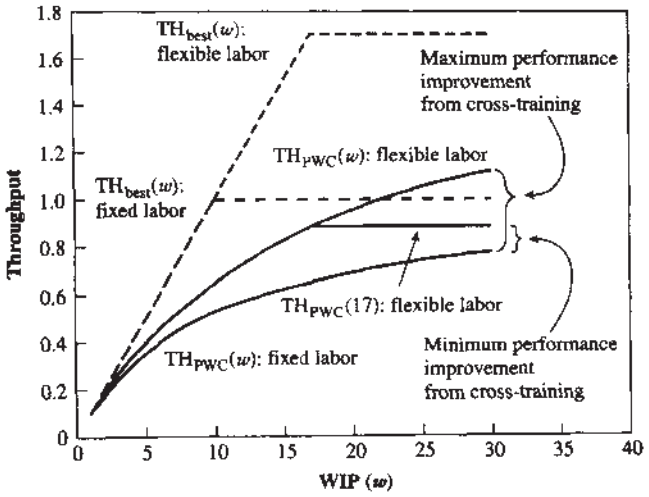


FIGURE 7.15
CONWIP line using bucket brigade with job dropping

FIGURE 7.16
Performance improvement in a CONWIP line through use of flexible labor



with bottleneck rate of $r_b = 1$ per hour and raw process time $t_0 = 10$ hours, which is staffed by $n = 17$ workers. Currently, the line is behaving as the practical worst case (the TH-versus-WIP curve in Figure 7.16 is labeled "$TH_{PWC}(w)$: Fixed Labor"). But suppose that we were to cross-train the workers so that they could staff any task. This would enable workers to shift to stations where they are needed when fluctuations in work require it. From the labor capacity law, we know that this could increase the effective capacity up to as much as $n/T_0 = 17/10 = 1.7$ jobs per hour. If it does and the line still behaves as in the practical worst case, then the throughput curve will shift upward accordingly. By the CONWIP with flexible labor law, the actual throughput will lie between $TH_{PWC}(n)$ and $TH_{PWC}(w)$. So while we cannot say exactly how large the performance improvement will be, it is clear that it is significant. The conclusion is that by dynamically balancing the line, the cross-trained workers are able to increase its effective capacity and thereby achieve increased output.

From our previous analyses, we know that systems with high process variability and a high degree of balance will tend to look more like the practical worst case than low-variability/low-balance systems. Hence, these conditions will tend to make cross-training attractive. The reason is that balanced, high-variability systems tend to starve workers who are tied to stations. Therefore, allowing workers to follow jobs prevents some of this starvation and hence increases throughput.

We should note, however, that while single-machine stations also tend to make systems behave as in the practical worst case, they do not generally make cross-training more attractive. Parallel machine stations actually facilitate flexible work policies by reducing the frequency with which workers are blocked for lack of a machine. In the extreme case, where there is sufficient parallel capacity to prevent blocking, the system can approach the behavior of the ample capacity case where labor becomes the only constraint.

7.5 Conclusions

In this chapter we examined the fundamental behavior of a single production line by studying the relationships among cycle time, WIP, throughput, and capacity. We observed the following:

1. A single line can be reasonably summarized by two independent parameters: the bottleneck rate r_b and the raw process time T_0. However, as we observed, a wide range of behavior is possible for lines with the same r_b and T_0. We will investigate the causes of this disparity in the next two chapters.

2. Little's law (WIP = TH × CT) provides a fundamental relationship between three long-term average measures of the performance of *any* production station, line, or system.

3. The best case defines the maximum throughput and minimum cycle time for a given WIP level for any line with specified values of r_b and T_0. The worst case defines the minimum throughput and maximum cycle time for any line with specified values of r_b and T_0. The practical worst case provides an intermediate scenario that serves as a useful demarcation between "good" and "bad" systems.

4. The critical WIP level, defined as $W_0 = r_b T_0$, represents a realistic ideal WIP level (as opposed to the unrealistic ideal of zero inventory, which would result in zero throughput). At W_0, a best-case (i.e., zero-variability) line achieves both maximum throughput (i.e., r_b) and minimum cycle time (i.e., T_0).

5. Both the best case and the worst case occur in systems with zero randomness. The worst case results from high variability caused by bad control rather than randomness. The practical worst case represents the maximum randomness situation.

6. When WIP levels are high, reducing raw process time T_0 has little effect on cycle times, while increasing r_b can have a great impact.

7. Other things being equal (that is, r_b and T_0 are the same), unbalanced lines exhibit less congestion than balanced lines.

8. Production lines can be constrained by a combination of equipment and labor. Equipment capacity is bounded by the bottleneck rate r_b, while labor capacity is bounded by n/T_0, where n is the number of workers in the line.

9. Systems with high process variability and balanced stations are most amenable to cross-training and flexible labor policies. In addition, parallel machine stations help facilitate flexible work policies.

A thread that has emerged from this analysis of basic factory dynamics is that a line can achieve the same thoughput at a lower WIP level by either increasing capacity or improving the efficiency of the line. As we hinted in our treatment of the practical worst case, a primary way of increasing line efficiency is by reducing variability at individual stations. To be able to evaluate the relative effectiveness of capacity increases versus variability reduction, we must further develop the science of factory physics to describe the behavior of production systems involving randomness. We do this next in Chapters 8 and 9.

Study Questions

1. Suppose throughput TH is near capacity r_b. Using Little's law, relate
   a. WIP and cycle time in a production line
   b. Finished goods inventory and time spent in finished goods inventory
   c. The number of cars waiting at a toll booth and the average wait time
2. Is it possible for a line to have the same throughput with both high WIP with high cycle time and low WIP with low cycle time? Which would you rather have? Why?
3. For a given set of production line characteristics (i.e., raw process time T_0 and bottleneck rate r_b) and a given WIP level w, what is the best cycle time that can be achieved? What is the "worst"? What is the corresponding throughput for these two cases?

4. What are the conditions for the practical worst-case throughput? What types of behavior can lead to performance worse than that in this case? What would this do to throughput? To cycle times?
5. Can the critical WIP level W_0 ever exceed the number of machines in the line?

Problems

1. Consider a four-station line in which all stations consist of single machines. Station 2 has average processing times of two hours per job, while the remaining stations have average processing times of one hour per job. Answer the following, under the assumption that the line behaves according to the best case.
   a. What are r_b and T_0 for this line?
   b. How do r_b and T_0 change if a second identical machine is added to station 2? What effects will this have on performance?
   c. How do r_b and T_0 change if the machine at station 2 is speeded up to have average processing times of one hour? What effects will this have on performance?
   d. How do r_b and T_0 change if a second identical machine is added to station 1? What effects will this have on performance?
   e. How do r_b and T_0 change if the machine at station 1 is speeded up to have average processing times of one-half hour? What effects will this have on performance? Do your results agree or disagree with the statement "An hour saved at a nonbottleneck is a mirage (i.e., of no value)"?
2. Repeat Problem 1 under the assumption that the line behaves according to the worst case.
3. Repeat Problem 1 under the assumption that the line behaves according to the practical worst case.
4. Consider the following three-station production line with a single product that must visit stations 1, 2, and 3 in sequence:
   - Station 1 has 5 identical machines with average processing times of 15 minutes per job.
   - Station 2 has 12 identical machines with average processing times of 30 minutes per job.
   - Station 3 has 1 machine with average processing times of 3 minutes per job.

   a. What are the bottleneck rate r_b, the raw process time T_0, and the critical WIP w_0?
   b. Compute the average cycle time when the WIP level is set at 20 jobs, under the assumptions of
      i. the best case
      ii. the worst case
      iii. the practical worst case
   c. We desire the throughput to be 90 percent of the bottleneck rate. Find the WIP level required to achieve this under the assumptions of
      i. the best case
      ii. the worst case
      iii. the practical worst case
   d. If the cycle time at the critical WIP is 100 minutes, where does performance fall relative to the three cases? Is there much room for improvement?
5. Positively Rivet Inc. is a small machine shop that produces sheet metal products. It had one line dedicated to the manufacture of light-duty vent hood shells, but because of strong demand it recently added a second line. The new line makes use of higher-capacity automated equipment but consists of the same basic four processes as the old line. In addition, the new line makes use of one machine per workstation, while the old line has parallel machines at the workstations. The processes, along with their machine rates, number of machines per station, and average times for a lone job to go through a station (i.e., not including queue time), are given for each line in the following table:

Process	Old Line			New Line		
	Rate per Machine (parts/hour)	**Number Machines per Station**	**Time (minute)**	**Rate per Machine (parts/hour)**	**Number Machines per Station**	**Time (minute)**
Punching	15	4	4.0	120	1	0.50
Braking	12	4	5.0	120	1	0.50
Assembly	20	2	3.0	125	1	0.48
Finishing	50	1	1.2	125	1	0.48

Over the past three months, the old line has averaged 350 parts per day, where one day consists of one eight-hour shift, and has had an average WIP level of 400 parts. The new line has averaged 680 parts per eight-hour day with an average WIP level of 350 parts. Management has been dissatisfied with the performance of the old line because it is achieving lower throughput with higher WIP than the new line. Your job is to evaluate these two lines to the extent possible with the above data and identify potentially attractive improvement paths for each line by addressing the following questions.

a. Compute r_b, T_0, and W_0 for both lines. Which line has the larger critical WIP? Explain why.

b. Compare the performance of the two lines to the practical worst case. What can you conclude about the relative performance of the two lines compared to their underlying capabilities? Is management correct in criticizing the old line for inefficiency?

c. If you were the manager in charge of these lines, what option would you consider first to improve throughput of the old line? Of the new line?

6. Floor-On, Ltd., operates a line that produces self-adhesive tiles. This line consists of single-machine stations and is almost balanced (i.e., station rates are nearly equal). A manufacturing engineer has estimated the bottleneck rate of the line to be 2,000 cases per 16-hour day and the raw process time to be 30 minutes. The line has averaged 1,700 cases per day, and cycle time has averaged 3.5 hours.

   a. What would you estimate average WIP level to be?

   b. How does this performance compare to the practical worst case?

   c. What would happen to the throughput of the line if we increased capacity at a nonbottleneck station and held WIP constant at its current level?

   d. What would happen to the throughput of the line if we replaced a single-machine station with four machines whose capacity equaled that of the single machine and held the WIP constant at its current level?

   e. What would happen to the throughput of the line if we began moving cases of tiles between stations in large batches instead of one at a time?

7. T&D Electric manufactures high-voltage switches and other equipment for electric utilities. One line that is staffed by three workers assembles a particular type of switch. Currently the three workers have fixed assignments; each worker fastens a specific set of components onto the switch and passes it downstream on a rolling conveyor. The conveyor has capacity to allow a queue to build up in front of each worker. The bottleneck is the middle station with a rate of 11 switches per hour. The raw process time is 15 minutes. To improve the efficiency of the line, management is considering cross-training the workers and implementing some sort of flexible labor system.

   a. If current throughput is 10.5 switches per hour with an average WIP level of five jobs, how much potential do you think there is for a flexible work system?

   b. If current throughput is eight switches per hour with an average WIP level of seven jobs, how much potential do you think there is for a flexible work system?

   c. If all three workers were fully cross-trained and equipped to assemble the entire switch in parallel (i.e., no passing of jobs to one another) and were able to maintain the current work

pace of each operation, what would the capacity of the system be? What real-world problems might make such a policy unattractive?

d. Suggest a flexible work system that could improve the efficiency of a line like this with less than full cross-training (i.e., with workers trained and equipped to assemble only certain components).

8. Consider a balanced line consisting of five single-machine stations with exponential process times. Suppose the utilization is 75 percent and the line runs under the CONWIP protocol (i.e., a new job is started each time a job is completed).
   a. What is the WIP level in the line?
   b. What is the cycle time as a percentage of T_0?
   c. What happens to WIP, CT, and TH relative to the original system if you make each of the following changes (one at a time)?
      i. Increase the WIP level
      ii. Decrease the variability of one station
      iii. Decrease the capacity at one station
      iv. Increase the capacity of all stations

Intuition-Building Exercises

1. Simulate Penny Fab Two by taking a piece of paper and drawing a schematic of the line (see Figure 7.17). Draw the squares large enough to contain a penny. To the right of each square, write the time of the completion of the job occupying that square (as the simulation progresses, you will cross out the old time and replace it with the next time). The simulation progresses by setting the current "simulated time" to be the earliest completion time and moving the pennies accordingly.
   a. Run your simulation for several simulated hours with seven pennies. Note how the second station sometimes starves.

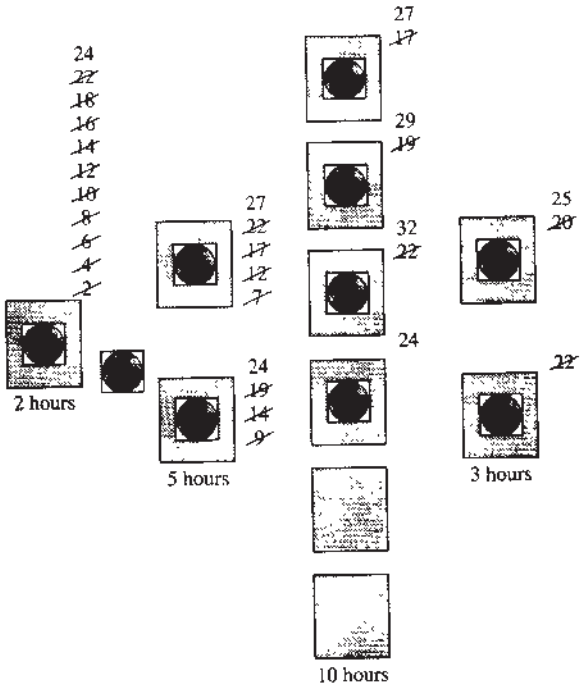


FIGURE 7.17
Penny Fab Two with $w = 9$, 22 hours into the simulation

   b. Run your simulation for several simulated hours with eight pennies. Observe that station 2 never starves and there is never any queueing once the initial transient queue is dissipated in front of the first station.
   c. Run your simulation for several simulated hours with nine pennies (Figure 7.17 illustrates this scenario after 22 simulated hours). Note that after the initial transient, there is always a queue in front of second station.

2. Simulate Penny Fab Two for 25 hours starting with an empty line and eight pennies in front. Record the cycle time of each penny that finishes during this time (i.e., record its start time and finish time and compute cycle time as the difference).
   a. What is the average cycle time CT?
   b. How many jobs finish during the 25 hours?
   c. What is the average throughput TH over 25 hours? Does average WIP equal CT times TH? Why or why not? (*Hint:* Did Little's law hold for the first two hours of our simulation of Penny Fab One?) What does this tell you about the use of Little's law over short time intervals?

CHAPTER

8 VARIABILITY BASICS

God does not play dice with the universe.
Albert Einstein

Stop telling God what to do.
Niels Bohr

8.1 Introduction

Little's law (TH = CT/WIP) implies that it is possible to achieve the same throughput with long cycle time and large WIP or short cycle time and small WIP. Of course, the short-cycle-time, low-WIP system is preferable. But what causes the difference? The answer, in a great many instances, is *variability*.

Penny Fab One from Chapter 7 achieves full throughput (one-half job per hour) at a WIP level of $W_0 = 4$ jobs (the critical WIP) if it behaves like the best case. But if it behaves like the practical worst case, it requires a WIP level of 27 jobs to achieve 90 percent of capacity (57 jobs to achieve 95 percent of capacity). If it behaves like the worst case, 90 percent of capacity is not even feasible. Why the big difference? *Variability!*

Briar Patch Manufacturing has two very similar workstations as part of its plant. Both are composed of a single machine that runs at a rate of four jobs per hour (when it is not down). Both are subject to the same pattern of demand with an average work load of 69 jobs per day (2.875 jobs per hour). And both are subject to periodic unpredictable outages. However, for one workstation, consisting of a Hare X19 machine, outages are rather infrequent but tend to be quite long when they occur. For the other station, consisting of a Tortoise 2000 machine, outages are much more frequent and correspondingly shorter. Both machines have an *availability* (i.e., the long-term fraction of the time that the machine is not down for repair) of 75 percent. Thus, the capacity of both stations is $4(0.75) = 3$ jobs per hour. Since the two stations have the same capacity and are subject to the same demand, they should have the same performance—cycle time, WIP, lead time, and customer service—right? Wrong! It turns out that the Hare X19 is substantially worse on all measures than the Tortoise 2000. Why? Again, the answer is *variability!*

Variability exists in all production systems and can have an enormous impact on performance. For this reason, the ability to measure, understand, and manage variability

is critical to effective manufacturing management. In this chapter we will develop basic tools and intuition for characterizing variability in production systems. In the next chapter, we probe more deeply into the manner in which variability degrades system performance and how it can be managed.

8.2 Variability and Randomness

What, exactly, is variability? A formal definition is the *quality of nonuniformity of a class of entities.* For example, a group of individuals who all weigh exactly the same have no variability in weight, while a group with vastly different weights is highly variable in this regard. In manufacturing systems, there are many attributes in which variability is of interest. Physical dimensions, process times, machine failure/repair times, quality measures, temperatures, material hardness, setup times, and so on are examples of characteristics that are prone to nonuniformity.

Variability is closely associated with (but not identical to) **randomness.** Therefore, to understand the causes and effects of variability, one must understand the concept of randomness and the related subject of **probability.** In this chapter we develop the necessary ideas in as loose and intuitive a manner as possible. However, for precision, there are points at which we must invoke the formal language of probability. In particular, the concept of a **random variable** and its characterization via its **mean** and **standard deviation** are essential. The reader for whom this terminology is new or rusty should refer to the review of basic probability in Appendix 2A before proceeding with this chapter.

As mentioned above, both the worst and practical worst cases represent systems whose performance is degraded by variability. However, the variability in the worst case is completely predictable—a consequence of *bad control*—while the variability in the practical worst case is due to unpredictable randomness. To understand the difference, we must distinguish between controllable variation and random variation.

Controllable variation occurs as a direct result of decisions. For instance, if several products are produced in a plant, there will be variability in the product descriptors (e.g., their physical dimensions, time to manufacture, etc.). Likewise, if material is moved in batches from one process to the next, the first part to finish will have to wait longer to move than the last part, and so waiting times will be more variable than if moved one at a time.

In contrast, **random variation** is a consequence of events beyond our immediate control. For example, the times between customer demands are not generally under our control. Thus, we should expect the load at any particular workstation to fluctuate. Likewise, we do not know when a machine might fail. Such downtime adds to the effective process time of a job, since the job must wait for the machine to be repaired before completing processing. Since such contingencies cannot be predicted or controlled (at least in the immediate term), machine outages increase the variability of effective process times in a random fashion.

Although both types of variation can be disruptive to a plant, the effects of random variation are more subtle and require more sophisticated tools to describe. For this reason, we will focus mainly on random variation in this chapter.

8.2.1 The Roots of Randomness

Unfortunately, the very notion of randomness gives most people (including philosophers) trouble. How can something occur that is independent of its initial conditions? Does this not violate the notion of cause and effect? While it is beyond our scope to discuss

this philosophical dilemma thoroughly, it is interesting to make some basic observations about the nature of randomness.

One interpretation of randomness is that because we have imperfect (or incomplete) information, systems *appear* to behave randomly. The underlying premise of this view is that if we knew all the laws of physics and had a complete description of the universe at some time, then, in theory, we could predict every detail of its evolution from then on with certainty.

A second interpretation is that the universe actually *behaves* randomly. In other words, having a complete description of the universe and the laws of physics is not enough to predict the future. At best, these can provide only *statistical* estimates of what will happen. Furthermore, identical starting conditions may not yield identical futures. Because of the apparent violation of the principle of cause and effect, this viewpoint has been roundly criticized in philosophy circles. However, its proponents have pointed out that the cause-and-effect principle can be recovered by defining other, more fundamental quantities that are not affected by randomness.[1]

The debate between these two schools of thought became quite heated within the physics community during the early part of the 20th century. Einstein sided with the first view (incomplete knowledge) and stated emphatically that "God does not play dice." Bohr and others believed in the second (random universe) view and suggested that Einstein "not tell God what to do" (see Planck 1936 for a discussion of this controversy). In recent years, experimental evidence has tended to side with the random universe view, much to the distaste of some philosophers.

Regardless of whether randomness is elemental or due to a lack of knowledge, the effects are the same—many facets of life, including manufacturing management, are inherently unpredictable. This means that the results of management actions can never be guaranteed. In fact, starting with the same conditions and using the same control policy on different days may well lead to different outcomes.

This does not mean that we should give up on managing the factory, only that we need to be concerned with finding *robust* policies. A robust policy is one that works well *most of the time*. This differs from an *optimal* policy, which is the best policy for a specific set of conditions. A robust policy is almost never optimal but is usually "pretty good." In contrast, an optimal policy may work extremely well for the set of conditions for which it was designed, but perform very poorly for many others. The most powerful tool a manager can have for identifying effective and robust policies in the face of randomness is *good probabilistic intuition*. Unfortunately, such intuition appears to be rare. A major goal of this chapter is to develop this critical skill.

8.2.2 Probabilistic Intuition

Intuition plays an important part in many aspects of our everyday lives. Most decisions we make are based upon some form of intuition. For instance, we slow down when making turns in an automobile because of our intuition developed after driving for some time, rather than our detailed understanding of automotive physics. We decide whether to refinance our house by appealing to our intuition about the economy, rather than a formal economic analysis. We time our request for a raise according to our intuitive sense of the boss's mood, rather than deep theory about his or her psychological profile.

In many situations, our intuition is quite good with respect to "first-order" effects. For example, if we speed up the bottleneck (busiest workstation) in a production line,

[1] Quantities known as *quantum numbers* are well-defined and determine the probability distributions of random observables, such as location and velocity, instead of actual outcomes.

without changing anything else, we expect to get out more product. This type of intuition typically comes from acting as though the world were **deterministic,** that is, without randomness. In the language of probability and statistics, such reasoning is based on the **first moment** or the **mean** (average) of the random variables involved. As long as the change in the mean quantity (e.g., increase in average speed of a machine) is large relative to the randomness involved, first-order intuition usually works well.

Our intuition tends to be much less developed for second moments (i.e., for quantities involving the variance of random variables). For instance, which is more variable, the time to process an individual part or the time to process a batch of parts? Which are more disruptive, short, frequent machine failures or long, infrequent ones? Which will result in a greater improvement in line performance, reducing the variability of process times at stations near the front of the line or near the back? These and other variability-related questions concerning plant behavior require much more subtle intuition than that required to see that speeding up the bottleneck will improve throughput.

Because people frequently lack well-developed intuition regarding second moments, they often misinterpret random phenomena. A typical example occurs in the classroom when students who made low grades on a first examination show relative improvement on the second examination, while students who made high scores on the first examination do worse on the second. This is an example of the phenomenon known as **regression to the mean.** An extreme score (high or low) on the first examination is likely to be at least partially due to randomness (e.g., lucky or unlucky guesses, a headache on test day, etc.). Since the random effects for a given student are unlikely to be extreme twice in a row, the student with an extreme score on the first examination is likely to have a more moderate score on the second. Unfortunately, many teachers interpret these results as a sign that they have finally reached the slower students and are beginning to lose the better ones. In reality, simple randomness may well account for the effect.

Misinterpretation of the general tendency for regression to the mean also occurs among manufacturing managers. After a particularly slow period of output, a manager may react with harsh appraisals and disciplinary action. Sure enough, production goes up. Similarly, after outstanding performance and much praise, production declines—clear evidence that the workers have grown complacent. Of course, the same behavior—better following bad and worse following good—is likely to happen *even when there has been no change,* whenever randomness is present.

In addition to the first two moments (mean and variance), random phenomena are influenced by the third (skewness), the fourth (kurtosis), and higher moments. The effects of these higher moments are generally much less pronounced than those associated with the first two, so we will focus on only the mean and the variance. Furthermore, as noted above, since effects associated with the mean are fairly intuitive, while effects associated with the variance are much more subtle, we will devote particular attention to understanding variance.

8.3 Process Time Variability

The random variable of primary interest in factory physics is the **effective process time** of a job at a workstation. We use the label *effective* because we are referring to the total time "seen" by a job at a station. We do this because from a logistical point of view, if machine B is idle because it is waiting for a job to finish on machine A, it does not matter whether the job is actually being processed or is being held up because machine A is being repaired, undergoing a setup, reworking the part due to a quality problem, or

waiting for its operator to return from a break. To machine B, the effects are the same. For this reason, we will combine these and other effects into one aggregate measure of variability.

8.3.1 Measures and Classes of Variability

To effectively analyze variability, we must be able to quantify it. We do this by using standard *measures* from statistics to define a set of factory physics variability *classes*.

Variance, commonly denoted by σ^2 (sigma squared), is a measure of *absolute* variability, as is the **standard deviation** σ, defined as the square root of the variance. Often, however, absolute variability is less important than *relative* variability. For instance, a standard deviation of 10 micrometers (μm) would indicate extremely low variability in the length of bolts with a nominal length of two inches, but would represent a very high level of variation for line widths on a chip whose mean width is five micrometers. A reasonable relative measure of the variability of a random variable is the standard deviation divided by the mean, which is called the **coefficient of variation (CV).** If we let t denote the mean (we use t because the primary random variables we are considering here are times) and σ denote the variance, the coefficient of variation c can be written

$$c = \frac{\sigma}{t}$$

In many cases, it turns out to be more convenient to use the **squared coefficient of variation (SCV)**

$$c^2 = \frac{\sigma^2}{t^2}$$

We will make extensive use of the CV and the SCV for representing and analyzing variability in production systems. We will say that a random variable has **low variability (LV)** if its CV is less than 0.75, that it has **moderate variability (MV)** if its CV is between 0.75 and 1.33, and that it has **high variability (HV)** if the CV is greater than 1.33. Table 8.1 presents these cases and provides examples.

8.3.2 Low and Moderate Variability

When we think of process times, we tend to think of the actual time that a machine or an operator spends on the job (i.e., not including failures or setups). Such times tend to have probability distributions that look like the classic bell-shaped curve. Figure 8.1 shows the probability distribution for process times with a mean of 20 minutes and a standard deviation of 6.3 minutes. Notice how most of the area under the curve is symmetrically distributed around 20. The CV for this case is around 0.32, so it is in the low variability (LV) range. It is a characteristic of most LV process times to have a bell-shaped probability density.

TABLE 8.1 Classes of Variability

Variability Class	Coefficient of Variation	Typical Situation
Low (LV)	$c < 0.75$	Process times without outages
Moderate (MV)	$0.75 \leq c < 1.33$	Process times with short adjustments (e.g., setups)
High (HV)	$c \geq 1.33$	Process times with long outages (e.g., failures)

FIGURE 8.1

A low-variability distribution

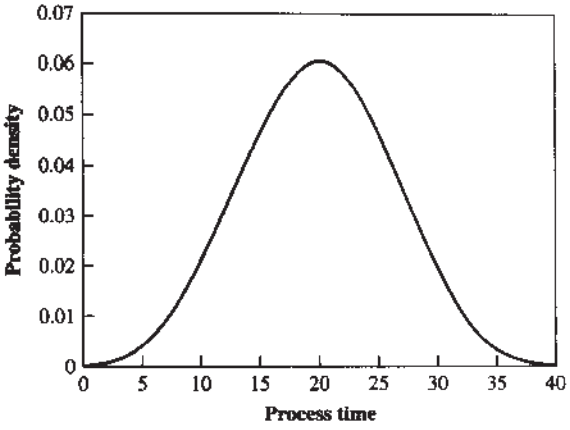
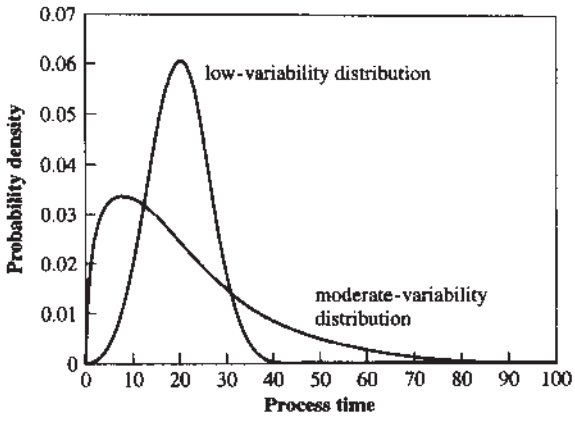


FIGURE 8.2

Low- and moderate-variability distributions



Now consider a situation with a mean process time of 20 minutes but for which the CV is around 0.75, the beginning of the moderate-variability case. An example might be process times of a manual operation in which most of the time the operation is easy but occasionally difficulties occur. Figure 8.2 compares the two distributions. Notice that the LV case has most of its probability concentrated near the mean of 20. In the moderate-variability (MV) case, the most likely times are actually lower than the mean, around nine minutes. However, while the LV plot tails off around 40, the MV plot does not do so until around 80. Thus the means are the same, but the variances are much different. As we will see, this difference is critical to the operational performance of a workstation.

To get a sense of the operational effects of variability, suppose the LV process is feeding the MV process. For a while, the MV process will be able to keep up easily. However, once a long process time occurs, a queue of work begins to build in front of the second process. Offhand we might think that the long process times will be offset by the short process times, but this does not happen. A string of short process times at the second station might deplete the queue, causing the second station to become idle. When this occurs, capacity is lost and cannot be "saved up" for the next period of longer process times.[2]

Another way to look at this is to note that when one process feeds another, what comes in must go out; that is, there is **conservation of material.** Unless we turn off the stream of work from the first process whenever the second process is full (a procedure called *blocking* and one which we will discuss later), the amount of work in front of the second process can grow freely. Since there are times when the second station runs much faster than the first and since the *average* rate out must equal the average rate in, there will tend to be a queue of work.

We will discuss this more fully in Section 8.6. For now, we note that the greater the variability in effective process times, the greater the average queue. Given Little's law, this also implies that the greater the variability, the longer the cycle time.

[2] In the moderate-variability process shown in Figure 8.2, 20 percent of the process times are nine minutes or less, and another 20 percent are 31 minutes or more. For the mean to remain at 20, both have to occur.

8.3.3 Highly Variable Process Times

It may be hard to imagine process times whose CV is greater than 1.33. However, it is easy to construct *effective* process times with this much variability. Suppose a machine has an average process time of 15 minutes with a CV of 0.225 when there are no outages. This would be less variable than the previous low-variability case. But now suppose the machine has outages that average 248 minutes and occur, on average, after 744 minutes of production. We can show (details are given later) that this results in an effective mean process time of 20 minutes (as before) and an effective CV of a whopping 2.5! Figure 8.3 compares this high-variability (HV) distribution with the previous LV distribution. Because the HV distribution is taller and thinner, at first glance, it might appear less variable than the LV distribution. This is because we cannot see what is happening farther out in time. Once past 40 minutes or so, the picture changes. Figure 8.4 compares the distributions on a different scale for time greater than 40 minutes. Here we see the LV distribution immediately drops to almost no probability while the HV distribution appears almost uniform. It is going down very slowly indeed. This implies that there is a small probability that the process times will be extremely long. It is also the reason that the distribution for the highly variable process times appears to have a lower mean on the other plot. Most of the time, it takes around 15 minutes. However, about 1 out of every 50 jobs takes around 17 times as long. This inflates the mean to around 20 and drives the CV up to 2.5.

The effect of this level of variability on the production line can be severe. For instance, suppose the throughput is one job every 22 minutes. There should be no problem from a capacity perspective since the average process time *including* outages is 20 minutes. However, an outage of 250 minutes will build up a queue of almost 12 jobs. When the machine comes back up, the rate at which this queue is depleted is $\frac{1}{15} - \frac{1}{22} \simeq \frac{1}{47}$. Thus, the time to clear the queue formed would be around 536 minutes, *assuming no more outages occur!* If an outage occurs during this time, it adds to the queue. Under conditions commonly found with complex equipment (i.e., times to failure that are exponentially distributed), the probability of such an outage is $1 - e^{-536/744} = 0.51$. This means that more than 50 percent of the time an outage occurs before the queue would be cleared. Thus the average queue will be greater than 12 jobs and is, in fact, around 20 (as we will see in Section 8.6).

FIGURE 8.3

Comparison of high- and low-variability distributions

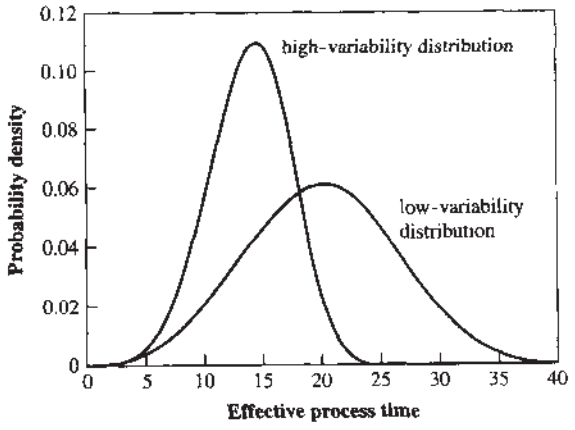
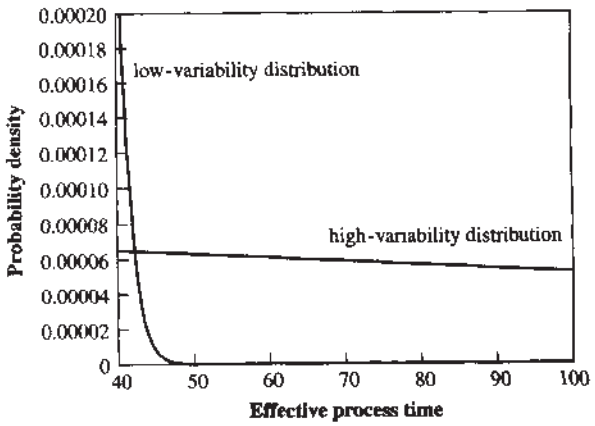


FIGURE 8.4

Comparison of high- and low-variability distributions above 40 minutes



8.4 Causes of Variability

To identify strategies for managing production systems in the face of variability, it is important to first understand the causes of variability. The most prevalent sources of variability in manufacturing environments are:

- "Natural" variability, which includes minor fluctuations in process time due to differences in operators, machines, and material.
- Random outages.
- Setups.
- Operator availability.
- Recycle.

We discuss each of these separately below.

8.4.1 Natural Variability

Natural variability is the variability inherent in **natural process time,** which excludes random downtimes, setups, or any other external influences. In a sense, this is a catch-all category, since it accounts for variability from sources that have not been explicitly called out (e.g., a piece of dust in the operator's eye). Because many of these unidentified sources of variability are operator-related, there is typically more natural variability in a manual process than in an automated one. But even in the most tightly controlled processes, there is always some natural variability. For instance, in fully automated machining operations, the composition of the material might differ, causing processing speed to vary slightly.

We let t_0 and σ_0 denote the mean and standard deviation, respectively, of natural process time. Thus, we can express the coefficient of variation of natural process time as

$$c_0 = \frac{\sigma_0}{t_0}$$

In most systems, natural process times are LV and so $c_0 < 0.75$.

Natural process times are only the starting point for evaluating effective process times. In any real production system, workstations are subject to various **detractors,** including machine downtime, setups, operator unavailability, and so on. As discussed earlier, these detractors serve to inflate both the mean *and* the standard deviation of effective process time. We now provide a way to quantify this effect.

8.4.2 Variability from Preemptive Outages (Breakdowns)

In the high-variability example discussed earlier, we saw that unscheduled downtimes can greatly inflate both the mean and the CV of effective process times. Indeed, in many systems, this is the single largest cause of variability. Fortunately, there are often practical ways to reduce its effects. Since this is a common problem, we will discuss it in detail.

We refer to breakdowns as **preemptive outages** because they occur whether we want them to or not (e.g., they can occur right in the middle of a job). Power outages, operators being called away on emergencies, and running out of consumables (e.g., cutting oil) are other possible sources of preemptive outages. Since these have similar effects on the behavior of production lines, it makes sense to combine them and treat them all as

machine breakdowns in the fashion discussed (i.e., include outages due to these other sources, as well as true machine breakdowns, when computing MTTF and MTTR). We discuss **nonpreemptive outages** (i.e., stoppages that occur between, rather than during, jobs) in the next section.

To see how machine outages cause variability, let us return to the Briar Patch Manufacturing example and provide some numerical detail. Both the Hare X19 and the Tortoise 2000 have a natural process time mean of $t_0 = 15$ minutes and a *natural* standard deviation of $\sigma_0 = 3.35$ minutes. Thus, both stations have a natural CV of $c_0 = \sigma_0/t_0 = 3.35/15.0 = 0.05$. Both machines are subject to failures and have the same long-term availability (i.e., fraction of uptime) of 75 percent. However, the Hare X19 has long but infrequent outages, while the Tortoise 2000 has short, frequent ones. Specifically, the Hare X19 has a mean time to failure (MTTF), denoted by m_f, of 12.4 hours, or 744 minutes, and a mean time to repair (MTTR), denoted by m_r, of 4.133 hours, or 248 minutes. The Tortoise 2000 has an MTTF of $m_f = 1.90$ hours, or 114.0 minutes, and MTTR of $m_r = 0.633$ hours, or 38.0 minutes. Note that the times to failure and times to repair are both three times greater for the Hare X19 than for the Tortoise 2000. Finally, we suppose that repair times are variable and have CV = 1.0 (moderate variability) for both machines.

Most capacity planning tools used in industry account for random outages when computing *average* capacity. This is done by computing the **availability,** which is given in terms of m_f and m_r by

$$A = \frac{m_f}{m_f + m_r} \tag{8.1}$$

Hence, for both machines, the availability A is

$$A = \frac{744}{744 + 248} = \frac{114}{114 + 38} = 0.75$$

Adjusting the natural process time t_0 to account for the fraction of time the machine is unavailable results in an **effective mean process time** t_e of

$$t_e = \frac{t_0}{A} \tag{8.2}$$

So in both cases, $t_e = 20$ minutes. Recall that in Chapter 7 we derived the capacity of a workstation to be the number of machines m divided by the effective mean process time. So if r_0 is the natural capacity (rate), then the **effective capacity** (rate) r_e is

$$r_e = \frac{m}{t_e} = A\frac{m}{t_0} = Ar_0 = 0.75(4 \text{ jobs/hour}) = 3 \text{ jobs/hour} \tag{8.3}$$

So the effective capacity of the Hare X19 and the Tortoise 2000 is the same. Since almost all maintenance systems used in industry to analyze breakdowns only consider the effects on availability and capacity, the two workstations would generally be regarded as equivalent.

However, when we include variability effects, the workstations are very different. To see why, consider how they will behave as part of a production line. If the Hare X19 fails for 12.4 hours (its average failure duration), it will need 12.4 hours of WIP to keep from starving. On the other hand, the Tortoise 2000 needs less than one-sixth as much WIP to be covered for an average-length failure. Since failures are, by their very nature, random, the WIP in the downstream buffer must be maintained at all times to provide protection against throughput loss. Clearly, a line with the Tortoise 2000 will be able to achieve the same level of protection, and hence the same level of throughput, with less

WIP, than same line with the Hare X19.[3] The net effect is that the line with the Hare X19 will be less efficient (i.e., will achieve lower throughput for a given WIP level or will have higher WIP and cycle time for the same throughput) than the line with the Tortoise 2000.

Earlier, we stated that the CV for the Hare X19 was 2.5. We obtained this by using a mathematical model, which we now describe. We assume the times to failures are exponentially distributed (i.e., they are MV).[4] However, we make no particular assumptions about the repair times other than that they are from some probability distribution. We define σ_r to be the standard deviation of these repair times and $c_r = \sigma_r/m_r$ to be the CV. In our example c_r is 1.0 (i.e., we assume repair times have moderate variability).

Under these assumptions we can calculate the mean, variance, and squared coefficient of variation (SCV) of the effective process time (t_e, σ_e^2, and c_e^2, respectively) as

$$t_e = \frac{t_0}{A} \tag{8.4}$$

$$\sigma_e^2 = \left(\frac{\sigma_0}{A}\right)^2 + \frac{(m_r^2 + \sigma_r^2)(1 - A)t_0}{Am_r} \tag{8.5}$$

$$c_e^2 = \frac{\sigma_e^2}{t_e^2} = c_0^2 + (1 + c_r^2)A(1 - A)\frac{m_r}{t_0} \tag{8.6}$$

The CV of effective process time c_e can be computed by taking the square root of c_e^2.

Notice that the mean effective process time, given by Equation (8.4), depends only on the mean natural process time and the availability and is hence the same for both stations:

$$t_e = \frac{t_0}{A} = \frac{15}{0.75} = 20.0 \text{ minutes}$$

However, the SCV of effective process time in Equation (8.6) depends on more than the mean process time and availability. To understand the effects involved, we can rewrite (8.6) as

$$c_e^2 = c_0^2 + A(1 - A)\frac{m_r}{t_0} + c_r^2 A(1 - A)\frac{m_r}{t_0}$$

The first term is due to the natural (unaccounted for) variability in the process. The second term is due to the fact that there are random outages. Note that this term would be there even if the outages themselves (i.e., the repair times) were constant (i.e., even if $c_r = 0$). For instance, a periodic adjustment that always takes the same time to complete would have $c_r^2 = 0$. Thus eliminating variability in repair time will do nothing to reduce this term. However, the last term is due explicitly to the variability of the repair times and would vanish if this variability were eliminated. Notice that both of the second two terms are increasing in m_r for a fixed availability. Hence, all other things being equal, long repair times induce more variability than short ones.

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[3] Actually, the line with the Hare X19 will require more than 12.4 hours of WIP, and the line with Tortoise 2000 will require more than 4.133 hours of WIP, because these are only *average* downtimes. But the point remains the same: The line with the Hare X19 requires substantially more WIP to achieve the same throughput as the line with the Tortoise 2000.

[4] This is frequently a good assumption in practice, particularly for complex equipment since such machines tend to be combinations of old and new components. Thus, the memoryless property of the exponential tends to hold for the time between *any* outage, which could be caused by failure of an old component or a new one.

Substituting numbers into these equations yields

$$c_e^2 = 0.05 + (1 + 1)0.75(1 - 0.75)\frac{248}{15} = 6.25$$

or $c_e = 2.5$, which shows that the Hare X19 is well up in the HV range. However, the Tortoise 2000 has

$$c_e^2 = 0.05 + (1 + 1)0.75(1 - 0.75)\frac{38}{15} = 1.0$$

and so $c_e = 1$, which shows that it is in the MV range.

Hence a line with the Hare X19 will exhibit much more variability than one with the Tortoise 2000. How this affects WIP and cycle time will be explored more fully in Section 8.6.

This analysis leads to the conclusion that a machine with frequent but short outages is preferable to one with infrequent but long outages, provided that the availabilities are the same. This may be somewhat contrary to our nonprobabilistic intuition, which might suggest that we would be better off with a major headache once per month than a minor throb every day. But logistically speaking, the daily throb is easier to manage.

This is a potentially valuable insight, since in practice we may be able to convert long, infrequent failures to shorter, more frequent ones (e.g., through preventive maintenance procedures). However, lest the reader become complacent—no failures at all are even better than short, frequent ones. Nothing here should be construed to deflect attention from efforts to improve overall reliability.

8.4.3 Variability from Nonpreemptive Outages

Nonpreemptive outages represent downtimes that will inevitably occur but for which we have some control as to exactly when. In contrast, a preemptive outage, which might be caused by catastrophic failure of a machine or when the machine becomes radically out of adjustment, forces a stoppage whether or not the current job is completed. An example of a nonpreemptive outage occurs when a tool starts to become dull and needs to be replaced or when the mask used to expose a circuit board begins to wear out. In situations like these we can wait until the current piece or job is finished before stopping production.

Process changeovers (setups) can be regarded as nonpreemptive outages when they occur due to changes in the production process (such as changing a mask) as opposed to changes in the product. Changeovers due to changes in product (e.g., setting up for a new part) are more under our control (we decide how many to make before changing to a new part) and are the subject of Chapters 9 and 15. Other nonpreemptive outages include preventive maintenance, breaks, operator meetings, and (we hope) shift changes. These typically occur *between* jobs, rather than *during* them. Nonpreemptive outages require somewhat different treatment than preemptive outages. Since the most common source of nonpreemptive outages is machine setups, we will frame our discussion in these terms. However, the approach is applicable to any form of nonpreemptive outage, just as our analysis of breakdowns is applicable to any form of preemptive outage.

As with preemptive outages, ordinary capacity calculations do not fully analyze the impacts of nonpreemptive setups. Average capacity analysis only tells us that short setups are better than long ones. It cannot evaluate the differences between a slow machine with short setups and a fast one with long setups that have the same effective capacity.

For example, consider the decision of whether to replace a relatively fast machine requiring periodic setups with a slower flexible machine that does not require setups. Machine 1, the fast one, can do an average of one part per hour, but requires a two-hour setup every four parts on average. Machine 2, the flexible one, requires no setups but is slower, requiring an average of 1.5 hours per part. The effective capacity r_e for machine 1 is

$$r_e = \frac{4 \text{ parts}}{6 \text{ hours}} = \frac{2}{3} \text{ parts/hour}$$

Since this is a single-machine workstation, the effective process time is simply the reciprocal of the effective capacity, so $t_e = 1.5$ hours. Thus, machines 1 and 2 have the same effective capacity.

Traditional capacity analysis, which considers only mean capacity, would consider the two machines equivalent and hence would offer no support for replacing machine 1 with machine 2. However, our previous factory physics treatment of machine breakdowns showed that considering variability can be important in evaluating machines with breakdowns. All other things being equal, machine 2 will have less variable effective process times than machine 1 (i.e., because every fourth job at machine 1 will have a long setup time included in its effective process time). Thus, replacing machine 1 with machine 2 will serve to reduce the process time CV and therefore will make the line more efficient. This *variability reduction* effect provides further support for the JIT preference for short setups and is a clear motivation for *flexible manufacturing* technology.

However, the evaluation of the benefits of flexibility can be subtle. The above condition of "all other things being equal" requires that the natural variability of both machines 1 and 2 be the same (i.e., so that the setups for machine 1 will unambiguously increase the CV of effective process times). But what if the flexible machine also has more natural variability? In this case, we must compute and compare the CV of effective process times for both machines.

To compute the CV of effective process times for a machine with setups, we first require data on the natural process times, namely, the mean t_0 and variance σ_0^2. (Equivalently, we could use the mean t_0 and the CV c_0, since $\sigma_0^2 = c_0^2 t_0^2$.) Next we must describe the setups, which we do by assuming that the machine processes an average of N_s parts (or jobs) between setups, where the setup times have a mean duration of t_s and a CV of c_s. We also assume that the probability of doing a setup after any part is equal.[5] That is, if an average of 10 parts are processed between setups, there will be a 1-in-10 chance that a setup will be performed after the current part, regardless of how many have been done since the last setup.

Under these assumptions, the equations for the mean, variance, and SCV of effective process time are, respectively,

$$t_e = t_0 + \frac{t_s}{N_s} \tag{8.7}$$

$$\sigma_e^2 = \sigma_0^2 + \frac{\sigma_s^2}{N_s} + \frac{N_s - 1}{N_s^2} t_s^2 \tag{8.8}$$

$$c_e^2 = \frac{\sigma_e^2}{t_e^2} \tag{8.9}$$

To illustrate the usefulness of these equations, consider another example that compares two machines. Machine 1 is a flexible machine, with no setups, but has somewhat

[5]This assumption implies that the number of parts processed between setups is moderately variable (i.e., the mean and standard deviation are equal). Similar analysis can be done for other assumptions regarding the variability of the time between setups.

variable process times. Specifically, the natural process time has a mean of $t_0 = 1.2$ hours and a CV of $c_0 = 0.5$. Machine 2 performs an average of $N_s = 10$ parts between setups and has natural process times with a mean of $t_0 = 1.0$ hours and a CV of $c_0 = 0.25$. The average setup time is $t_s = 2$ hours with a CV of $c_s = 0.25$. Which machine is better?

First, consider the effective capacity. Machine 1 has

$$r_e = \frac{1}{t_0} = \frac{1}{1.2} = 0.833$$

while machine 2 has

$$r_e = \frac{1}{t_e} = \frac{1}{1 + \frac{2}{10}} = 0.833$$

so the two machines are equivalent in this regard. Therefore, the question of which is better becomes, Which machine has less variability?

Using Equation (8.9), we can compute $c_e^2 = 0.31$ for machine 2, as compared to $c_e^2 = c_0^2 = 0.25$ for machine 1. Thus, machine 1, the more variable machine without setups, has less overall variability than machine 2, the less variable machine with setups.

Of course, this conclusion was a consequence of the specific numbers in the example. Flexible machines do not always have less variability. For instance, consider what happens if machine 2 has a shorter setup ($t_s = 1$ hour) after an average of $N_s = 5$ parts. The effective capacity remains unchanged. However, the effective variability for machine 2 is significantly less, with $c_e^2 = 0.16$. In this case, machine 2 with setups would be the better choice.

8.4.4 Variability from Recycle

Another major source of variability in manufacturing systems is quality problems. The simplest quality case to analyze is that of rework on a single workstation. This happens when a workstation performs a task and then checks to see whether the task was done correctly. If it was not, the task is repeated. If we think of the additional processing time spent "getting the job right" as an outage, it is easy to see that this situation is equivalent to the nonpreemptive outage case. Hence, rework has analogous effects to those of setups, namely, that it both robs capacity and contributes greatly to the variability of the effective process times.

As with breakdowns and setups, the traditional reason for reducing rework is to prevent a loss of effective capacity (i.e., reduce waste). Of course, as with traditional analyses of breakdowns and setups, this perspective would regard two machines with the same effective capacity but different rework fractions as equivalent. However, an analysis like that done above for setups shows that the CV of effective process times increases as the fraction of rework increases. Hence, more rework implies more variability. More variability causes more congestion, WIP, and cycle time. Hence, these variability impacts, coupled with the loss of capacity, make rework a disruptive problem indeed. We will return to this important interface between quality and operations in greater detail in Chapter 12.

8.4.5 Summary of Variability Formulas

The computations for t_e, σ_e^2, and c_e^2 for both the preemptive and the nonpreemptive cases are summarized in Table 8.2. Note that if we have a situation involving both preemptive and nonpreemptive outages (e.g., both breakdowns and setups), then these formulas must be applied consecutively. For instance, we begin with the natural process time

TABLE 8.2 Summary of Formulas for Computing Effective Process Time Parameters

Situation	Natural	Preemptive	Nonpreemptive
Examples	**Reliable Machine**	**Random Failures**	**Setups; Rework**
Parameters	t_0, c_0^2 (basic)	Basic plus m_f, m_r, c_r^2	Basic plus N_s, t_s, c_s^2
t_e	t_0	$\frac{t_0}{A}, A = \frac{m_f}{m_f + m_r}$	$t_0 + \frac{t_s}{N_s}$
σ_e^2	$t_0^2 c_0^2$	$\frac{\sigma_0^2}{A^2} + \frac{(m_r^2 + \sigma_r^2)(1 - A)t_0}{Am_r}$	$\sigma_0^2 + \frac{\sigma_s^2}{N_s} + \frac{N_s - 1}{N_s^2} t_s^2$
c_e^2	c_0^2	$c_0^2 + (1 + c_r^2)A(1 - A)\frac{m_r}{t_0}$	$\frac{\sigma_e^2}{t_e^2}$

parameters t_0 and c_0^2. Then we incorporate the effects of failures by computing t_e, σ_e, and c_e^2 for the effective process times, using the preemptive outage formulas. Finally, we incorporate the effects of setups by using these values of t_e, σ_e, and c_e^2 in place of t_0, σ_e, and c_0^2 in the nonpreemptive outage formulas. The final mean t_e, standard deviation σ_e, and SCV c_e^2 will thus be "inflated" to reflect both types of outage.

8.5 Flow Variability

All the above discussion focused solely on process time variability at individual workstations. But variability at one station can affect the behavior of other stations in a line by means of another type of variability, which we call **flow variability.** Flows refer to the transfer of jobs or parts from one station to another. Clearly if an upstream workstation has highly variable process times, the flows it feeds to downstream workstations will also be highly variable. Therefore, to analyze the effect of variability on the line, we must characterize the variability in flows.

8.5.1 Characterizing Variability in Flows

The starting point for studying flows is the arrival of jobs to a single workstation. The departures from this workstation will in turn be arrivals to other workstations. Therefore, once we have described the variability of arrivals to one workstation and determined how this affects the variability of departures from that workstation (and hence arrivals to other workstations), we will have characterized the flow variability for the entire line.

The first descriptor of arrivals to a workstation is the **arrival rate,** measured in jobs per unit time. For consistency, the units of arrival rate must be the same as those of capacity. For instance, if we state capacities of workstations in units of jobs per hour, then arrival rates must also be stated in jobs per hour. Then just as we can characterize capacity by either the mean process time t_e or the average rate of the station r_e, we can characterize the arrival rate to the station by either the **mean time between arrivals,** which we denote by t_a, or the average arrival rate, denoted by r_a. These two measures

are simply the inverse of each other

$$r_a = \frac{1}{t_a}$$

and so are entirely equivalent as information.

In order for the workstation to be able to keep up with arrivals, it is essential that capacity exceed the arrival rate, that is,

$$r_e > r_a$$

In virtually all realistic cases (i.e., those with variability present), the capacity must be *strictly* greater than the arrival rate to keep the station from becoming overloaded. We will examine why more precisely below.

Just as there is variability in process times, there is also variability in interarrival times. A reasonable variability measure for interarrival times can be defined in exactly the same way as for process times. If σ_a is the standard deviation of the time between arrivals, then the coefficient of variation of the interarrival times c_a is

$$c_a = \frac{\sigma_a}{t_a}$$

We refer to this as the **arrival CV,** to distinguish it from the **process time CV,** denoted by c_e. Intuitively, a low arrival CV indicates regular, or evenly spaced, arrivals, while a high arrival CV indicates uneven, or "bursty" arrivals. The difference is illustrated in Figure 8.5. The arrival CV c_a, along with the mean interarrival time t_a, summarizes the essential aspects of the arrival process to a workstation.

The next step is to characterize the departures from a workstation. We can use measures analogous to those used to describe arrivals, namely, the **mean time between departures** t_d, the **departure rate** $r_d = 1/t_d$, and the departure CV c_d. In a serial production line, where all the output from workstation i becomes input to workstation $i + 1$, the departure rate from i must equal the arrival rate to $i + 1$, so

$$t_a(i + 1) = t_d(i)$$

Indeed, in a serial production line without yield loss or rework, the arrival rate to *every* workstation is equal to the throughput TH. Also, in a serial line where departures from i become arrivals to $i + 1$, the departure CV of workstation i is the same as the arrival CV of workstation $i + 1$

$$c_a(i + 1) = c_d(i)$$

These relationships are depicted graphically in Figure 8.6.

FIGURE 8.5
Arrival processes with low and high CVs

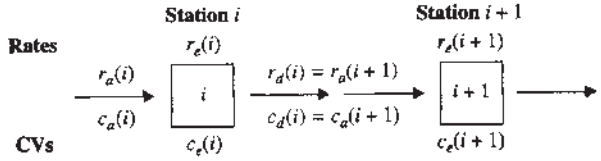

FIGURE 8.6
Propagation of variability between workstations in series

The one remaining issue to resolve concerning flow variability is how to characterize the variability of departures from a station in terms of information about the variability of arrivals and process times. Variability in departures from a station are the result of both variability in arrivals to the station and variability in the process times. The relative contribution of these two factors depends on the **utilization** of the workstation. Recall that the utilization of a workstation, denoted by u, is the fraction of time it is busy over the long run and is defined formally for a workstation consisting of m identical machines as

$$u = \frac{r_a t_e}{m}$$

Notice that u increases with both the arrival rate and the mean effective process time. An obvious upper limit on the utilization is one (that is, 100 percent), which implies that the effective process times must satisfy

$$t_e < \frac{m}{r_a}$$

If u is close to one, then the station is almost always busy. Therefore, under these conditions, the interdeparture times from the station will be essentially identical to the process times. Thus, we would expect the departure CV to be the same as the process time CV (that is, $c_d = c_e$).

At the other extreme, when u is close to zero, the station is very lightly loaded. Virtually every time a job is finished, the station has to wait a long time for another arrival to work on. Because process time is a small fraction of the time between departures, interdeparture times will be almost identical to interarrival times. Thus, under these conditions we would expect the arrival and departure CVs to be the same (that is, $c_d = c_a$).

A good, simple method for interpolating between these two extremes is to use the square of the utilization as follows:[6]

$$c_d^2 = u^2 c_e^2 + (1 - u^2) c_a^2 \tag{8.10}$$

If the workstation is always busy, so that $u = 1$, then $c_d^2 = c_e^2$. Similarly, if the machine is (almost) always idle, so that $u = 0$, then $c_d^2 = c_a^2$. For intermediate utilization levels, $0 < u < 1$, the departure SCV c_d^2 is a combination of the arrival SCV c_a^2 and the process time SCV c_e^2.

When there is more than one machine at a station (that is, $m > 1$), the following is a reasonable way to estimate c_d^2 (although there are others; see Buzacott and Shanthikumar 1993):

$$c_d^2 = 1 + (1 - u^2)(c_a^2 - 1) + \frac{u^2}{\sqrt{m}}(c_e^2 - 1) \tag{8.11}$$

Note that this reduces to Equation (8.10) when $m = 1$.

The net result is that flow variability, like process time variability, can vary widely in practical situations. Using the same classification scheme we used for process time variability, we can classify arrivals according to the arrival CV c_a as follows:

Low variability (LV)	$c_a \leq 0.75$
Moderate variability (MV)	$0.75 < c_a \leq 1.33$
High variability (HV)	$c_a > 1.33$

Departures can be classified in the same manner according to the departure CV c_d.

For example, departures from a heavily loaded LV workstation will tend to be LV, while departures from a heavily loaded HV workstation will tend to be HV. MV

[6]Notice that once again an equation involving CVs is written in terms of their SCVs.

workstations fed by MV arrivals will produce MV departures. All these departures in turn become arrivals to other stations, so all types of arrivals can occur in practice.

Another way that MV arrivals can arise in practice is when a workstation is fed by many sources. For instance, a heat-treating operation may receive jobs from many different lines. When this is the case, the time since the last arrival does not provide much information about when the next arrival is likely to occur (because it could come from many places). Thus, the interarrival times will tend to be *memoryless* (i.e., exponential), and therefore c_a will be close to one. Even when the arrivals from any given source are quite regular (i.e., LV), the *superposition* of all the arrivals tends to look MV.

8.5.2 Batch Arrivals and Departures

One important cause of flow variability is **batch arrivals.** These happen whenever jobs are batched together for delivery to a station. For example, suppose a forklift brings 16 jobs once per shift (eight hours) to a workstation. Since arrivals always occur in this way with no randomness whatever, one might reasonably interpret the variability and the CV to be zero.

However, a very different picture results from looking at the interarrival times of the jobs in the batch from the perspective of the individual jobs. The interarrival time (i.e., time since the previous arrival) for the first job in the batch is eight hours. For the next 15 jobs it is zero. Therefore, the mean time between arrivals t_a is one-half hour (eight hours divided by 16 jobs), and the variance of these times is given by

$$\sigma_a^2 = [\tfrac{1}{16}(8^2) + \tfrac{15}{16}(0^2)] - t_a^2 = \tfrac{1}{16}(8^2) - 0.5^2 = 3.75$$

The arrival SCV is therefore

$$c_a^2 = \frac{3.75}{(0.5)^2} = 15$$

In general, if we have a batch size k, this analysis will yield $c_a^2 = k - 1$.

So which is correct, $c_a^2 = 15$ or $c_a^2 = 0$? The answer is that the system will behave "somewhere in between." The reason is that batching confounds two different effects. The first effect is due to the batching itself. This is not really a randomness issue, but rather one of *bad control*, like that we discussed for the worst case in Chapter 7. The second is the variability in the batch arrivals themselves (i.e., as characterized by the arrival CV for the batches). We will examine the relationship between batching and variability more carefully in Chapter 9.

8.6 Variability Interactions—Queueing

The above results for process time variability and flow variability are building blocks for characterizing the effects of variability in the overall production line. We now turn to the problem of evaluating the impact of these types of variability on the key performance measures for a line, namely, WIP, cycle time, and throughput.

To do this, we first observe that actual process time (including setups, downtime, etc.) typically represents only a small fraction (5 to 10 percent) of the total cycle time in a plant. This has been documented in numerous published surveys (e.g., Bradt 1983). The majority of the extra time is spent *waiting* for various resources (e.g., workstations, transport devices, machine operators, etc.). Hence, a fundamental issue in factory physics is to understand the underlying causes of all this waiting.

The science of waiting is called **queueing theory.** In Great Britain, people do not stand in line, they stand in a **queue.** So, queueing theory is the theory of standing in lines.[7] Since jobs "stand in line" while waiting to be processed, waiting to move, waiting for parts, and so on, queueing theory is a powerful tool for analyzing manufacturing systems.

A **queueing system** combines the components that have been considered so far: an arrival process, a service (i.e., production) process, and a queue. Arrivals can consist of individual jobs or batches. Jobs can be identical or have different characteristics. Interarrival times can be constant or random. The workstation can have a single machine or several machines in parallel, which can have constant or random process times. The queueing discipline can be first-come first-served (FCFS), last-come first-served (LCFS), earliest due date (EDD), shortest process time (SPT), or any of a host of priority schemes. The queue space can be unlimited or finite. The variety of queueing systems is almost endless.

Regardless of the queueing system under consideration, the job of queueing theory is to characterize performance measures in terms of descriptive parameters. We do this below for a few queueing systems that are most applicable to manufacturing settings.

8.6.1 Queueing Notation and Measures

To use queueing theory to describe the performance of a single workstation, we will assume we know the following parameters:

r_a = rate of arrivals in jobs per unit time to station. In a serial line without yield loss or rework, r_a = TH at every workstation.

$t_a = 1/r_a$ = average time between arrivals

c_a = arrival CV

m = number of parallel machines at station

b = buffer size (i.e., maximum number of jobs allowed in system)

t_e = mean effective process time. The rate (capacity) of the workstation is given by $r_e = m/t_e$.

c_e = CV of effective process time

The performance measures we will focus on are

p_n = probability there are n jobs at station

CT_q = expected waiting time spent in queue

CT = expected time spent at station (i.e., queue time plus process time)

WIP = average WIP level (in jobs) at station

WIP_q = expected WIP (in jobs) in queue

In addition to the above parameters, a queueing system is characterized by a host of specific assumptions, including the type of arrival and process time distributions, dispatching rules, balking protocols, batch arrivals or processing, whether it consists of a network of queueing stations, whether it has single or multiple job classes, and many others. A partial classification of single-station, single-job-class queueing systems is given by *Kendall's notation,* which characterizes a queueing station by means of four parameters:

$$A/B/m/b$$

[7] **Queueing** is also the only word we can think of with five vowels in a row, which could be useful if one is a contestant on a game show.

where A describes the distribution of interarrival times, B describes the distribution of process times, m is the number of machines at the station, and b is the maximum number of jobs that can be in the system. Typical values for A and B, along with their interpretations, are

D: constant (deterministic) distribution

M: exponential (Markovian) distribution

G: completely general distribution (e.g., normal, uniform)

In many situations, queue size is not explicitly restricted (e.g., the buffer is very large). We indicate this case as $A/B/m/\infty$ or simply as $A/B/m$.

For example, the $M/G/3$ queueing system refers to a three-machine station with exponentially distributed interarrival times and generally distributed process times and an infinite buffer.

We will focus initially on the $M/M/1$ and $M/M/m$ queueing systems because they yield important intuition and serve as building blocks for more general systems. We will then consider the $G/G/1$ and $G/G/m$ queueing systems because they are directly useful for modeling manufacturing workstations. Finally, we discuss what happens when we limit the buffer in the $M/M/1/b$ and the $G/G/1/b$ cases.

For simplicity, we will restrict our consideration to systems with a single job class (i.e., a single product). Of course, most manufacturing systems have multiple products. But we can develop the key insights into the role of variability in production systems with single-job-class models. Moreover, these models can sometimes be used to approximate the behavior of multiple-job-class systems. Details on how to do this and the development of more sophisticated multiple-job-class models are given in Buzacott and Shanthikumar (1993).

8.6.2 Fundamental Relations

Before considering specific queueing systems, we note that some important relationships hold for all single-station systems (i.e., regardless of the assumptions about arrival and process time distributions, number of machines, etc.). First is the expression for **utilization,** which is the probability that the station is busy, and is given by

$$u = \frac{r_a}{r_e} = \frac{r_a t_e}{m} \tag{8.12}$$

Second is the relation between mean total time spent at the station CT and mean time spent in queue CT_q. Since means are additive,

$$\mathrm{CT} = \mathrm{CT}_q + t_e \tag{8.13}$$

Third, applying Little's law to the station yields a relation among WIP, CT, and the arrival rate:

$$\mathrm{WIP} = \mathrm{TH} \times \mathrm{CT} \tag{8.14}$$

And fourth, applying Little's law to the queue alone yields a relation among WIP_q, CT_q, and the arrival rate:

$$\mathrm{WIP}_q = r_a \times \mathrm{CT}_q \tag{8.15}$$

Using the above relations and knowledge of any one of the four performance measures (CT, CT_q, WIP, or WIP_q), we can compute the other three.

8.6.3 The *M/M/1* Queue

One of the simplest queueing systems to analyze is the $M/M/1$. This model assumes exponential interarrival times, a single machine with exponential process times, a first-come first-served protocol, and unlimited space for jobs waiting in queue. While not an accurate representation of most manufacturing workstations, the $M/M/1$ queue is tractable and offers valuable insight into more complex and realistic systems.

The key to analyzing the $M/M/1$ queue is the *memoryless property* of the exponential distribution. To see why, consider what information is needed to characterize the future (probabilistic) evolution of the system. That is, what do we need to know about the current status of the system in order to answer such questions as How likely is it that the system will be empty by a certain time? or How likely is it that a job will wait less than a specified amount of time before being served? The issue is not *how* to compute the answers to such questions, but simply *what information* about the system would be needed to do so.

To begin, we require information about the interarrival and process times. Since both are assumed to be exponential, all we need to know are the means (i.e., because the standard deviation is equal to the mean for the exponential distribution). The mean time between arrivals is t_a, so that the arrival rate is $r_a = 1/t_a$. The mean process time is t_e, so the process rate is $r_e = 1/t_e$.

Beyond these, the *only* other information we need is how many jobs are currently in the system. Because the interarrival and process time distributions are memoryless, the time since the last arrival and the time the current job has been in process are irrelevant to the future behavior of the system. Because of this, the **state** of the system can be expressed as a single number n, representing the number of jobs currently in the system. By computing the long-run probability of being in each state, we can characterize all the long-term (steady state) performance measures, including CT, WIP, CT_q, and WIP_q. We do this for the $M/M/1$ queue in the following Technical Note.

Technical Note

Define p_n to be the long-run probability of finding the system in state n (i.e., with a total of n jobs in process and in queue).[8] Since jobs arrive one at a time and the machine works on only one job at a time, the system state can only change by one unit at a time. For instance, if there are currently n jobs at the station, then the only possible state changes are an increase to $n + 1$ (an arrival) or a decrease to $n - 1$ (a departure). The rate the system moves from state n to state $n + 1$, **given it is currently in state** n, is r_a, the arrival rate. Likewise, the conditional rate to move from n to $n - 1$, **given the system is currently in state** n, is r_e, the process rate. The dynamics of the system are graphically illustrated in Figure 8.7.

It follows that the unconditional (i.e., steady-state) **rate** at which the system moves from state $n - 1$ to state n is given by $p_{n-1}r_a$, that is, the probability of being in state $n - 1$ times the rate from $n - 1$ to n, given the system is in state n. Similarly, the rate at which the system moves from state n to state $n - 1$ is $p_n r_e$. In order for the system to be stable, these two rates must be equal (i.e., otherwise the probability of being in any given state would "drift" over time). Hence,

$$p_{n-1}r_a = p_n r_e$$

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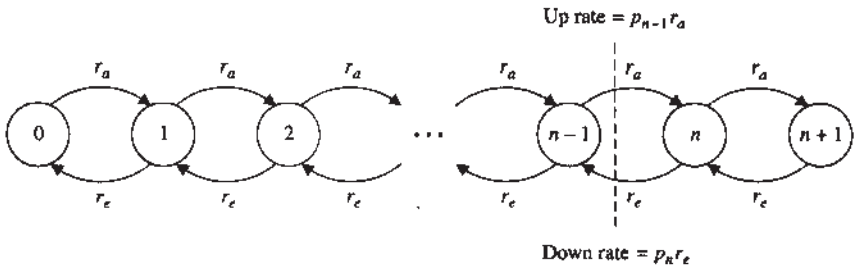
[8] These probabilities are only meaningful in **steady state** (i.e., after the system has been running so long that the current state does not depend on the starting conditions). This means that we can only compute long-term measures from the p_n values. Fortunately, our key measures CT, WIP, CT_q, and WIP_q are long-term measures. Analysis of the **transient** (i.e., short-term) behavior of queueing systems is difficult and will not be discussed here.

FIGURE 8.7
State transition diagram for $M/M/1$ queue



or
$$p_n = \frac{r_a}{r_e} p_{n-1} = u p_{n-1} \tag{8.16}$$

where $u = r_a t_e = r_a / r_e$ is the utilization which, if there is no blocking, will be the long-run fraction of time the machine is busy.

By the definition of utilization, it follows that the probability (long-run fraction of time) that the station is not busy is $1 - u$. Since the machine is only idle when there are no jobs in the system, this implies that $p_0 = 1 - u$. This gives us one of the p_n values. To get the rest, we write out Equation (8.16) for $n = 1, 2, 3, \ldots$, which yields

$$\begin{aligned} p_1 &= u p_0 = u(1-u) \\ p_2 &= u p_1 = u \cdot u(1-u) = u^2(1-u) \\ p_3 &= u p_2 = u \cdot u^2(1-u) = u^3(1-u) \\ &\vdots \end{aligned}$$

Continuing in this manner shows that for any state

$$p_n = u^n(1-u) \qquad n = 0, 1, 2, \ldots \tag{8.17}$$

These p_n values are probabilities and therefore must sum to 1, so

$$p_0 + p_1 + p_2 + \cdots = (1 + u + u^2 + \cdots) p_0 = 1$$

or
$$p_0 = 1 - u \tag{8.18}$$

However, if $u \geq 1$, then the sum in the parentheses will be infinite, which violates the properties of probabilities. Therefore, in order for the station to have stable long-run behavior (i.e., not have a queue that "blows up"), we must have $u < 1$ (i.e., utilization *strictly* less than 100 percent).[9]

The most straightforward performance measure to compute is WIP (i.e., expected number in the system). For the $M/M/1$ case

$$\begin{aligned} \text{WIP} &= \sum_{n=0}^{\infty} n p_n \\ &= (1-u) \sum_{n=0}^{\infty} n u^n \\ &= u(1-u) \sum_{n=1}^{\infty} n u^{n-1} \end{aligned} \tag{8.19}$$

[9] If $u < 1$, then by noting that $1 + u + u^2 + \cdots = 1 + u(1 + u + u^2 + \cdots)$ and letting $x = 1 + u + u^2 + \cdots$, we see that $x = 1 + ux$. Solving for x yields $1 - ux = 1$, or $x = (1-u)^{-1}$. Since $x p_0 = 1$, this shows that $p_0 = 1 - u$, as we showed above by considering utilization.

It is easy to show that $\sum_{n=1}^{\infty} nu^{n-1} = (1-u)^{-2}$, so Equation (8.19) yields a concise expression for WIP.[10]

8.6.4 Performance Measures

The various steady-state performance measures can be computed from the results derived in the Technical Note. The expression for expected WIP follows from Equation (8.19) and is given by

$$\text{WIP}(M/M/1) = \frac{u}{1-u} \tag{8.20}$$

Using this and Little's law yields a relation for average cycle time

$$\text{CT}(M/M/1) = \frac{\text{WIP}(M/M/1)}{r_a} = \frac{t_e}{1-u} \tag{8.21}$$

Then from Equation (8.13) we can compute the average time in queue

$$\text{CT}_q(M/M/1) = \text{CT}(M/M/1) - t_e = \frac{u}{1-u} t_e \tag{8.22}$$

Finally, for the WIP in queue, Little's law again yields

$$\text{WIP}_q(M/M/1) = r_a \times \text{CT}_q(M/M/1) = \frac{u^2}{1-u} \tag{8.23}$$

Observe that WIP, CT, CT_q, and WIP_q are all increasing in u. Not surprisingly, busy systems exhibit more congestion than lightly loaded systems. Also, for a fixed u, CT and CT_q are increasing in t_e. Hence, for a given level of utilization, slower machines cause more waiting time. Finally, notice that since these expressions have the term $1-u$ in the denominator, all the congestion measures "explode" as u gets close to one. What this means is that WIP levels and cycle times increase very rapidly (i.e., nonlinearly) as utilization approaches 100 percent. We will discuss the implications of this in greater detail in Chapter 9.

Example:
Recall that in the Briar Patch Manufacturing example, the arrival rate to the Tortoise 2000 was 2.875 jobs per hour ($r_a = 2.875$). Assume now that times between arrival are exponentially distributed (not a bad assumption if jobs are arriving from many different locations). Also, recall that the production rate is three jobs per hour (or $t_e = \frac{1}{3}$) and that $c_e = 1.0$. Since the effective process times have a CV of one, just as the exponential distribution does, it is reasonable to use the $M/M/1$ model to represent the Tortoise 2000.[11] The utilization is computed as $u = 2.875/3 = 0.9583$, and the performance measures are given below:

$$\text{WIP} = \frac{u}{1-u} = \frac{0.9583}{1-0.9583} = 23 \text{ jobs}$$

$$\text{CT} = \frac{\text{WIP}}{\text{TH}} = \frac{23}{2.875} = 8 \text{ hours}$$

[10] This is because $\sum_{n=1}^{\infty} nu^n$ is the derivative of $\sum_{n=0}^{\infty} u^n$, which we saw is equal to $1/(1-u)$. Since the derivative of the sum is the sum of the derivatives, $\sum_{n=1}^{\infty} nu^n$ is equal to the derivative of $1/(1-u)$, which is $1/(1-u)^2$. Notice that this is only valid as long as $u < 1$, which was already required for the queue to be stable.

[11] The process times are not actually exponential, however, since $c_e = 1$ was the result of failures superimposed on low-variability natural process times. So the $M/M/1$ queue is not exact, but will be a reasonable approximation.

$$\text{CT}_q = \text{CT} - t_e = 8 - 0.3333 = 7.6667 \text{ hours}$$

$$\text{WIP}_q = \text{TH} \times \text{CT}_q = 2.875 \times 7.6667 = 22.0417 \text{ jobs}$$

We see that WIP and CT are much smaller than those for the Hare X19 under the same demand conditions. However, to model the nonexponential Hare X19, we need a more general model than the $M/M/1$.

8.6.5 Systems with General Process and Interarrival Times

Most real-world manufacturing systems do not satisfy the assumptions of the $M/M/1$ queueing model. Process times are seldom exponential. When workstations are fed by upstream stations whose process times are not exponential, interarrival times are also unlikely to be exponential. To address systems with nonexponential interarrival and process time distributions, we must turn to the $G/G/1$ queue.

Unfortunately, without the memoryless property of the exponential to facilitate analysis, we cannot compute exact performance measures for the $G/G/1$ queue. But we can estimate them by means of a "two-moment" approximation, which makes use of only the mean and standard deviation (or CV) of the interarrival and process time distributions. Although cases can be constructed for which this approximation works poorly, it is reasonably accurate in typical manufacturing systems (i.e., for most cases except those with c_e and c_a much larger than one, or u larger than 0.95 or smaller than 0.1). Because it works well, this approximation is the basis of several commercially available manufacturing queueing analysis packages.

As we did for the $M/M/1$ case, we will proceed by first developing an expression for the waiting time in queue CT_q and then computing the other performance measures. The approximation for CT_q, which was first investigated by Kingman (1961) (see Medhi 1991 for a derivation), is given by

$$\text{CT}_q(G/G/1) = \left(\frac{c_a^2 + c_e^2}{2}\right)\left(\frac{u}{1-u}\right)t_e \tag{8.24}$$

This approximation has several nice properties. First, it is exact for the $M/M/1$ queue.[12] It also happens to be exact for the $G/G/1$ queue, although this is not evident from our discussion here. Finally, it neatly separates into three terms: a dimensionless **variability term** V, a **utilization term** U, and a **time term** T, as

$$\text{CT}_q(G/G/1) = \underbrace{\left(\frac{c_a^2 + c_e^2}{2}\right)}_{V}\underbrace{\left(\frac{u}{1-u}\right)}_{U}\underbrace{t_e}_{T}$$

or

$$\text{CT}_q = VUT \tag{8.25}$$

We refer to this as **Kingman's equation** or as the ***VUT* equation.** From it, we see that if the V factor is less than one, then the queue time, and hence other congestion measures, for the $G/G/1$ queue will be smaller than those for the $M/M/1$ queue. Conversely, if V is greater than one, congestion will be greater than in the $M/M/1$ queue. Thus, the VUT equation shows that the $M/M/1$ case represents an intermediate case for single stations analogous to that represented by the practical worst case for lines.

[12] When c_a and c_e are both equal to one, the first fraction becomes one and the other term is the waiting time in queue for the $M/M/1$ queue $\text{CT}_q(M/M/1)$.

Example:
Let us return to the Briar Patch Manufacturing example and consider the Hare X19. Recall that this machine has high variability ($c_e^2 = 6.25$). Again, assume the time between job arrivals is exponential (that is, $c_a^2 = 1$). Utilization of the Hare X19 is $u = 0.9583$. Hence, we can use the VUT equation to compute the expected queue time as

$$\begin{aligned} \mathrm{CT}_q &= \left(\frac{c_a^2 + c_e^2}{2}\right)\left(\frac{u}{1-u}\right)t_e \\ &= \left(\frac{1+6.25}{2}\right)\left(\frac{0.9583}{1-0.9583}\right)20 \\ &= 1{,}667.5 \text{ minutes} = 27.79 \text{ hours} \end{aligned}$$

which is what we reported in the introduction to the chapter.

Now suppose that the Hare X19 feeds the Tortoise 2000. There is no yield loss, so the rate into the Tortoise 2000 is the same as that into the Hare X19; and since the two machines have the same effective rate, they will have the same utilization $u = 0.9583$. However, to use the VUT equation, we must find the arrival CV c_a to the Tortoise 2000. We do this by first finding the departure CV from the Hare c_d by using linking Equation (8.10)

$$\begin{aligned} c_d^2 &= c_e^2 u^2 + c_a^2(1-u^2) \\ &= 6.25(0.9583^2) + 1.0(1-0.9583^2) \\ &= 5.8216 \end{aligned}$$

Since the Hare X19 feeds the Tortoise 2000, c_a^2 for the Tortoise 2000 is equal to c_d^2 for the Hare X19. Hence, the expected queue time at the Tortoise 2000 will be

$$\begin{aligned} \mathrm{CT}_q &= \left(\frac{c_a^2 + c_e^2}{2}\right)\left(\frac{u}{1-u}\right)t_e \\ &= \left(\frac{5.82+1.0}{2}\right)\left(\frac{0.9583}{1-0.9583}\right)20 \\ &= 1{,}568.97 \text{ minutes} = 26.15 \text{ hours} \end{aligned}$$

which again is what we reported in the introduction.

Notice that the queue time at the Tortoise 2000 is almost as large as that for the Hare X19, even though the Hare X19 has much higher process variability. The reason for this is the high variability of arrivals to the Tortoise 2000 ($c_a = \sqrt{5.8216} = 2.41$). If the Tortoise 2000 were fed by moderately variable arrivals (with $c_a = 1.0$), then its performance would be represented by the $M/M/1$ queue, which predicts average queue time of 7.67 hours. The excess time (and congestion) is a consequence of the propagation of variability from the upstream Hare X19.

8.6.6 Parallel Machines

The VUT equation gives us a tool for analyzing workstations consisting of single machines. However, in real-world systems, workstations often consist of multiple machines in parallel. The reason, of course, is that often more than a single machine is required to achieve the desired workstation capacity. To analyze and understand the behavior of parallel machine stations, we need a more general model.

The simplest type of parallel machine station is the case in which interarrival times are exponential ($c_a = 1$) and process times are exponential ($c_e = 1$). This corresponds

to the $M/M/m$ queueing system. In this model, all jobs wait in a single queue for the next available machine (unlike in most grocery stores where each server has a separate queue, but like in most banks where there is a single queue for all the servers). Although the steady-state probabilities for the $M/M/m$ queue *can* be computed exactly, they are messy and provide little additional intuition. More useful is the following closed-form approximation for the waiting time in queue proposed by Sakasegawa (1977) that both offers intuition and is quite accurate (see Whitt (1993) for a discussion of its merits and uses):

$$\mathrm{CT}_q(M/M/m) = \frac{u^{\sqrt{2(m+1)}-1}}{m(1-u)} t_e \tag{8.26}$$

Note that when $m = 1$, this expression reduces to Equation (8.22), which is the exact expression for queue time in the $M/M/1$ queue. Using this expression, along with universal relations (8.13) to (8.15), we can obtain expressions for $\mathrm{CT}(M/M/m)$, $\mathrm{WIP}(M/M/m)$, and $\mathrm{WIP}_q(M/M/m)$.

Example:
Consider the Briar Patch Manufacturing example again. Recall that the Tortoise 2000 had process times with $c_e = 1$ and hence is well approximated by an exponential model. Suppose now, however, that arrivals to the Tortoise 2000 occur at a rate of 207 jobs per day and have exponential interarrival times ($c_a = 1$). Since this is beyond the capacity of a single Tortoise 2000, we now assume that Briar Patch Manufacturing has three machines.

First, consider what would happen if each of the three machines had its own arrival stream. That is, each machine sees one-third of the total demand, or 69 jobs per day (2.875 jobs per hour). Since process times are one-third hour, the utilization of each machine is $u = 2.875(\frac{1}{3}) = 0.958$. Hence, the situation for each machine is precisely that which we modeled in Section 8.6.4, where we computed the average time in queue to be 7.67 hours.

Now suppose that the three Tortoise 2000s are combined into a single station so that the entire demand of 207 jobs per day, or 8.625 jobs per hour, arrives to a single queue that is serviced by the three machines in parallel. Utilization is the same, since

$$u = \frac{r_a t_e}{m} = \frac{(8.625)(\frac{1}{3})}{3} = 0.958$$

However, average time in queue is now

$$\begin{aligned}\mathrm{CT}_q &= \frac{u^{\sqrt{2(m+1)}-1}}{m(1-u)} t_e \\ &= \frac{(0.958)^{\sqrt{2(3+1)}-1}}{3(1-0.958)} \left(\frac{1}{3}\right) = 2.467 \text{ hours}\end{aligned}$$

which is significantly lower than the case where the three machines had separate queues. We conclude that when variability and utilization are the same, a station with parallel machines will outperform one with dedicated machines. The reason, as anyone who has ever chosen the wrong line at the grocery store knows, is that a long process time will delay everyone waiting in the queue at a dedicated machine. When the queue is combined, as at the bank, the machine experiencing a long process time gets bypassed and therefore does not have such a damaging effect on average queue time. This is an example of the more general property of **variability pooling,** which we discuss in Section 8.8.

8.6.7 Parallel Machines and General Times

A parallel machine station with general (nonexponential) process and interarrival times is represented by a $G/G/m$ queue. To develop an approximation for this situation, note that approximation (8.24) can be rewritten as

$$\mathrm{CT}_q(G/G/1) = \left(\frac{c_a^2 + c_e^2}{2}\right) \mathrm{CT}_q(M/M/1)$$

where $\mathrm{CT}_q(M/M/1) = [u/(1-u)]t_e$ is the waiting time in queue for the $M/M/1$ queue. This suggests the following approximation for the $G/G/m$ queue (see Whitt 1983 for a discussion)

$$\mathrm{CT}_q(G/G/m) = \left(\frac{c_a^2 + c_e^2}{2}\right) \mathrm{CT}_q(M/M/m) \tag{8.27}$$

Using Equation (8.26) to approximate $\mathrm{CT}_q(M/M/m)$ in Equation (8.27) yields the following closed-form expression for the waiting time in the $G/G/m$ queue:

$$\mathrm{CT}_q(G/G/m) = \left(\frac{c_a^2 + c_e^2}{2}\right)\left(\frac{u^{\sqrt{2(m+1)}-1}}{m(1-u)}\right) t_e \tag{8.28}$$

Expression (8.28) is the parallel machine version of the VUT equation. The V and T terms are identical to the single-machine version given in expression (8.25), but the U term is different. Although it may appear complicated, it does not require any type of iterative algorithm to solve and is therefore easily implementable in a spreadsheet program. This makes it possible to couple the single-station approximation (8.28) with the multimachine "linking equation" (8.11) to create a spreadsheet tool for analyzing the performance of a line.

8.7 Effects of Blocking

Thus far, we have considered only systems in which there is no limit to how large the queue can grow. Indeed, in every system we have examined, the average queue (and cycle time) grows to infinity as utilization approaches 100 percent. But in the real world, queues never become infinite. They are bounded by limitations of space, time, or operating policy. Therefore, an important topic in the science of factory physics is the behavior of systems with finite queueing space.

8.7.1 The *M/M/1/b* Queue

Consider the case where process and interarrival times are exponential, as they are in the $M/M/1$ queue, but where there is only enough space for b units in the system (in queue and in process). In Kendall's notation this corresponds to the $M/M/1/b$ queue. This system behaves in much the same way as the $M/M/1$ queue except now whenever the system becomes full, the arrival process is stopped. When this happens, the machine is said to be **blocked.** This model represents a very common situation in manufacturing applications.

For instance, consider a manufacturing cell consisting of two stations with a finite buffer in between. The first machine processes raw material and delivers it to the buffer of the second machine. If we can assume that raw material is always available (e.g.,

raw material is bar stock or sheet metal, which is in ample supply), then the $M/M/1/b$ model can be a good approximation of the behavior of the second machine. Indeed, if both machines have exponential process times, the model will be exact. This type of configuration is not uncommon. In fact, by their very nature all kanban systems exhibit blocking behavior.

In a queueing model with blocking, like the $M/M/1/b$, the arrival rate r_a takes on a different meaning than it does in models with unbounded queues. Here it represents the rate of *potential* arrivals, assuming that the system is not full. Thus, $u = r_a t_e$, is no longer the long-run probability that the machine is busy, but instead represents what the utilization would be if no arrivals were turned away. Consequently, u can equal or exceed one. We compute the probabilities and measures for the $M/M/1/b$ queue in the next Technical Note.

Technical Note

As in the $M/M/1$ queue, we define the state of the $M/M/1/b$ queue to be the number of jobs in the system. However, unlike in the $M/M/1$ case, the $M/M/1/b$ queue has a finite number of states $n = 0, 1, 2, \ldots, b$. Proceeding as we did for the $M/M/1$ queue, we can show that the long-run probability of being in state n is

$$p_n = u^n p_0$$

for the $M/M/1/b$ queue. A little algebra shows that in order to have $p_0 + \cdots + p_b = 1$, we must have

$$p_0 = \frac{1-u}{1-u^{b+1}} \tag{8.29}$$

Thus,

$$p_n = \frac{u^n(1-u)}{1-u^{b+1}} \tag{8.30}$$

Note that Equations (8.29) and (8.30) reduce to those for the $M/M/1$ queue as b goes to infinity (because $u^{b+1} \to 0$ as $b \to \infty$).

Equation (8.30) is valid as long as $u \neq 1$. For the special case where $u = 1$, all states of the system are equally likely and have the same probability, so

$$p_n = \frac{1}{b+1} \quad \text{for } n = 0, 1, \ldots, b \tag{8.31}$$

We can compute the average WIP level from

$$\text{WIP} = \sum_{n=0}^{b} n p_n \tag{8.32}$$

Since the system accepts arrivals whenever it is not full and the rate in equals the rate out, we can compute throughput from

$$\text{TH} = (1 - p_b) r_a \tag{8.33}$$

For the case where $u \neq 1$, the average WIP and throughput are

$$\text{WIP}(M/M/1/b) = \frac{u}{1-u} - \frac{(b+1)u^{b+1}}{1-u^{b+1}} \tag{8.34}$$

$$\text{TH}(M/M/1/b) = \frac{1-u^b}{1-u^{b+1}} r_a \tag{8.35}$$

For the case where $u = 1$, WIP and throughput simplify to

$$\text{WIP}(M/M/1/b) = \frac{b}{2} \tag{8.36}$$

$$\text{TH}(M/M/1/b) = \frac{b}{b+1} r_a = \frac{b}{b+1} r_e \tag{8.37}$$

For either case, we can use Little's law to compute the cycle time, queue time, and queue length as

$$\text{CT}(M/M/1/b) = \frac{\text{WIP}(M/M/1/b)}{\text{TH}(M/M/1/b)} \tag{8.38}$$

$$\text{CT}_q(M/M/1/b) = \text{CT}(M/M/1/b) - t_e \tag{8.39}$$

$$\text{WIP}_q(M/M/1/b) = \text{TH}(M/M/1/b) \times \text{CT}_q(M/M/1/b) \tag{8.40}$$

We can gain some useful insights from these formulas by interpreting the $M/M/1/b$ model as a system of two machines in series. The first machine is assumed to have enough raw material so that it never starves. Similarly, the second machine can always move its product out (i.e., it is never blocked). However, the buffer between the two machines is finite and is equal to B. If both machines have exponential process times, the model for the behavior of the second machine and the buffer is given by the $M/M/1/b$ queue, where $b = B + 2$. The two extra buffer spaces are the two machines themselves.

Notice that the WIP for the $M/M/1/b$ queue will *always* be less than that for the $M/M/1$ system. This is because the second machine has blocking, which prevents the WIP level from growing beyond b. If b is small, the effect can be dramatic. Indeed, kanban, which acts just like a finite buffer, is specifically intended to prevent WIP buildup.

However, WIP has a price—lost throughput. Recall that in the $M/M/1$ case the arrival rate is equal to the output rate. This is because, in steady state, whatever comes in must go out. This is not so in the case with blocking since the input rate is equal to the output rate (throughput) plus the balking rate (rate at which arrivals are rejected). Using Equations (8.35) and (8.37), we see that

$$\text{TH} = \frac{1 - u^b}{1 - u^{b+1}} u r_e < u r_e$$

if $u \neq 1$, and

$$\text{TH} = \frac{b}{b+1} r_e < r_e$$

if $u = 1$. These last expressions show that the throughput in a system with blocking will always be less than that in a system without blocking. Furthermore, the smaller the buffer size b, the greater the reduction in throughput.

Example:
Consider a line consisting of two machines in series. The first machine takes, on average, $t_e(1) = 21$ minutes to complete a job. The second machine takes $t_e(2) = 20$ minutes. Both machines have exponential process times ($c_e(1) = c_e(2) = 1$). Between the two machines there is enough room for two jobs, so $b = 4$ (two in the buffer and two at the machines themselves).

First consider what would happen if there were an infinite buffer. Since the first machine runs constantly, the arrival rate to the second machine is simply the rate of the first machine. Hence, utilization of the second machine is $u = r_a/r_e = \frac{1}{21}/\frac{1}{20} = 0.9524$.

The other performance measures for the second machine can be computed by using the $M/M/1$ formulas to be

$$\text{WIP} = \frac{u}{1-u} = \frac{0.9524}{1-0.9524} = 20 \text{ jobs}$$

$$\text{TH} = r_a = \tfrac{1}{21} \text{ minute} = 0.0476 \text{ job/minute}$$

$$\text{CT} = \frac{\text{WIP}}{\text{TH}} = 420.18 \text{ minutes}$$

Now, consider the finite buffer case. We first compute TH, using the $M/M/1/b$ queueing model.

$$\begin{aligned}\text{TH} &= \frac{1-u^b}{1-u^{b+1}} r_a \\ &= \frac{1-0.9524^4}{1-0.9524^5}\left(\frac{1}{21}\right) \\ &= 0.039 \text{ job/minute}\end{aligned}$$

We can now compute the *partial WIP* (denoted by WIPP) in the system represented by the $M/M/1/b$ model, namely, the second machine, the two-job buffer, and the buffer involving the first machine. We note that WIP at the first machine is only included in WIPP if it is in queue (i.e., when the first machine is blocked). WIP that is being processed at the first machine is not included, since it is viewed as "on its way" to the system represented by the $M/M/1/b$ model. From Equation (8.34), the partial WIP is

$$\begin{aligned}\text{WIPP} &= \frac{u}{1-u} - \frac{(b+1)u^{b+1}}{1-u^{b+1}} \\ &= 20 - \frac{5(0.9524^5)}{1-0.9524^5} = 20 - 18.106 = 1.894 \text{ jobs}\end{aligned}$$

The cycle time for the line is the time spent in partial WIP at the second machine plus the time in process at the first machine. Note that we do not consider any queue time at the first machine since it would be infinite due to the assumption of unlimited raw materials.

$$\text{CT} = \frac{\text{WIPP}}{\text{TH}} + t_e(1) = \frac{1.894}{0.039} + 21 = 69.57 \text{ minutes}$$

A second application of Little's law shows that the WIP in the system line is

$$\text{WIP} = \text{TH} \times \text{CT} = 0.039 \text{ job/minute} \times 69.57 \text{ minutes} = 2.71 \text{ jobs}$$

Comparison of the buffered and unbuffered cases is revealing. Limiting the interstation queue greatly reduces WIP and CT (by more than 83 percent) but also reduces TH (but by only 18 percent). However, a decline in throughput of 18 percent could more than offset the savings in inventory costs. This highlights why kanban cannot be implemented simply by reducing buffer sizes. The loss in throughput is typically too great. The only way to reduce WIP and CT without sacrificing too much throughput is to also reduce variability (i.e., we have to remove the rocks, not just lower the water). Unfortunately, we cannot examine variability reduction with the $M/M/1/b$ model because it assumes exponential process times. We discuss nonexponential models in the next section.

A second observation we can make using the $M/M/1/b$ model is that finite buffers force stability regardless of r_a and r_e. The reason is that WIP, and consequently CT, cannot "blow up" in a system with a finite buffer. For instance, suppose the speeds of the two machines above were reversed with the faster one feeding the slower one. If the

buffer were infinite, WIP would go to infinity (in the long run), as would CT. But in the finite buffer case $u = 21/20 = 1.05$, so

$$\text{TH} = \frac{1-u^b}{1-u^{b+1}} r_a = \frac{1-1.05^4}{1-1.05^5}\left(\frac{1}{20}\right) = 0.0390 \text{ job/minute}$$

The partial WIP is

$$\begin{aligned}\text{WIPP} &= \frac{u}{1-u} - \frac{(b+1)u^{b+1}}{1-u^{b+1}} \\ &= \frac{1.05}{1-1.05} - \frac{5(1.05^5)}{1-1.05^5} \\ &= 2.097 \text{ jobs}\end{aligned}$$

and cycle time is

$$\text{CT} = \frac{\text{WIP}}{\text{TH}} + t_e(1) = \frac{2.097}{0.0390} + 20 = 73.78 \text{ minutes}$$

Finally, WIP in the line is

$$\text{WIP} = \text{TH} \times \text{CT} = 0.0390 \times 73.78 = 2.88 \text{ jobs}$$

which is somewhat larger than in the case with the faster machine in second position, because the rate of arrival to the system is greater. However, throughput is unaffected by the order of the machines. This latter result is known as *reversibility* and holds for lines with more than two machines and general process times (see Muth 1979 for a proof). It is a fascinating theoretical result, but since firms seldom get the opportunity to run their lines backward, it does not often come up in practice.

8.7.2 General Blocking Models

To analyze variability effects, we need to extend the $M/M/1/b$ model to more general process and interarrival time distributions. In general, this is very difficult. We refer the interested reader to Buzacott and Shanthikumar (1993, Chapter 4) for a more complete treatment. However, we can make some useful approximations by modifying the $M/M/1/b$ queue in a manner analogous to the way we modified the $M/M/1$ queue to model the $G/G/1$ queue.

We consider three cases: (1) when the arrival rate is less than the production rate ($u < 1$), (2) when the arrival rate exceeds the production rate ($u > 1$), and (3) when the arrival and production rates are the same ($u = 1$).

Arrival Rate Less than Production Rate. First we compute the expected WIP in the system without any blocking, denoted by WIP_{nb}, by using Kingman's equation and Little's law.

$$\begin{aligned}\text{WIP}_{\text{nb}} &\approx r_a\left(\frac{c_a^2 + c_e^2}{2}\right)\left(\frac{u}{1-u}\right)t_e + t_e \\ &= \left(\frac{c_a^2 + c_e^2}{2}\right)\left(\frac{u^2}{1-u}\right) + u \qquad (8.41)\end{aligned}$$

Now recall that for the $M/M/1$ queue, $\text{WIP} = u/(1-u)$, so that

$$u = \frac{\text{WIP} - u}{\text{WIP}}$$

We can use WIP_{nb} in analogous fashion to compute a "corrected" utilization ρ

$$\rho = \frac{\text{WIP}_{nb} - u}{\text{WIP}_{nb}} \tag{8.42}$$

Then we substitute ρ for (almost) all the u terms in the $M/M/1/b$ expression for TH to obtain

$$\text{TH} \approx \frac{1 - u\rho^{b-1}}{1 - u^2\rho^{b-1}} r_a \tag{8.43}$$

By combining Kingman's equation (to compute ρ) with the $M/M/1/b$ model, we incorporate the effects of both variability and blocking. Although this expression is significantly more complex than that for the $M/M/1/b$ queue, it is straightforward to evaluate by using a spreadsheet. Furthermore, because we can easily show that $\rho = u$ if $c_a = c_e = 1$, Equation (8.43) reduces to the exact expression (8.35) for the case in which interarrival and process times are exponential.

Unfortunately, the expressions for expected WIP and CT become much more messy. However, for small buffers, WIP will be close to (but always less than) the size of the buffer (that is, $b - 1$). For large buffers, WIP will approach (but always be less than) that for the $G/G/1$ queue. Thus,

$$\text{WIP} < \min\{\text{WIP}_{nb}, b - 1\} \tag{8.44}$$

From Little's law, we obtain an approximate bound on CT

$$\text{CT} > \frac{\min\{\text{WIP}_{nb}, b - 1\}}{\text{TH}} \tag{8.45}$$

with TH computed as above. It is only an approximate bound because the expression for TH is an approximation.

Arrival Rate Greater than Production Rate. In the earlier example for the $M/M/1/b$ queue, we saw that the average WIP level was different, but not too different, when the order of the machines was reversed. This motivates us to approximate the WIP in the case in which the arrival rate is greater than the production rate by the WIP that results from having the machines in reverse order. When we switch the order of the machines, the production process becomes the arrival process and vice versa, so that utilization is $1/u$ (which will be less than 1 since $u > 1$). The average WIP level of the reversed line is approximated by

$$\text{WIP}_{nb} \approx \left(\frac{c_a^2 + c_e^2}{2}\right)\left(\frac{1/u^2}{1 - 1/u}\right) + \frac{1}{u} \tag{8.46}$$

We can compute a "corrected" utilization ρ_R for the reversed line in the same fashion as we did for the case where $u < 1$, which yields

$$\rho_R = \frac{\text{WIP}_{nb} - 1/u}{\text{WIP}_{nb}}$$

We then define $\rho = 1/\rho_R$ and compute TH as before. Once we have an approximation for TH, we can use inequalities (8.44) and (8.45) for bounds on WIP and CT, respectively.

Arrival Rate Equal to Production Rate. Finally, the following is a good approximation of TH for the case in which $u = 1$ (Buzacott and Shanthikumar 1993):

$$\text{TH} \approx \frac{c_a^2 + c_e^2 + 2(b - 1)}{2(c_a^2 + c_e^2 + b - 1)} \tag{8.47}$$

Again, with this approximation of TH, we can use inequalities (8.44) and (8.45) for bounds on WIP and CT.

Example:
Let us return to the example of Section 8.7.1, in which the first machine (with 21-minute process times) fed the second machine (with 20-minute process times) and there is an interstation buffer with room for two jobs (so that $b = 4$). Previously, we assumed that the process times were exponential and saw that limiting the buffer resulted in an 18 percent reduction in throughput. One way to offset the throughput drop resulting from limiting WIP is to reduce variability. So let us reconsider this example with reduced process variability, such that the effective coefficients of variation (CVs) for both machines are equal to 0.25.

Utilization is still $u = r_a/r_e = \frac{1}{21}/\frac{1}{20} = 0.9524$, so we can compute the WIP without blocking to be

$$\begin{aligned} \text{WIP}_{nb} &= \left(\frac{c_a^2 + c_e^2}{2}\right)\left(\frac{u^2}{1-u}\right) + u \\ &= \left(\frac{0.25^2 + 0.25^2}{2}\right)\left(\frac{0.9524^2}{1-0.9524}\right) + 0.9524 \\ &= 2.143 \end{aligned}$$

The corrected utilization is

$$\rho = \frac{\text{WIP}_{nb} - u}{\text{WIP}_{nb}} = \frac{2.143 - 0.9524}{2.143} = 0.556$$

Finally, we compute the throughput as

$$\begin{aligned} \text{TH} &= \frac{1 - u\rho^{b-1}}{1 - u^2\rho^{b-1}} r_a \\ &= \frac{1 - 0.9524(0.556^3)}{1 - 0.9524^2(0.556^3)} \frac{1}{21} \\ &= 0.0473 \end{aligned}$$

Hence, the percentage reduction in throughput relative to the unbuffered rate ($\frac{1}{21} = 0.0476$) is now less than one percent. Reducing process variability in the two machines made it possible to reduce the WIP by limiting the interstation buffer without a significant loss in throughput. This highlights why variability reduction is such an important component of JIT implementation.

8.8 Variability Pooling

In this chapter we have identified a number of causes of variability (failures, setups, etc.) and have observed how they cause congestion in a manufacturing system. Clearly, as we will discuss more fully in Chapter 9, one way to reduce this congestion is to reduce variability by addressing its causes. But another, and more subtle, way to deal with congestion effects is by combining multiple sources of variability. This is known as **variability pooling,** and it has a number of manufacturing applications.

An everyday example of the use of variability pooling is financial planning. Virtually all financial advisers recommend investing in a diversified portfolio of financial instruments. The reason, of course, is to hedge against risk. It is highly unlikely that a wide

spectrum of investments will perform extremely poorly at the same time. At the same time, it is also unlikely that they will perform extremely well at the same time. Hence, we expect less variable returns from a diversified portfolio than from any single asset.

Variability pooling plays an important role in a number of manufacturing situations. Here we discuss how it affects batch processing, safety stock aggregation, and queue sharing.

8.8.1 Batch Processing

To illustrate the basic idea behind variability pooling, we consider the question, Which is more variable, the process time of an individual part or the process time of a batch of parts? To answer this question, we must define what we mean by *variable*. In this chapter we have argued that the coefficient of variation is a reasonable way to characterize variability. So we will frame our analysis in terms of the CV.

First, consider a single part whose process time is described by a random variable with mean t_0 and standard deviation σ_0. Then the process time CV is

$$c_0 = \frac{\sigma_0}{t_0}$$

Now consider a batch of n parts, each of which has a process time with mean t_0 and standard deviation σ_0. Then the mean time to process the batch is simply the sum of the individual process times

$$t_0(\text{batch}) = nt_0$$

and the variance of the time to process the batch is the sum of the individual variances

$$\sigma_0^2(\text{batch}) = n\sigma_0^2$$

Hence, the CV of the time to process the batch is

$$c_0(\text{batch}) = \frac{\sigma_0(\text{batch})}{t_0(\text{batch})} = \frac{\sqrt{n}\sigma_0}{nt_0} = \frac{\sigma_0}{\sqrt{n}t_0} = \frac{c_0}{\sqrt{n}}$$

Thus, the CV of the time to process decreases by one over the square root of the batch size. We can conclude that process times of batches are less variable than process times of individual parts (provided that all process times are independent and identically distributed). The reason is analogous to that for the financial portfolio. Having extremely long or short process times for all n parts is highly unlikely. So the batch tends to "average out" the variability of individual parts.

Does this mean that we should process parts in batches to reduce variability? Not necessarily. As we will see in Chapter 9, batching has other negative consequences that may offset any benefits from lower variability. But there are times when the variability reduction effect of batching is very important, for instance, in sampling for quality control. Taking a quality measurement on a batch of parts reduces the variability in the estimate and hence is a standard practice in the construction of statistical control charts (see Chapter 12).

8.8.2 Safety Stock Aggregation

Variability pooling is also of enormous importance in inventory management. To see why, consider a computer manufacturer that sells systems with three different choices each of processor, hard drive, CD ROM, removable media storage device, RAM configurations, and keyboard. This makes a total of $3^6 = 729$ different computer configurations. To make the example simple, we suppose that all components cost \$150, so that the cost

Using the model in Section 8.6.6, we can model both the case with dedicated queues and the case with a single combined queue. In the dedicated queue case, average cycle time is 5.8 hours, while in the combined-queue case it is 1.27 hours, a 78 percent reduction (see Problem 6). Here the reason for the big difference is clear. The combined queue protects jobs against long failures. It is unlikely that all the machines will be down simultaneously, so if the machines are fed by a shared queue, jobs can avoid a failed machine by going to the other machines. This can be a powerful way to mitigate variability in processes with shared machines.

However, if the separate queues are actually different job types and combining them entails a time-consuming setup to switch the machines from one job type to another, then the situation is more complex. The capacity savings by avoiding setups through the use of dedicated queues might offset the variability savings possible by combining the queues. We will examine the tradeoffs involved in setups and batching in systems with variability in Chapter 9.

8.9 Conclusions

This chapter has traversed the complex and subtle topic of variability all the way from the fundamental nature of randomness to the propagation and effects of variability in a production line. Points that are fundamental from a factory physics perspective include the following:

1. *Variability is a fact of life.* Indeed, the field of physics is increasingly indicating that randomness may be an inescapable aspect of existence itself. From a management point of view, it is clear that the ability to deal effectively with variability and uncertainty will be an important skill for the foreseeable future.

2. *There are many sources of variability in manufacturing systems.* Process variability is created by things as simple as work procedure variations and by more complex effects such as setups, random outages, and quality problems. Flow variability is created by the way work is released to the system or moved between stations. As a result, the variability present in a system is the consequence of a host of process selection, system design, quality control, and management decisions.

3. *The coefficient of variation is a key measure of item variability.* Using this unitless ratio of the standard deviation to the mean, we can make consistent comparisons of the level of variability in both process times and flows. At the workstation level, the CV of *effective* process time is inflated by machine failures, setups, recycle, and many other factors. Disruptions that cause long, infrequent outages tend to inflate CV more than disruptions that cause short, frequent outages, given constant availability.

4. *Variability propagates.* Highly variable outputs from one workstation become highly variable inputs to another. At low utilization levels, the flow variability of the output process from a station is determined largely by the variability of the arrival process to that station. However, as utilization increases, flow variability becomes determined by the variability of process times at the station.

5. *Waiting time is frequently the largest component of cycle time.* Two factors contribute to long waiting times: high utilization levels and high levels of variability. The queueing models discussed in this chapter clearly illustrate that both increasing effective capacity (i.e., to bring down utilization levels) and decreasing variability (i.e., to decrease congestion) are useful for reducing cycle time.

6. *Limiting buffers reduces cycle time at the cost of decreasing throughput.* Since limiting interstation buffers is logically equivalent to installing kanban, this property is

the key reason that variability reduction (via production smoothing, improved layout and flow control, total preventive maintenance, and enhanced quality assurance) is critical in just-in-time systems. It also points up the manner in which capacity, WIP buffering, and variability reduction can act as substitutes for one another in achieving desired throughput and cycle time performance. Understanding the tradeoffs among these is fundamental to designing an operating system that supports strategic business goals.

7. *Variability pooling reduces the effects of variability.* Pooling variability tends to dampen the overall variability by making it less likely that a single occurrence will dominate performance. This effect has a variety of factory physics applications. For instance, safety stocks can be reduced by holding stock at a generic level and assembling to order. Also, cycle times at multiple-machine process centers can be reduced by sharing a single queue.

In the next chapter, we will use these insights, along with the concepts and formulas developed, to examine how variability degrades the performance of a manufacturing plant and to provide ways to protect against it.

Study Questions

1. What is the rationale for using the coefficient of variation c instead of the standard deviation σ as a measure of variability?
2. For the following random variables, indicate whether you would expect each to be LV, MV or HV.
   a. Time to complete this set of study questions
   b. Time for a mechanic to replace a muffler on an automobile
   c. Number of rolls of a pair of dice between rolls of seven
   d. Time until failure of a recently repaired machine by a good maintenance technician
   e. Time until failure of a recently repaired machine by a not-so-good technician
   f. Number of words between typographical errors in the book *Factory Physics*
   g. Time between customer arrivals to an automatic teller machine
3. What type of manufacturing workstation does the $M/G/2$ queue represent?
4. Why must utilization be *strictly* less than 100 percent for the $M/M/1$ queueing system to be stable?
5. What is meant by *steady state?* Why is this concept important in the analysis of queueing models?
6. Why is the number of customers at the station an adequate state for summarizing current status in the $M/M/1$ queue but not the $G/G/1$ queue?
7. What happens to CT, WIP, CT_q, and WIP_q as the arrival rate r_a approaches the process rate r_e?

Problems

1. Consider the following sets of interoutput times from a machine. Compute the coefficient of variation for each sample, and suggest a situation under which such behavior might occur.
   a. 5, 5, 5, 5, 5, 5, 5, 5, 5, 5
   b. 5.1, 4.9, 5.0, 5.0, 5.2, 5.1, 4.8, 4.9, 5.0, 5.0
   c. 5, 5, 5, 35, 5, 5, 5, 5, 5, 42
   d. 10, 0, 0, 0, 0, 10, 0, 0, 0, 0
2. Suppose jobs arrive at a single-machine workstation at a rate of 20 per hour and the average process time is two and one-half minutes.
   a. What is the utilization of the machine?

b. Suppose that interarrival and process times are exponential,
   i. What is the average time a job spends at the station (i.e., waiting plus process time)?
   ii. What is the average number of jobs at the station?
   iii. What is the long-run probability of finding more than three jobs at the station?

c. Process times are not exponential, but instead have a mean of two and one-half minutes and a standard deviation of five minutes
   i. What is the average time a job spends at the station?
   ii. What is the average number of jobs at the station?
   iii. What is the average number of jobs in the queue?

3. The mean time to expose a single panel in a circuit-board plant is two minutes with a standard deviation of one-half minutes.
   a. What is the natural coefficient of variation?
   b. If the times remain independent, what will be the mean and variance of a job of 60 panels? What will be the coefficient of variation of the job of 60?
   c. Now suppose times to failure on the expose machine are exponentially distributed with a mean of 60 hours and the repair time is also exponentially distributed with a mean of two hours. What are the *effective* mean and CV of the process time for a job of 60 panels?

4. Reconsider the expose machine of Problem 3 with mean time to expose a single panel of two minutes with a standard deviation of one and one-half minutes and jobs of 60 panels. As before, failures occur after about 60 hours of run time, but now happen only between jobs (i.e., these failures do not *preempt* the job). Repair times are the same as before. Compute the effective mean and CV of the process times for the 60 panel jobs. How do these compare with the results in Problem 3?

5. Consider two different machines A and B that could be used at a station. Machine A has a mean effective process time t_e of 1.0 hours and an SCV c_e^2 of 0.25. Machine B has a mean effective process time of 0.85 hour and an SCV of four. (*Hint:* You may find a simple spreadsheet helpful in making the calculations required to answer the following questions.)
   a. For an arrival rate of 0.92 job per hour with $c_a^2 = 1$, which machine will have a shorter average cycle time?
   b. Now put two machines of type A at the station and double the arrival rate (i.e., double the capacity and the throughput). What happens to cycle time? Do the same for machine B. Which type of machine produces shorter average cycle time?
   c. With only one machine at each station, let the arrival rate be 0.95 job per hour with $c_a^2 = 1$. Recompute the average time spent at the stations using both machines A and B. Compare with *a*.
   d. Consider the station with one machine of type A.
      i. Let the arrival rate be one-half. What is the average time spent at the station? What happens to the average time spent at the station if the arrival rate is increased by one percent (i.e., to 0.505)? What percentage increase in wait time does this represent?
      ii. Let the arrival rate be 0.95. What is the average time spent at the station? What happens to the average time spent at the station if the arrival rate is increased by one percent (i.e., to 0.9595)? What percentage increase in wait time does this represent?

6. Consider the example in Section 8.8. The arrival rate of jobs is 13.5 jobs per hour (with $c_a^2 = 1$) to a workstation consisting of five machines. Each machine nominally takes 0.3 hour per job with a natural CV of $\frac{1}{2}$ (that is, $c_0^2 = 0.25$). The mean time to failure for any machine is 36 hours, and repair times are exponential with a mean time to repair of four hours.
   a. Show that the SCV of effective process times is 2.65.
   b. What is the utilization of a single machine when it is allocated one-fifth of the demand (that is, 2.7 jobs per hour) assuming c_a is still equal to one?
   c. What is the utilization of the battery with an arrival rate of 13.5 jobs per hour?
   d. Compute the mean cycle time at a single machine when allocated one-fifth of the demand.
   e. Compute the mean cycle time at the station serving 13.5 jobs per hour.

7. A car company sells 50 different basic models (additional options are added at the dealership after purchases are made). Customers are of two basic types: (1) those who are willing to order the configuration they desire from the factory and wait several weeks for delivery and (2) those who want the car quickly and therefore buy off the lot. The traditional mode of handling customers of the second type is for the dealerships to hold stock of models they think will sell. A newer strategy is to hold stock in regional distribution centers, which can ship cars to dealerships within 24 hours. Under this strategy, dealerships only hold show inventory and a sufficient variety of stock to facilitate test drives.

   Consider a region in which total demand for each of the 50 models is Poisson with a rate of 1,000 cars per month. Replenishment lead time from the factory (to either a dealership or the regional distribution center) is one month.

   a. First consider the case in which inventory is held at the dealerships. Assume that there are 200 dealerships in the region, each of which experiences demand of $1{,}000/200 = 5$ cars of each of the 50 model types per month (and demand is still Poisson). The dealerships monitor their inventory levels in continuous time and order replenishments in lots of one (i.e., they make use of a base stock model). How many vehicles must each dealership stock to guarantee a fill rate of 99 percent?

   b. Now suppose that all inventory is held at the regional distribution center, which also uses a base stock model to set inventory levels. How much inventory is required to guarantee a 99 percent fill rate?

8. Frequently, natural process times are made up of several distinct stages. For instance, a manual task can be thought of as being comprised of individual motions (or "therbligs" as Gilbreth termed them).

   Suppose a manual task takes a single operator an average of one hour to perform. Alternatively, the task could be separated into 10 distinct six-minute subtasks performed by separate operators. Suppose that the subtask times are independent (i.e., uncorrelated), and assume that the coefficient of variation is 0.75 for both the single large task and the small subtasks. Such an assumption will be valid if the relative shapes of the process time distributions for both large and small tasks are the same. (Recall that the variances of independent random variables are additive.)

   a. What is the coefficient of variation for the 10 subtasks taken together?

   b. Write an expression relating the SCV of the original tasks to the SCV of the combined task.

   c. What are the issues that must be considered before dividing a task into smaller subtasks? Why not divide it into as many as possible? Give several pros and cons.

   d. One of the principles of JIT is to standardize production. How does this explain some of the success of JIT in terms of variability reduction?

9. Consider a workstation with 11 machines (in parallel), each requiring one hour of process time per job with $c_e^2 = 5$. Each machine costs \$10,000. Orders for jobs arrive at a rate of 10 per hour with $c_a^2 = 1$ and must be filled. Management has specified a maximum allowable average response time (i.e., time a job spends at the station) of two hours. Currently it is just over three hours (check it).

   Analyze the following options for reducing average response time.

   a. Perform more preventive maintenance so that m_r and m_f are reduced, but m_r/m_f remains the same. This costs \$8,000 and does not improve the average process time but does reduce c_e^2 to one.

   b. Add another machine to the workstation at a cost of \$10,000. The new machine is identical to existing machines, so $t_e = 1$ and $c_e^2 = 5$.

   c. Modify the existing machines to make them faster without changing the SCV, at a cost of \$8,500. The modified machines would have $t_e = 0.96$ and $c_e^2 = 5$.

   What is the best option?

10. (This problem is fairly involved and could be considered a small project.) Consider a simple two-station line as shown in Figure 8.8. Both machines take 20 minutes per job and have

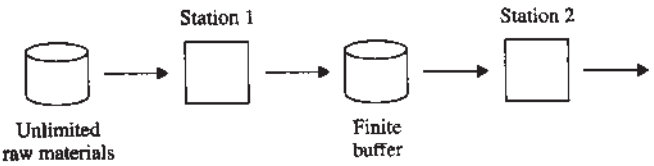


FIGURE 8.8
Two-station line with a finite buffer

SCV = 1. The first machine can always pull in material, and the second machine can always push material to finished goods. Between the two machines is a buffer that can hold only 10 jobs (see Sections 8.7.1 and 8.7.2).

a. Model the system using an $M/M/1/b$ queue. (Note that $b = 12$ considering the two machines.)
   i. What is the throughput?
   ii. What is the partial WIP (i.e., WIP waiting at the first machine or at the second machine, but not in process at the first machine)?
   iii. What is the total cycle time for the line (not including time in raw material)? (*Hint:* Use Little's law with the partial WIP and the throughput and then add the process time at the first machine.)
   iv. What is the total WIP in the line? (*Hint:* Use Little's law with the total cycle time and the throughput.)

b. Reduce the buffer to one (so that $b = 3$) and recompute the above measures. What happens to throughput, cycle time, and WIP? Comment on this as a strategy.

c. Set the buffer to one and make the process time at the second machine equal to 10 minutes. Recompute the above measures. What happens to throughput, cycle time, and WIP? Comment on this as a strategy.

d. Keep the buffer at one, make the process times for both stations equal to 20 minutes (as in the original case), but set the process CVs to 0.25 (SCV = 0.0625).
   i. What is the throughput?
   ii. Compute an upper bound on the partial WIP at the second machine and waiting at the first machine.
   iii. Compute an (approximate) upper bound on the total cycle time. (*Hint:* Use Little's law with the partial WIP upper bound and the throughput, and then add the time at the first machine.) Is even this upper bound an acceptable cycle time?
   iv. Compute an (approximate) upper bound on the total WIP in the line. (*Hint:* Use Little's law with the upper bound on the total cycle time and the throughput estimate.) Is this upper bound an acceptable WIP level?
   v. Comment on reducing variability as a strategy.

CHAPTER

9 THE CORRUPTING INFLUENCE OF VARIABILITY

When luck is on your side, you can do without brains.
Giordano Bruno, burned at the stake in 1600

The more you know the luckier you get.
J. R. Ewing of *Dallas*

9.1 Introduction

The previous chapter developed tools for characterizing and evaluating variability in process times and flows. In this chapter, we use these tools to describe fundamental behavior of manufacturing systems involving variability.

As we did in Chapter 7, we state our main conclusions as laws of factory physics. Some of these "laws" are *always* true (e.g., the Conservation of Material Law), while others hold *most of the time*. On the surface this may appear unscientific. However, we point out that physics laws, such as Newton's second law $F = ma$ and the law of the conservation of energy, hold only approximately. But even though they have been replaced by deeper results of quantum mechanics and relativity, these laws are still very useful. So are the laws of factory physics.

9.1.1 Can Variability Be Good?

The discussions of Chapters 7 and 8 (and the title of this chapter) may give the impression that variability is evil. Using the jargon of lean manufacturing (Womack and Jones 1996), one might be tempted to equate variability with *muda* (waste) and conclude that it should always be eliminated.[1]

But we must be careful not to lose sight of the fundamental objective of the firm. As we observed in Chapter 1, Henry Ford was something of a fanatic about reducing variability. A customer could have any color desired as long as it was *black*. Car models

[1] *Muda* is the Japanese word for "waste" and is defined as "any human activity that absorbs resources but creates no *value*." Ohno gave seven examples of *muda:* defects in products, overproduction of goods, inventories of goods awaiting further processing or consumption, unnecessary processing, unnecessary movement, unnecessary transport, and waiting.

were changed infrequently with little variety within models. By stabilizing products and keeping operations simple and efficient, Ford created a major revolution by making automobiles affordable to the masses. However, when General Motors under Alfred P. Sloan offered greater product variety in the 1930s and 1940s, Ford Motor Company lost much of its market share and nearly went under. Of course, greater product variety meant greater variability in GM's production system. Greater variability meant GM's system could not run as efficiently as Ford's. Nonetheless, GM did better than Ford. Why?

The answer is simple. Neither GM nor Ford were in business to reduce variability or even to reduce *muda*. They were in business to *make a good return on investment over the long term*. If adding product variety increases variability and hence *muda* but increases revenues by an amount that more than offsets the additional cost, then it can be a sound business strategy.

9.1.2 Examples of Good and Bad Variability

To highlight the manner in which variability can be good (a necessary implication of a business strategy) or bad (an undesired side effect of a poor operating policy), we consider a few examples.

Table 9.1 lists several causes of undesirable variability. For instance, as we saw in Chapter 8, unplanned outages, such as machine breakdowns, can introduce an enormous amount of variability into a system. While such variability may be unavoidable, it is not something we would deliberately introduce into the system.

In contrast, Table 9.2 gives some cases in which effective corporate strategies consciously introduced variability into the system. As we noted above, at GM in the 1930s and 1940s the variability was a consequence of greater product variety. At Intel in the 1980s and 1990s, the variability was a consequence of rapid product introduction in an environment of changing technology. By aggressively pushing out the next generation of microprocessor before processes for the last generation had stabilized, Intel stimulated demand for new computers and provided a powerful barrier to entry by competitors. At Jiffy Lube, where offering while-you-wait oil changes is the core of the firm's business strategy, demand variability is an unavoidable result. Jiffy Lube could reduce this variability by scheduling oil changes as in traditional auto shops, but doing so would forfeit the company's competitive edge.

Regardless of whether variability is good or bad in business strategy terms, it causes operating problems and therefore must be managed. The specific strategy for dealing with variability will depend on the structure of the system and the firm's strategic goals.

TABLE 9.1 Examples of Bad Variability

Cause	Example
Planned outages	Setups
Unplanned outages	Machine failures
Quality problems	Yield loss and rework
Operator variation	Skill differences
Inadequate design	Engineering changes

TABLE 9.2 Examples of (Potentially) Good Variability

Cause	Example
Product variety	GM in the1930s and 1940s
Technological change	INTEL in the 1980s and 1990s
Demand variability	Jiffy Lube

In this chapter, we present laws governing the manner in which variability affects the behavior of manufacturing systems. These define key tradeoffs that must be faced in developing effective operations.

9.2 Performance and Variability

In the systems analysis terminology of Chapter 6, management of any system begins with an **objective.** The decision maker manipulates **controls** in an attempt to achieve this objective and evaluates performance in terms of **measures.** For example, the objective of an airplane trip is to take passengers from point A to point B in a safe and timely manner. To do this, the pilot makes use of many controls while monitoring numerous measures of the plane's performance. The links between controls and measures are well known through the science of aeronautical engineering. Analogously, the objective of a plant manager is to contribute to the firm's long-term profitability by efficiently converting raw materials to goods that will be sold. Like the pilot, the plant manager has many controls and measures to consider. Understanding the relationships between the controls and measures available to a manufacturing manager is the primary goal of factory physics.

A concept at the core of how controls affect measures in production systems is variability. As we saw in Chapter 7, best-case behavior occurs in a line with no variability, while worst-case behavior occurs in a line with maximum variability. In Chapter 8 we observed that several important measures of station performance, such as cycle time and work in process (WIP), are increasing functions of variability.

To understand how variability impacts performance in more general production systems than the idealized lines of Chapter 7 or the single stations of Chapter 8, we need to be more precise about how we define performance. We do this by first discussing *perfect* performance in a production system. Then, by observing the dimensions along which this performance can degrade, we define a set of measures. Finally, we discuss the manner in which the relative weights of these measures depend on both the manufacturing environment and the firm's business strategy.

9.2.1 Measures of Manufacturing Performance

Anyone who has ever peeked into a cockpit knows that the performance of an airplane is not evaluated by a single measure. The impressive array of gauges, dials, meters, LED readouts, etc., is proof that even though the objective is simple (travel from point A to point B), measuring performance is not. Altitude, direction, thrust, airspeed, groundspeed, elevator settings, engine temperature, etc., must be monitored carefully in order to attain the fundamental objective.

In the same fashion, a manufacturing enterprise has a relatively simple fundamental objective (make money) but a wide array of potential performance measures, such as throughput, inventory, customer service, and quality (see Figure 9.1). Appropriate numerical definitions of performance measures depend on the environment. For example, a styrene plant might measure throughput in straightforward units of pounds per day. A manufacturer of seed planters (devices pulled behind tractors to plant and fertilize as few as 4 or as many as 30 rows at once) might not want to measure throughput in the obvious units of planters per day. The reason is that there is wide variability in size among planters. Measuring throughput in row units per day might be a better measure of aggregate output. Indeed in some systems with many products and complex flows,

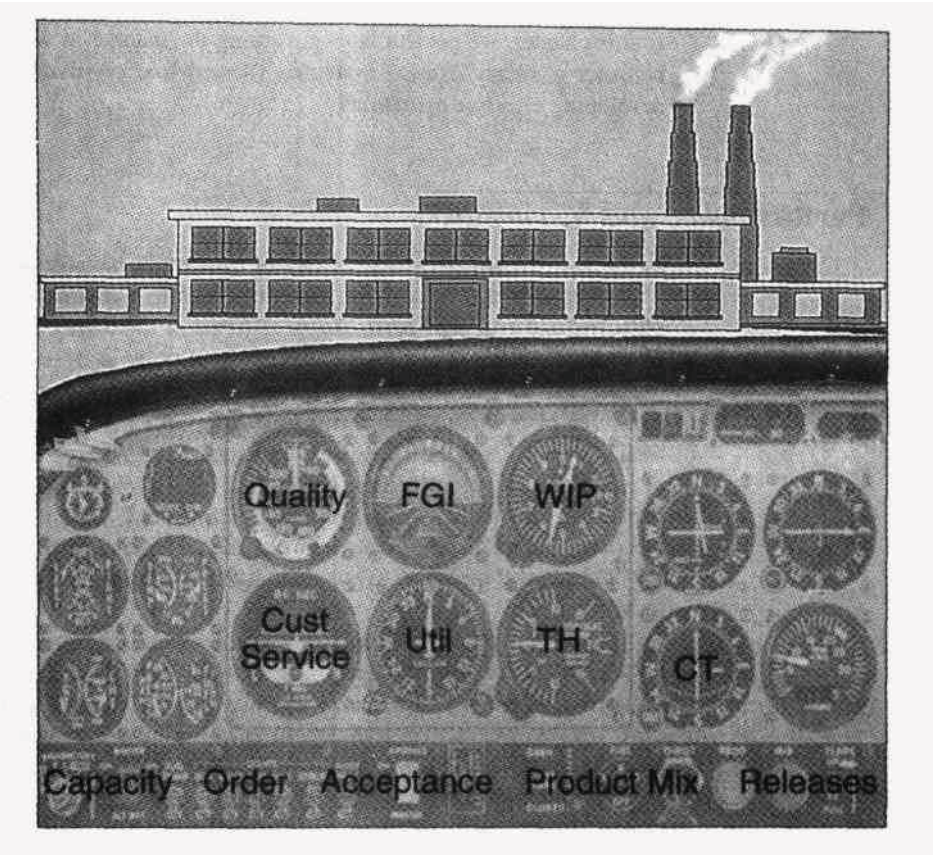


FIGURE 9.1
The manufacturing control panel

throughput is measured in dollars per day in order to aggregate output into a single number.

The relative importance of performance measures also depends on the specific system and its business strategy. For example, Federal Express, whose competitive advantage is delivery speed and traceability, places a great deal of weight on measures of responsiveness (lead time) and customer service (on-time delivery). The U.S. Postal Service, in contrast, competes largely on price and therefore emphasizes cost-related measures, such as equipment utilization and amount of material handling. Even though both organizations are in the package delivery industry, they have different business strategies targeted at different segments of the market and therefore require different measures of performance.

Given the broad range of production environments and business strategies, it is not possible to define a single set of performance measures for all manufacturing systems. However, to get a sense of what types of measures are possible and to see how these relate to variability, it is useful to consider performance of a simple single-product production line. In principle, measures for more complex multiproduct lines can be developed as extensions of the single-product line measures, and measures for systems made up of many lines can be constructed as weighted combinations of the line measures.

Chapter 7 used throughput, cycle time, and WIP to characterize performance of a simple serial production line. Clearly these are important measures, but they are not comprehensive. Because cost matters, we must also consider equipment utilization. Since the line is fed by a procurement process, another measure of interest is raw material

inventory. When we consider customers, lead time, service and finished goods inventory become relevant measures. Finally, since yield loss and rework are often realities, quality is a key performance measure. A perfect single-product line would have throughput exactly equal to demand, full utilization of all equipment, average cycle and lead times as short as possible, perfect customer service (no late or backordered jobs), perfect quality (no scrap or rework), zero raw material or finished goods inventory, and minimum (critical) WIP.

We can characterize each of these measures more precisely in terms of a quantitative efficiency value. For each efficiency, a value of one indicates perfect performance, while zero represents the worst possible performance. To do this, we make use of the following notation, where for specificity we will measure inventories in units of parts and time in days:

$r_e(i)$ = effective rate of station i including detractors such as downtime, setups, and operator efficiency (parts/day)

$r^*(i)$ = ideal rate of station i not including detractors (parts/day)

r_b = bottleneck rate of line including detractors (parts/day)

r_b^* = bottleneck rate of line not including detractors (parts/day)

T_0 = raw process time including detractors (days)

T_0^* = raw process time not including detractors (days)

$W_0 = r_b T_0$ = critical WIP including detractors (parts)

$W_0^* = r_b^* T_0^*$ = critical WIP not including detractors (parts)

D = average demand rate (parts/day)

WIP = average work in process level in line (parts)

FGI = average finished goods inventory level (parts)

RMI = average raw material inventory level (parts)

CT = average cycle time from release to stock point, which is either finished goods or an interline buffer (days)

LT = average lead time quoted to customer; in systems where lead time is fixed, LT is constant; where lead times are quoted individually to customers, it represents an average (days)

TH = average throughput given by ouput rate from line (parts/day)

TH(i) = average throughput (output rate) at station i, which could include multiple visits by some parts due to routing or rework considerations (parts/day)

Notice that the starred parameters, $r^*(i)$, r_b^*, T_0^*, and W_0^* are ideal versions of $r_e(i)$, r_b, T_0, and W_0. The reason we need them is that a line running at the bottleneck rate and raw process time may actually *not* be exhibiting perfect performance because r_b and T_0 can include many inefficiencies. Perfect performance, therefore, involves two levels. First, the line must attain the best possible performance given its parameters; this is what the best case of Chapter 7 represents. Second, its parameters must be as good as they can be. Thus, perfect performance represents the *best of the best.*

Using the above parameters, we can define seven efficiencies that measure the performance of a single-product line.

Throughput is defined as the rate of parts produced by the line that are *used.* Ideally, this should exactly match demand. Too little production, and we lose sales; too much, and we build up unnecessary finished goods inventory (FGI). Since we

will have another measure to penalize excess inventory, we define **throughput efficiency** in terms of whether output is adequate to satisfy demand, so that

$$E_{\mathrm{TH}} = \frac{\min\{\mathrm{TH}, D\}}{D}$$

If throughput is greater than or equal to demand, then throughput efficiency is equal to one. Any shortage will degrade this measure.

Utilization of a station is the fraction of time it is busy. Since unused capacity implies excess cost, an ideal line will have all workstations 100 percent utilized.[2] Furthermore, since a perfect line will not be plagued by detractors, utilization will be 100 percent relative to the best possible (no detractors) rate. Thus, for a line with n stations, we define **utilization efficiency** as

$$E_u = \frac{1}{n}\sum_{i=1}^{n}\frac{\mathrm{TH}(i)}{r^*(i)}$$

Inventory includes RMI, FGI, and WIP. A perfect line would have no raw material inventory (suppliers would deliver literally just-in-time), no finished goods inventory (deliveries to customers would also be made just-in-time), and only the minimum WIP needed for the given throughput, which by Little's Law is $\sum_i \mathrm{TH}(i)/r^*(i)$. Thus a measure of **inventory efficency** is,

$$E_{\mathrm{inv}} = \frac{\sum_i \mathrm{TH}(i)/r^*(i)}{\mathrm{RMI} + \mathrm{WIP} + \mathrm{FGI}}$$

Cycle time is important to both costs and revenue. Shorter cycle time means less WIP, better quality, better forecasting, and less scrap—all of which reduce costs. It also means better responsiveness, which improves sales revenue. By Little's Law, average cycle time is fully determined by throughput and WIP. Hence, a line with perfect throughput efficiency and inventory efficiency is guaranteed to have perfect cycle time efficiency. However, for imperfect lines WIP is not completely characterized by inventory efficiency (since it involves RMI and FGI), and hence cycle time becomes an independent measure. We define **cycle time efficiency** as the ratio of the best-possible cycle time (raw process time with no detractors) to actual cycle time:

$$E_{\mathrm{CT}} = \frac{T_0^*}{\mathrm{CT}}$$

Lead time is the time quoted to the customer, which should be as short as possible for competitive reasons. Indeed, in make-to-stock systems, lead time is zero, which is clearly as short as possible. However, zero is not a reasonable target for a make-to-order system. Therefore, we define **lead time efficiency** as the ratio of the ideal raw process time to the actual lead time, provided lead time (LT) is at least as large as the ideal raw process time. If lead time is less than this, then we define the lead time efficiency to be one. We can write this as follows:

$$E_{\mathrm{LT}} = \frac{T_0^*}{\max\{\mathrm{LT}, T_0^*\}}$$

Notice that in a make-to-order system we could quote unreasonably short lead times (less than T_0^*) and ensure that this measure is one. But if the line is not capable of delivering product this quickly, the measure of customer service will suffer.

[2]Note that 100 percent utilization is only possible in *perfect* lines. In realistic lines containing variability, pushing utilization close to one will seriously degrade other measures. It is critical to remember that system performance is measured by *all* the efficiencies, not by any single number.

Customer service is the fraction of demands that are satisfied on time. In a make-to-stock situation, this is the fill rate (fraction of demands filled from stock, rather than backordered). In a make-to-order system, customer service is the fraction of orders that are filled within their lead times (i.e., cycle time is less than or equal to lead time). Hence, we define **service efficiency** as the customer service itself:

$$E_S = \begin{cases} \text{fraction of demand filled from stock in make-to-stock system} \\ \text{fraction of orders filled within lead time in make-to-order system} \end{cases}$$

Quality is a complex characteristic of the product, process, and customer (see Chapter 12 for a discussion). For operational purposes, the essential aspect of quality is captured by the fraction of parts that are made correctly the first time through the line. Any scrap or rework decreases this value. Hence, we measure **quality efficiency** as

$$E_Q = \text{fraction of jobs that go through line with no defects on first pass}$$

These efficiencies are stated specifically for a single-product line. However, one could extend these measures to a multiproduct line by aggregating the flows and inventories (e.g., in dollars) and measuring cycle time, lead time, and service individually by product (see Problem 1).

A perfect single-product line will have all seven of the above efficiencies equal to one. For example, Penny Fab One of Chapter 7 has no detractors, so $r_b = r_b^*$ and $T_0 = T_0^*$. If raw materials are delivered just in time (one penny blank every two hours), customer orders are promised (and shipped) every two hours, and the CONWIP level is set at WIP $= W_0^*$, then inventory, lead time, and service efficiencies will all be one. Finally, since there are no quality problems, quality efficiency is also one. Obviously we would not expect to see such perfect performance in the real world. All realistic production systems will have some efficiencies less than one.

In less than perfect lines, performance is a composite of these efficiencies (or similar ones suited to the specific environment of the line). In theory, we could construct a single-number measure of efficiency as a weighted average of these efficiencies. As we noted, however, the individual weights would be highly dependent on the nature of the line and its business. For instance, a commodity producer with expensive capital equipment would stress utilization and service efficiency much more than inventory efficiency, while a specialty job shop would stress lead time efficiency at the expense of utilization efficiency.

Consider the example shown in Figure 9.2, which represents a card stuffing operation line feeding an assembly operation in a "box plant" making personal computers. In this case, finished goods inventory is really intermediate stock for the final assembly operation controlled by a kanban system. The five percent rework through the last station represents cards that must be touched up. Cards that are reworked never need to be reworked again.

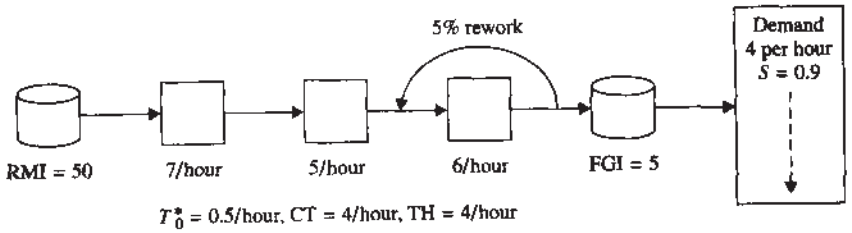


FIGURE 9.2
Operational efficiency example

Since TH is equal to demand, throughput efficiency E_{TH} is equal to one. Cycle time efficiency is given by $E_{CT} = T_0^*/CT = 0.5/4 = 0.125$. Utilization efficiency is the average of the individual station utilizations. To get this, we must first compute the throughput at each station. Because there is five percent rework at station 3,

$$TH(3) = TH + 0.05TH = 1.05(4) = 4.2$$

Since there is no rework at stations 1 and 2, $TH(1) = TH(2) = 4$. Thus, utilization efficiency is

$$E_u = \frac{1}{3}\sum_{i=1}^{3}\frac{TH(i)}{r^*(i)} = \frac{\frac{4}{7}+\frac{4}{5}+\frac{4.2}{6}}{3} = 0.6905$$

According to the problem data, service efficiency E_S is 0.9. Since production is controlled by a kanban system, lead time is zero so that $E_{LT} = 1.0$. Quality efficiency E_Q is also given as part of the data and is 0.95. To compute inventory efficiency, we must first compute WIP from Little's Law: WIP = TH × CT = (4 cards per hour)(4 hours) = 16 cards; and the ideal WIP is given by $\sum_i TH(i)/r^*(i) = \frac{4}{7}+\frac{4}{5}+\frac{4.2}{6} = 2.071$. Then we compute

$$E_{inv} = \frac{\sum_i TH(i)/r^*(i)}{RMI + WIP + FGI} = \frac{2.071}{50+16+5} = 0.0292$$

Now suppose we increase the kanban level so that, on average, there are 15 cards in FGI; and suppose that this change causes the service level to increase to 0.999. While the other efficiencies stay the same, E_S becomes 0.999 and E_{inv} goes down to 0.0256. Table 9.3 compares the two systems.

Which system is better? It depends on whether the firm's business strategy deems it more important to have high customer service or low inventory. Most likely in this environment the modified system is better, since the stuffing line's customer is the assembly line and shutting it down 10 percent of the time would probably result in unacceptable service to the ultimate customer.

9.2.2 Variability Laws

Now that we have defined performance in reasonably concrete terms, we can characterize the effect of variability on performance. Variability can affect supplier deliveries, manufacturing process times, or customer demand. If we examine these carefully, we see that increasing any source of variability will degrade at least one of the above efficiency measures. For instance, if we increase the variability of process times while holding throughput constant, we know from the *VUT* equation of Chapter 8 that WIP will

TABLE 9.3 System Efficiency Comparison

Measure	Card Stuffing System	Modified Card Stuffing System
Cycle time	0.1250	0.1250
Utilization	0.6905	0.6905
Service	0.9000	0.9990
Quality	0.9500	0.9500
Inventory	0.0292	0.0256

increase, thereby degrading inventory efficiency. If we place a restriction on WIP (via kanban or CONWIP), then by our analysis of queueing systems with blocking we know that, in general, throughput will decline (because the bottleneck will starve), thereby degrading throughput efficiency.

These observations are specific instances of the following fundamental law of factory physics.

Law (Variability): *Increasing variability always degrades the performance of a production system.*

This is an extremely powerful concept, since it implies that higher variability of any sort must harm some measure of performance. Consequently, variability reduction is central to improving performance, regardless of the specific weights a firm attaches to the individual performance measures. Indeed, much of the success of JIT methods was a consequence of recognizing the power of variability reduction and developing methods for achieving it (e.g., production smoothing, setup reduction, total quality management, and total preventive maintenance).

We can deepen the insight of the Variability Law by observing that increasing variability impacts the system along three general dimensions: inventory, capacity, and time. Clearly, inventory efficiency measures the inventory impact. Production and utilization efficiency are measures of the capacity impact. Cycle time and lead time efficiency measure the time impact, as does service efficiency, since the customer must wait for parts that are not ready. Finally, quality efficiency impacts the system in all three dimensions: Scrap or rework requires additional capacity, redoing an operation requires additional time, and parts being (or waiting to be) repaired or redone add inventory to the system.

Another way to view these three impacts is as **buffers** with which we control the system. Worse performance corresponds to more buffering. We can summarize this as the following factory physics law.

Law (Variability Buffering): *Variability in a production system* will *be buffered by some combination of*

1. *Inventory*
2. *Capacity*
3. *Time*

This law is an enormously important extension of the Variability Law because it enumerates the ways in which variability can impact a system. While there is no question that variability will degrade performance, we have a choice of *how* it will do so. Different strategies for coping with variability make sense in different business environments. For instance, in the earlier board-stuffing example, the modified system used a larger inventory buffer to enable a smaller time (service) buffer, a change that made good business sense in that environment. We offer some additional examples of the different ways to buffer variability.

9.2.3 Buffering Examples

The following examples illustrate (1) that variability must be buffered and (2) how the appropriate buffering strategy depends on the production environment and business strategy. We deliberately include some nonmanufacturing examples to emphasize that the variability laws apply to production systems for services as well as for goods.

Ballpoint pens. Suppose a retailer sells inexpensive ballpoint pens. Demand is unpredictable (variable). But since customers will go elsewhere if they do not find the item in stock (who is going to backorder a cheap ballpoint pen?), the retailer cannot buffer this variability with time. Likewise, because the instant-delivery requirement of the customer rules out a make-to-order environment, capacity cannot be used as a buffer. This leaves only inventory. And indeed, this is precisely what the retailer creates by holding a stock of pens.

Emergency service. Demand for fire or ambulance service is necessarily variable, since we obviously cannot get people to schedule their emergencies. We cannot buffer this variability with inventory (an inventory of trips to the hospital?). We cannot buffer with time, since response time is *the* key performance measure for this system. Hence, the only available buffer is capacity. And indeed, utilization of fire engines and ambulances is very low. The "excess" capacity is necessary to cover peaks in demand.

Organ transplants. Demand for organ transplants is variable, as is supply, since we cannot schedule either. Since the supply rate is fixed by donor deaths, we cannot (ethically) increase capacity. Since organs have a very short usable life after the donor dies, we cannot use inventory as a buffer. This leaves only time. And indeed, the waiting time for most organ transplants is very long. Even medical production systems must obey the laws of factory physics.

The Toyota Production System. The Toyota production system was the birthplace of JIT and remains the paragon of lean manufacturing. On the basis of its success, Toyota rose from relative obscurity to become one of the world's leading auto manufacturers. How did they do it?

First, Toyota reduced variability at every opportunity. In particular:

1. *Demand variability.* Toyota's product design and marketing were so successful that demand for its cars consistently exceeded supply (the Big Three in America also did their part by building particularly shoddy cars in the late 1970s). This helped in several ways. First, Toyota was able to limit the number of options of cars produced. A maroon Toyota would always have maroon interior. Many options, such as chrome packages and radios, were dealer installed. Second, Toyota could establish a production schedule months in advance. This virtually eliminated all demand variability seen by the manufacturing facility.
2. *Manufacturing variability.* By focusing on setup reduction, standardizing work practices, total quality management, error proofing, total preventive maintenance, and other flow-smoothing techniques, Toyota did much to eliminate variability inside its factories.
3. *Supplier variability.* The Toyota-supplier relationship in the early 1930s hinted of feudalism. Because Toyota was such a large portion of its suppliers' demand, it had enormous leverage. Indeed, Toyota executives often sat as directors on the boards of its suppliers. This ensured that (1) Toyota got the supplies it needed when it needed them, (2) suppliers adopted variability reduction techniques "suggested" to them by Toyota, and (3) the suppliers carried any necessary buffer inventory.

Second, Toyota made use of capacity buffers against remaining manufacturing variability. It did this by scheduling plants for less than three shifts per day and making use of preventive maintenance periods at the end of shifts to make up any

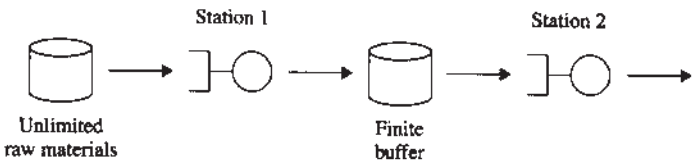


FIGURE 9.3
"Pay me now or pay me later" scenario

shortfalls relative to production quotas. The result was a very predictable daily production rate.

Third, despite the propensity of American JIT writers to speak in terms of "zero inventories" and "evil inventory," Toyota did carry WIP and finished goods inventories in its system. But because of its vigorous variability reduction efforts and willingness to buffer with capacity, the amount of inventory required was far smaller than was typical of auto manufacturers in the 1980s.

9.2.4 Pay Me Now or Pay Me Later

The Buffering Law could also be called the "law of pay me now or pay me later" because if you do not pay to reduce variability, you *will* pay in one or more of the following ways:

- Lost throughput.
- Wasted capacity.
- Inflated cycle times.
- Larger inventory levels.
- Long lead times and/or poor customer service.

To examine the implications of the Buffering Law in more concrete manufacturing terms, we consider the simple two-station line shown in Figure 9.3. Station 1 pulls in jobs, which contain 50 pieces, from an unlimited supply of raw materials, processes them, and sends them to a buffer in front of station 2. Station 2 pulls jobs from the buffer, processes them, and sends them downstream. Throughout this example, we assume station 1 requires 20 minutes to process a job and is the bottleneck. This means that the theoretical capacity is 3,600 pieces per day (24 hours/day × 60 minutes/hour × 1 job/20 minutes × 50 pieces/job).[3]

To start with, we assume that station 2 also has average processing times of 20 minutes, so that the line is balanced. Thus, the theoretical minimum cycle time is 40 minutes, and the minimum WIP level is 100 pieces (one job per station). However, because of variability, the system cannot achieve this ideal performance. Below we discuss the results of a computer simulation model of this system under various conditions, to illustrate the impacts of changes in capacity, variability, and buffer space. These results are summarized in Table 9.4.

Balanced, Moderate Variability, Large Buffer. As our starting point, we consider the balanced line where both machines have mean process times of 20 minutes per job and are moderately variable (i.e., have process CVs equal to one, so $c_e(1) = c_e(2) = 1$)

[3]This is the same system that was considered in Problem 10 of Chapter 8.

TABLE 9.4 Summary of Pay-Me-Now-or-Pay-Me-Later Simulation Results

Case	Buffer (Jobs)	$t_e(2)$ (Minutes)	CV	TH (per Day)		CT (Minutes)	WIP (Pieces)
				E_{TH}	E_u	E_{CT}	E_{inv}
1	10	20	1	3,321		150	347
				0.9225	0.9225	0.2667	0.2659
2	1	20	1	2,712		60	113
				0.7533	0.7533	0.6667	0.6667
3	1	10	1	3,367		36	83
				0.9353	0.7015	0.8333	0.8451
4	1	20	0.25	3,443		51	123
				0.9564	0.9564	0.7843	0.7776

and the interstation buffer holds 10 jobs (500 pieces).[4] A simulation of this system for 1,000,000 minutes (694 days running 24 hours/day) estimates throughput of 3,321 pieces/day, an average cycle time of 150 minutes, and an average WIP of 347 pieces. We can check Little's Law (WIP = TH × CT) by noting that throughput can be expressed as 3,321 pieces/day ÷ 1,440 minutes/day = 2.3 pieces/minute, so

$$347 \text{ pieces} \approx 2.3 \text{ pieces/minute} \times 150 \text{ minutes} = 345 \text{ pieces}$$

Because we are simulating a system involving variability, the estimates of TH, CT, and WIP are necessarily subject to error. However, because we used a long simulation run, the system was allowed to stabilize and therefore very nearly complies with Little's Law.

Notice that while this configuration achieves reasonable throughput (i.e., only 7.7 percent below the theoretical maximum of 3,600 pieces per day), it does so at the cost of high WIP and long cycle times. The reason is that fluctuations in the speeds of the two stations causes the interstation buffer to fill up regularly, which inflates both WIP and cycle time. Hence, the system is using WIP as the primary buffer against variability.

Balanced, Moderate Variability, Small Buffer. One way to reduce the high WIP and cycle time of the above case is by fiat. That is, simply reduce the size of the buffer. This is effectively what implementing a low-WIP kanban system without any other structural changes would do. To give a stark illustration of the impacts of this approach, we reduce buffer size from 10 jobs to 1 job. If the first machine finishes when the second has one job in queue, it will wait in a nonproductive *blocked* state until the second machine is finished.

[4]Note that because the line is balanced and has an unlimited supply of work at the front, utilization at both machines would be 100 percent if the interstation buffer were infinitely large. But this would result in an unstable system in which the WIP would grow to infinity. A finite buffer will occasionally become full and block station 1, choking off releases and preventing WIP from growing indefinitely. This serves to stabilize the system and makes it more representative of a real production system, in which WIP levels would never be allowed to become infinite.

Our simulation model confirms that the small buffer reduces cycle time and WIP as expected, with cycle time dropping to around 60 minutes and WIP dropping to around 113 pieces. However, throughput also drops to around 2,712 pieces per day (an 18 percent decrease relative to the first case). Without the high WIP level in the buffer to protect station 2 against fluctuations in the speed of station 1, station 2 frequently becomes starved for jobs to work on. Hence, throughput and revenue seriously decline. Because utilization of station 2 has fallen, the system is now using capacity as the primary buffer against variability. However, in most environments, this would not be an acceptable price to pay for reducing cycle time and WIP.

Unbalanced, Moderate Variability, Small Buffer. Part of the reason that stations 1 and 2 are prone to blocking and starving each other in the above case is that their capacities are identical. If a job is in the buffer and station 1 completes its job before station 2 is finished, station 1 becomes *blocked;* if the buffer is empty and station 2 completes its job before station 1 is finished, station 2 becomes *starved.* Since both situations occur often, neither station is able to run at anything close to its capacity.

One way to resolve this is to unbalance the line. If either machine were significantly faster than the other, it would almost always finish its job first, thereby allowing the other station to operate at close to its capacity. To illustrate this, we suppose that the machine at station 2 is replaced with one that runs twice as fast (i.e., has mean process times of $t_e(2) = 10$ minutes per job), but still has the same CV (that is, $c_e(2) = 1$). We keep the buffer size at one job.

Our simulation model predicts a dramatic increase in throughput to 3,367 pieces per day, while cycle time and WIP level remain low at 36 minutes and 83 pieces, respectively. Of course, the price for this improved performance is wasted capacity—the utilization of station 2 is less than 50 percent—so the system is again using capacity as a buffer against variability. If the faster machine is inexpensive, this might be attractive. However, if it is costly, this option is almost certainly unacceptable.

Balanced, Low Variability, Small Buffer. Finally, to achieve high throughput with low cycle time and WIP *without* resorting to wasted capacity, we consider the option of reducing variability. In this case, we return to a balanced line, with both stations having mean process times of 20 minutes per job. However, we assume the process CVs have been reduced from 1.0 to 0.25 (i.e., from the moderate-variability category to the low-variability category).

Under these conditions, our simulation model shows that throughput is high, at 3,443 pieces per day; cycle time is low, at 51 minutes; and WIP level is low, at 123 pieces. Hence, if this variability reduction is feasible and affordable, it offers the best of all possible worlds. As we noted in Chapter 8, there are many options for reducing process variability, including improving machine reliability, speeding up equipment repairs, shortening setups, and minimizing operator outages, among others.

Comparison. As we can see from the summary in Table 9.4, the above four cases are a direct illustration of the pay-me-now-or-pay-me-later interpretation of the Variability Buffering Law. In the first case, we "pay" for throughput by means of long cycle times and high WIP levels. In the second case, we pay for short cycle times and low WIP levels with lost throughput. In the third case we pay for them with wasted capacity. In the fourth case, we pay for high throughput, short cycle time, and low WIP through

variability reduction. While the Variability Buffering Law cannot specify which form of payment is best, it does serve warning that some kind of payment *will* be made.

9.2.5 Flexibility

Although variability always requires some kind of buffer, the effects can be mitigated somewhat with **flexibility.** A flexible buffer is one that can be used in more than one way. Since a flexible buffer is more likely to be available when and where it is needed than a fixed buffer is, we can state the following corollary to the buffering law.

Corollary (Buffer Flexibility): *Flexibility reduces the amount of variability buffering required in a production system.*

An example of flexible capacity is a cross-trained workforce. By floating to operations that need the capacity, flexible workers can cover the same workload with less total capacity than would be required if workers were fixed to specific tasks.

An example of flexible inventory is generic WIP held in a system with late product customization. For instance, Hewlett-Packard produced generic printers for the European market by leaving off the country-specific power connections. These generic printers could be assembled to order to fill demand from any country in Europe. The result was that significantly less generic (flexible) inventory was required to ensure customer service than would have been required if fixed (country-specific) inventory had been used.

An example of flexible time is the practice of quoting variable lead times to customers depending on the current work backlog (i.e., the larger the backlog, the longer the quote). A given level of customer service can be achieved with shorter average lead time if variable lead times are quoted individually to customers than if a uniform fixed lead time is quoted in advance. We present a model for lead time quoting in Chapter 15.

There are many ways that flexibility can be built into production systems, through product design, facility design, process equipment, labor policies, vendor management, etc. Finding creative new ways to make resources more flexible is the central challenge of the mass customization approach to making a diverse set of products at mass production costs.

9.2.6 Organizational Learning

The pay-me-now-or-pay-me-later example suggests that adding capacity and reducing variability are, in some sense, interchangeable options. Both can be used to reduce cycle times for a given throughput level or to increase throughput for a given cycle time. However, there are certain intangibles to consider. First is the ease of implementation. Increasing capacity is often an easy solution—just buy some more machines—while decreasing variability is generally more difficult (and risky), requiring identification of the source of excess variability and execution of a custom-designed policy to eliminate it. From this standpoint, it would seem that if the costs and impacts to the line of capacity expansion and variability reduction are the same, capacity increases are the more attractive option.

But there is a second important intangible to consider—*learning.* A successful variability reduction program can generate capabilities that are transferable to other parts of the business. The experience of conducting systems analysis studies (discussed in Chapter 6), the resulting improvements in specific processes (e.g., reduced setup times or rework), and the heightened awareness of the consequences of variability by the workforce are examples of benefits from a variability reduction program whose

impact can spread well beyond that of the original program. The mind-set of variability reduction promotes an environment of continual process capability improvement. This can be a source of significant competitive advantage—anyone can buy more machinery, but not everyone can constantly upgrade the ability to use it. For this reason, we believe that variability reduction is frequently the preferred improvement option, which should be considered seriously before resorting to capacity increases.

9.3 Flow Laws

Variability impacts the way material flows through the system and how much capacity can be actually utilized. In this section we describe laws concerning material flow, capacity, utilization, and variability propagation.

9.3.1 Product Flows

We start with an important law that comes directly from (natural) physics, namely *Conservation of Material.* In manufacturing terms, we can state it as follows:

Law (Conservation of Material): *In a stable system, over the long run, the rate out of a system will equal the rate in, less any yield loss, plus any parts production within the system.*

The phrase *in a stable system* requires that the input to the system not exceed (or even be equal to) its capacity. The next phrase, *over the long run,* implies that the system is observed over a significantly long time. The law can obviously be violated over shorter intervals. For instance, more material may come out of a plant than went into it—for awhile. Of course, when this happens, WIP in the plant will fall and eventually will become zero, causing output to stop. Thus, the law cannot be violated indefinitely. The last phrases, *less any yield loss* and *plus any parts production* are important caveats to the simpler statement, *input must equal output.* Yield losses occur when the number of parts in a system is reduced by some means other than output (e.g., scrap or damage). Parts production occurs whenever one part becomes multiple parts. For instance, one piece of sheet metal may be cut into several smaller pieces by a shearing operation.

This law links the utilization of the individual stations in a line with the throughput. For instance, in a serial line with no yield loss operating under an MRP (push) protocol, throughput at any station i, $\mathrm{TH}(i)$, plus the line throughput itself, TH, equals the release rate r_a into the line. The reason, of course, is that what goes in must come out (provided that the release rate is less than the capacity of the line, so that it is stable). Then the utilization at each station is given by the ratio of the throughput to the station capacity (for example, $u(i) = \mathrm{TH}(i)/r_e(i) = r_a/r_e(i)$ at station i).

Finally, this law is behind our choice to define the bottleneck as the *busiest* station, not necessarily the *slowest* station. For example, if a line has yield loss, then a slower station later in the line may have a lower utilization than a faster station earlier in the line (i.e., because the earlier station processes parts that are later scrapped). Since the earlier station will serve to constrain the performance of the line, it is rightly deemed the bottleneck.

9.3.2 Capacity

The Conservation of Material Law implies that the capacity of a line must be at least as large as the arrival rate to the system. Otherwise, the WIP levels would continue to grow

and never stabilize. However, when one considers variability, this condition is not strong enough. To see why, recall that the queueing models presented in Chapter 8 indicated that both WIP and cycle time go to infinity as utilization approaches one if there is no limit on how much WIP can be in the system. Therefore, to be stable, all workstations in the system must have a processing rate that is *strictly greater* than the arrival rate to that station. It turns out that this behavior is not some sort of mathematical oddity, but is, in fact, a fundamental principle of factory physics.

To see this, note that if a production system contains variability (and all real systems do), then regardless of the WIP level, we can always find a possible sequence of events that causes the system bottleneck to *starve* (run out of WIP). The only way to ensure that the bottleneck station does not starve is to *always* have WIP in the queue. However, no matter how much WIP we begin with, there exists a set of process and interarrival times that will eventually exhaust it. The only way to *always* have WIP is to start with an *infinite* amount of it. Thus, for r_a (arrival rate) to be equal to r_e (process rate), there must be an infinite amount of WIP in the queue. But by Little's Law this implies that cycle time will be infinite as well.

There is one exception to this behavior. When both c_a^2 and c_e^2 are equal to zero, then the system is completely deterministic. For this case, we have *absolutely no* randomness in either interarrival or process time, and the arrival rate is *exactly* equal to the service rate. However, since modern physics ("natural," not "factory") tells us that there is always some randomness present, this case will never arise in practice.

At this point, the reader with a practical bent may be skeptical, thinking something like, "Wait a minute. I've been in a lot of plants, many of which do their best to set work releases equal to capacity, and I've yet to see a single one with an *infinite* amount of WIP." This is a valid point, which brings up the important concept of **steady state.**

Steady state is related to the notion of a "stable system" and "long-run" performance, discussed in the conservation of material law. For a system to be in steady state, the parameters of the system must *never* change and the system must have been operating long enough that initial conditions no longer matter.[5] Since our formulas were derived under the assumption of steady state, the discrepancy between our analysis (which is correct) and what we see in real life (which is also correct) must lie in our view of the steady state of a manufacturing system.

The Overtime Vicious Cycle. What really happens in steady state is that a plant runs through a series of "cycles," in which system parameters are changed over time. A common type of behavior is the "overtime vicious cycle," which goes as follows:

1. Plant capacity is computed by taking into consideration detractors such as random outages, recycle, setups, operator unavailability, breaks, and lunches.
2. The master production schedule is filled according to this effective capacity. Release rates are now essentially the same as capacity.[6]
3. Sooner or later, due to randomness in job arrivals, in process times, or in both, the bottleneck process starves.

[5] Recall in the Penny Fab examples of Chapter 7 that the line had to run for awhile to work out of a transient condition caused by starting up with all pennies at the first station. There, steady state was reached when the line began to cycle through the same behavior over and over. In lines with variability, the actual behavior will not repeat, but the probability of finding the system in a given state will stabilize.

[6] Notice that if there has been some wishful thinking in computing capacity, release rates may well be *greater* than capacity.

4. More work has gone in than has gone out, so WIP increases.
5. Since the system is at capacity, throughput remains relatively constant. From Little's Law, the increase in WIP is reflected by a nearly proportional increase in cycle times.
6. Jobs become late.
7. Customers begin to complain.
8. After WIP and cycle times have increased enough and customer complaints grow loud enough, management decides to take action.
9. A "one-time" authorization of overtime, adding a shift, subcontracting, rejection of new orders, etc., is allowed.
10. As a consequence of step 9, effective capacity is now significantly greater than the release rate. For instance, if a third shift was added, utilization dropped from 100 percent to around 67 percent.
11. WIP level decreases, cycle times go down, and customer service improves. Everyone breaths a sigh of relief, wonders aloud how things got so out of hand, and promises to never let it happen again.
12. *Go to step 1!*

The moral of the overtime vicious cycle is that although management may *intend* to release work at the rate of the bottleneck, in steady state, it *cannot.* Whenever overtime, or adding a shift, or working on a weekend, or subcontracting, etc., is authorized, plant capacity suddenly jumps to a level significantly greater than the release rate. (Likewise, order rejection causes release rate to suddenly fall below capacity.) Thus, over the long run, *average* release rate is *always* less than *average* capacity. We can sum up this fact of manufacturing life with the following law of factory physics.

Law (Capacity): *In steady state, all plants* will *release work at an average rate that is strictly less than the average capacity.*

This law has profound implications. Since it is impossible to achieve true 100 percent utilization of plant resources, the real management decision concerns whether measures such as excess capacity, overtime, or subcontracting will be part of a planned strategy or will be used in response to conditions that are spinning out of control. Unfortunately, because many manufacturing managers fail to appreciate this law of factory physics, they unconsciously choose to run their factories in constant "fire-fighting" mode.

9.3.3 Utilization

The Buffering Law and the *VUT* equation suggest that there are two drivers of queue time: utilization and variability. Of these, utilization has the most dramatic effect. The reason is that the *VUT* equation (for single- or multiple-machine stations) has a $1 - u$ term in the denominator. Hence as utilization u approaches one, cycle time approaches infinity. We can state this as the following law.

Law (Utilization): *If a station increases utilization without making any other changes, average WIP and cycle time will increase in a highly nonlinear fashion.*

In practice, it is the phrase *in a highly nonlinear fashion* that generally presents the real problem. To illustrate why, suppose utilization is $u = 97$ percent, cycle time is two days, and the CVs of both process times c_e and interarrival times c_a are equal to one. If we increase utilization by one percent to $u = 0.9797$, cycle time becomes 2.96 days,

a 48 percent increase. Clearly, cycle time is very sensitive to utilization. Moreover, this effect becomes even more pronounced as u gets closer to one, as we can see in Figure 9.4. This graph shows the relationship between cycle time and utilization for $V = 1.0$ and $V = 0.25$, where $V = (c_a^2 + c_e^2)/2$. Notice that both curves "blow up" as u gets close to 1.0, but the curve corresponding to the system with higher variability ($V = 1.0$) blows up faster. From Little's Law, we can conclude that WIP similarly blows up as u approaches one.

A couple of technical caveats are in order. First, if $V = 0$, then cycle time remains constant for all utilization levels up to 100 percent and then becomes infinite (infeasible) when utilization becomes greater than 100 percent. In analogous fashion to the best-case line we studied in Chapter 7, a station with absolutely no variability can operate at 100 percent utilization without building a queue. But since all real stations contain some variability, this never occurs in practice.

Second, no real-world station has space to build an infinite queue. Space, time, or policy will serve to cap WIP at some finite level. As we saw in the blocking models of Chapter 8, putting a limit on WIP without any other changes causes throughput (and hence utilization) to decrease. Thus, the qualitative relationship in Figure 9.4 still holds, but the limit on queue size will make it impossible to reach the high utilization/high cycle time parts of the curve.

The extreme sensitivity of system performance to utilization makes it very difficult to choose a release rate that achieves both high station efficiency and short cycle times. Any errors, particularly those on the high side (which are likely to occur as a result of optimism about the system's capacity, coupled with the desire to maximize output), can result in large increases in average cycle time. We will discuss structural changes for addressing this issue in Chapter 10 in the context of push and pull production systems.

9.3.4 Variability and Flow

The Variability Law states that variability degrades performance of all production systems. But how much it degrades performance can depend on *where* in the line the variability is created. In lines without WIP control, increasing process variability at any station will (1) increase the cycle time at that station *and* (2) propagate more variability to downstream stations, thereby increasing cycle time at them as well. This observation

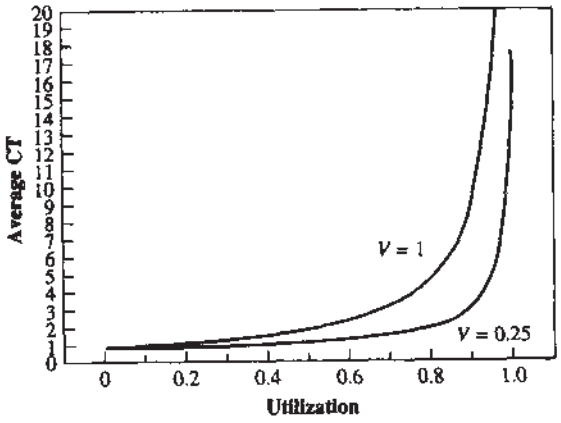


FIGURE 9.4
Relation between cycle time and utilization

motivates the following corollary of the Variability Law and the propagation property of Chapter 8.

Corollary (Variability Placement): *In a line where releases are independent of completions, variability early in a routing increases cycle time more than equivalent variability later in the routing.*

The implication of this corollary is that efforts to reduce variability should be directed at the front of the line first, because that is where they are likely to have the greatest impact (see Problem 12 for an illustration).

Note that this corollary applies *only* where releases are independent of completions. In a CONWIP line, where releases are directly tied to completions, the flow at the first station is affected by flow at the last station just as strongly as the flow at station $i + 1$ is affected by the flow at station i. Hence, there is little distinction between the front and back of the line and little incentive to reduce variability early as opposed to late in the line. The variability placement corollary, therefore, is applicable to push rather than pull systems.

9.4 Batching Laws

A particularly dramatic cause of variability is batching. As we saw in the worst-case performance in Chapter 7, maximum variability can occur when moving product in large batches even when process times themselves are constant. The reason in that example was that the effective interarrival times were large for the first part in a batch and zero for all others (because they arrived simultaneously). The result was that each station "saw" highly variable arrivals, hence the average cycle time was as bad as it could possibly be for a given bottleneck rate and raw process time. Because batching can have such a large effect on variability, and hence performance, setting batch sizes in a manufacturing system is a very important control. However, before we try to compute "optimal" batch sizes (which we will save for Chapter 15 as part of our treatment of scheduling), we need to understand the effects of batching on the system.

9.4.1 Types of Batches

An issue that sometimes clouds discussions of batching is that there are actually two kinds of batches. Consider a dedicated assembly line that makes only one type of product. After each unit is made, it is moved to a painting operation. What is the batch size?

On one hand, you might say it is *one* because after each item is complete, it can be moved to the painting operation. On the other hand, you could argue that the batch size is *infinity* since you never perform a changeover (i.e., the number of parts between changeovers is infinite). Since one is not equal to infinity, which is correct?

The answer is that both are correct. But there are two different kinds of batches: **process batches** and **transfer batches.**

Process batch. There are two types of process batches. The **serial batch** size is the number of jobs of a common family processed before the workstation is changed over to another family. We call these *serial* batches because the parts are produced serially (one at a time) on the workstation. **Parallel batch** size is the number of parts produced simultaneously in a true batch workstation, such as a furnace or heat treat operation. Although serial and parallel batches are very different physically, they have similar operational impacts, as we will see.

The size of a serial process batch is related to the length of a changeover or setup. The longer the setup, the more parts must be produced between setups to achieve a given capacity. The size of a parallel process batch depends on the demand placed on the station. To minimize utilization, such machines should be run with a full batch. However, if the machine is not a bottleneck, then minimizing utilization may not be critical, so running less than a full load may be the right thing to do to reduce cycle times.

Transfer batch. This is the number of parts that accumulate before being transferred to the next station. The smaller the transfer batch, the shorter the cycle time since there is less time waiting for the batch to form. However, smaller transfer batches also result in more material handling, so there is a tradeoff. For instance, a forklift might be needed only once per shift to move material between adjacent stations in a line if moves are made in batches of 3,000 units. However, the operator would have to make 30 trips per shift to move material between the stations in batches of 100 units.

Strictly speaking, if one considers the material handling operation between stations to be a process, a transfer batch is simply a parallel process batch. The forklift can transfer 10 parts as quickly as one, just as a furnace can bake 10 parts as quickly as one. Nonetheless, since it is intuitive to think of material handling as distinct from processing, we will consider transfer and process batching separately.

The distinction between process and transfer batches is sometimes overlooked. Indeed, from the time Ford Harris first derived the EOQ in 1913 until recently, most production planners simply assumed that these two batches should be equal. But this need not be so. In a system where setups are long but processes are close together, it might make good sense to keep process batches large and transfer batches small. This practice is called **lot splitting** and can significantly reduce the cycle time (we discuss this in greater detail in Section 9.5.3).

9.4.2 Process Batching

Recall from Chapter 4 that JIT advocates are fond of calling for batch sizes of one. The reason is that if processing is done one part at a time, no time is spent waiting for the batch to form and less time is spent waiting in a queue of large batches. However, in most real-world systems, setting batch sizes equal to one is not so simple. The reason is that batch size can affect *capacity*. It may well be the case that processing in batches of one will cause a workstation to become overutilized (due to excessive setup time or excessive parallel batch process time). The challenge, therefore, is to balance these capacity considerations with the delays that batching introduces (see Karmarkar (1987) for a more complete discussion). We can summarize the key dynamics of serial and parallel process batching in the following factory physics law.

Law (Process Batching): *In stations with batch operations or significant changeover times:*

1. *The minimum process batch size that yields a stable system may be greater than one.*
2. *As process batch size becomes large, cycle time grows proportionally with batch size.*
3. *Cycle time at the station will be minimized for some process batch size, which may be greater than one.*

We can illustrate the relationship between capacity and process batching described in this law with the following examples.

Example: Serial Process Batching

Consider a machining station that processes several part families. The parts arrive in batches where all parts within batches are of like family, but the batches are of different families. The arrival rate of batches is set so that parts arrive at a rate of 0.4 part per hour. Each part requires one hour of processing regardless of family type. However, the machine requires a five-hour setup between batches (because it is assumed to be switching to a different family). Hence, the choice of batch size will affect both the number of setups required (and hence utilization) and the time spent waiting in a partial batch. Furthermore, the cycle time will be affected by whether parts exit the station in a batch when the whole batch is complete or one at a time if lot splitting is used.

Notice that if we were to use a batch size of one, we could only process one part every six hours (five hours for the setup plus one hour for processing), which does not keep up with arrivals. The smallest batch size we can consider is four parts, which will enable a capacity of four parts every nine hours (five hours for setup plus four hours to process the parts), or a rate of 0.44 part per hour.

Figure 9.5 graphs the cycle time at the station for a range of batch sizes with and without lot splitting. Notice that minimum feasible batch size yields an average cycle time of approximately 70 hours without lot splitting and 68 hours with lot splitting. Without lot splitting, the minimum cycle time is about 31 hours and is achieved at a batch size of eight parts. With lot splitting, it is about 22 hours and is achieved at a batch size of nine parts. Above these minimal levels, cycle time grows in an almost straight-line fashion, with the lot splitting case outperforming (achieving smaller cycle times than) the nonsplitting case by an increasing margin.

The Process Batching Law implies that it may be necessary, even desirable, to use large process batches in order to keep utilization, and hence cycle time and WIP, under control. But one should be careful about accepting this conclusion without question. The need for large serial batch sizes is caused by long setup times. Therefore, the first priority should be to try to reduce setup times as much as economically practical. For instance, Figure 9.5 shows the behavior of the machining station example, but with average setup times of two and one-half hours instead of five hours. Notice that with shorter setup times, minimal cycle times are roughly 50 percent smaller (around 16 hours without lot splitting and 11 hours with lot splitting) and are attained at smaller batch sizes (four parts without lot splitting and five parts with lot splitting). So the full implication of the above law is that batching *and* setup time reduction must be used in concert to achieve high throughput and efficient WIP and cycle time levels.

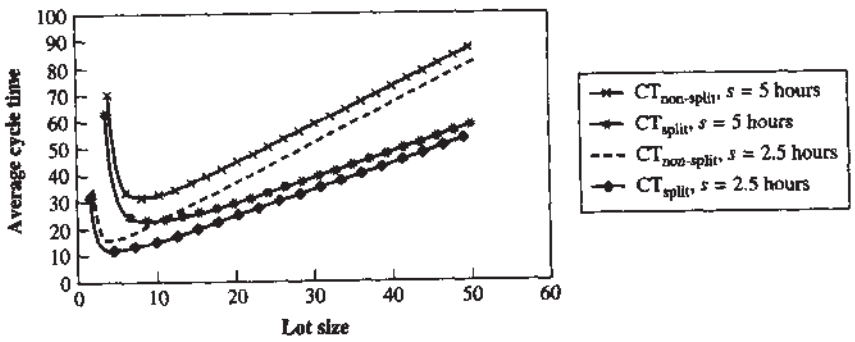


FIGURE 9.5

Cycle time versus serial batch size at a station with five-hour and two-and-one-half-hour setup times

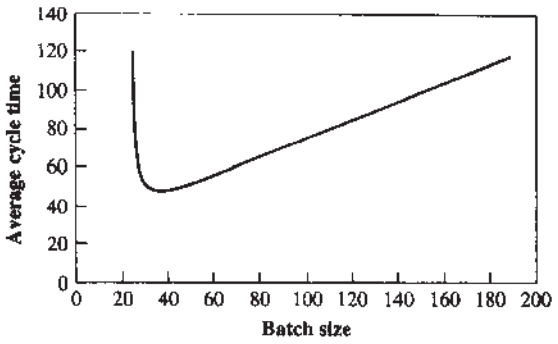


FIGURE 9.6
Cycle time versus parallel batch size in a batch operation

Example: Parallel Process Batching

Consider the burn-in operation of a facility that produces medical diagnostic units. The operation involves running a batch of units through multiple power-on and diagnostic cycles inside a temperature-controlled room, and it requires 24 hours regardless of how many units are being burned in. The burn-in room is large enough to hold 100 units at a time. Suppose units arrive to burn in at a rate of one per hour (24 per day). Clearly, if we were to burn in one unit at a time, we would only have capacity of $\frac{1}{24}$ per hour, which is far below the arrival rate. Indeed, if we burn in units in batches of 24, then we will have capacity of one per hour, which would make utilization equal to 100 percent. Since utilization must be less than 100 percent to achieve stability, the smallest feasible batch size is 25.

Figure 9.6 plots the cycle time as a function of batch size. It turns out that cycle time is minimized at a batch size of 32, which achieves a cycle time of 43 hours. Since 24 hours of this is process time, the rest is queue time and wait-to-batch time. We will develop the formulas for computing these quantities later.

Serial Batching. We can give a deeper interpretation of the batching–cycle time interactions underlying the process batching law by examining the models behind the above examples. We begin with the serial batching case of Figure 9.5 in the following technical note.

Technical Note—Serial Batching Interactions

To model serial batching, in which batches of parts arrive at a single machine and are processed with a setup between each batch, we make use of the following notation:

k = serial batch size
r_a = arival rate (parts per hour)
t = time to process a single part (hour)
s = time to perform a setup (hour)
c_e^2 = effective SCV for processing time of a batch, including both process time and setup time

Furthermore, we make these simplifying assumptions: (1) The SCV c_e^2 of the effective process time of a batch is equal to 0.5 regardless of batch size[7] and (2) the arrival SCV (of batches) is always one.

[7]We could fix the CV for processing individual jobs and compute the CV for a batch as a function of batch size. However, the model assuming a constant arrival CV for batches exhibits the same principal behavior—a sharp increase in cycle time for small batches and the linear increase for large batches—and is much easier to analyze.

Since r_a is the arrival rate of *parts*, the arrival rate of batches is r_a/k. The effective process time for a batch is given by the time to process the k parts in the batch plus the setup time

$$t_e = kt + s \tag{9.1}$$

so machine utilization is

$$u = \frac{r_a}{k}(kt + s) = r_a\left(t + \frac{s}{k}\right) \tag{9.2}$$

Notice that for stability we must have $u < 1$, which requires

$$k > \frac{sr_a}{1 - tr_a}$$

The average time in queue CT_q is given by the *VUT* equation

$$\mathrm{CT}_q = \left(\frac{1 + c_e^2}{2}\right)\left(\frac{u}{1 - u}\right)t_e \tag{9.3}$$

where t_e and u are given by Equations (9.1) and (9.2).

The total average cycle time at the station consists of queue time plus setup time plus wait-in-batch time (WIBT) plus process time. WIBT depends on whether lots are split for purposes of moving parts downstream. If they are not (i.e., the entire batch must be completed before any of the parts are moved downstream), then all parts wait for the other $k - 1$ parts in the batch, so

$$\mathrm{WIBT}_{\mathrm{nonsplit}} = (k - 1)t$$

and total cycle time is

$$\begin{aligned}\mathrm{CT}_{\mathrm{nonsplit}} &= \mathrm{CT}_q + s + \mathrm{WIBT}_{\mathrm{nonsplit}} + t \\ &= \mathrm{CT}_q + s + (k - 1)t + t \\ &= \mathrm{CT}_q + s + kt\end{aligned} \tag{9.4}$$

If lots are split (i.e., individual parts are sent downstream as soon as they have been processed, so that transfer batches of one are used), then wait-in-batch time depends on the position of the part in the batch. The first part spends no time waiting, since it departs immediately after it is processed. The second part waits behind the first part and hence spends t waiting in batch. The third part spends $2t$ waiting in batch, and so on. The average time for the k jobs to wait in batch is therefore

$$\mathrm{WIBT}_{\mathrm{split}} = \frac{k - 1}{2}t$$

so that

$$\begin{aligned}\mathrm{CT}_{\mathrm{split}} &= \mathrm{CT}_q + s + \mathrm{WIBT}_{\mathrm{split}} + t \\ &= \mathrm{CT}_q + s + \frac{k - 1}{2}t + t \\ &= \mathrm{CT}_q + s + \frac{k + 1}{2}t\end{aligned} \tag{9.5}$$

Equations (9.4) and (9.5) are the basis for Figure 9.5. We can give a specific illustration of their use by using the data from the Figure 9.5 example ($r_a = 0.4$, $c_a^2 = 1$, $t = 1$, $c_e^2 = 0.5$, $s = 5$) for $k = 10$, so that

$$t_e = s + kt = 5 + 10 \times 1 = 15 \text{ hours}$$

Machine utilization is

$$u = \frac{r_a t_e}{k} = \frac{(0.4 \text{ part/hour})(15 \text{ hours})}{10} = 0.6$$

The expected time in queue for a batch is

$$\mathrm{CT}_q = \left(\frac{1 + 0.5}{2}\right)\left(\frac{0.6}{1 - 0.6}\right)15 = 16.875 \text{ hours}$$

So if we do not use lot splitting, average cycle time is

$$\text{CT}_{\text{nonsplit}} = \text{CT}_q + s + kt = 16.875 + 5 + 10(1) = 31.875 \text{ hours}$$

If we do split process batches into transfer batches of size one, average cycle time is

$$\text{CT}_{\text{split}} = \text{CT}_q + s + \frac{k+1}{2}t = 16.875 + 5 + \frac{10+1}{2}(1) = 27.375 \text{ hours}$$

which is smaller, as expected.

The main conclusion of this analysis of serial batching is that if setup times can be made sufficiently short, then using serial process batch sizes of one is an effective way to reduce cycle times. However, if short setup times are not possible (at least in the near term), then cycle time can be sensitive to the choice of process batch size and the "best" batch size may be significantly greater than one.

Parallel Process Batching. Depending on the control policy, a serial batching operation can start on a batch before the entire batch is present at the station and can release jobs in the batch before the entire batch has been processed. (We will examine the manner in which this causes cycle time to "overlap" at stations in the next section.) But in a parallel batching operation, such as a heat treat furnace, a bake oven, or a burn-in room, the entire batch is processed at once and therefore must begin and end processing at the same time. This makes analysis of parallel process batching slightly different from analysis of serial process batching.

Total cycle time at a parallel batching station includes wait-to-batch time (the time to accumulate a full batch), queue time (the time full batches wait in queue), and processing time. We develop formulas for these in the following technical note.

Technical Note—Parallel Batching Interactions

We assume that parts arrive one at a time to the parallel batch operation. They wait to form a batch, may wait in a queue of batches, and then are processed as a batch. We make use of the following notation, which is similar to that used for the serial batching case.

k = parallel batch size
r_a = arrival rate (parts per hour)
c_a = CV of interarrival times
t = time to process batch (hour)
c_e = effective CV for processing time of batch
B = maximum batch size (number of parts that can fit into process)

To calculate the average wait-to-batch time (WTBT), note that the average time between arrivals is $1/r_a$. The first part in a batch waits for $k-1$ other parts to arrive and hence waits $(k-1)/r_a$ hour. The last part in a batch does not wait at all to form a batch. Hence, the average time a part waits to form a batch is the average of these two extremes, or

$$\text{WTBT} = \frac{k-1}{2r_a}$$

Once k arrivals have occurred, we have a full batch to move either into the queue or into the process. Hence, the interarrival times of batches are equal to the sum of k interarrival times of parts. As we saw in Chapter 8, adding k independent, identically distributed random

variables with SCVs of c^2 results in a random variable with an SCV of c^2/k. Therefore, the arrival SCV of batches is given by

$$c_a^2(\text{batch}) = \frac{c_a^2}{k}$$

The capacity of the process with batch size k is k/t, so the maximum capacity is B/t. To keep utilization below 100 percent, effective capacity must be greater than demand, so we require

$$u = \frac{r_a}{k/t} < 1$$

or

$$k > r_a t$$

If B is less than or just equal to $r_a t$, then there is insufficient capacity to meet demand.

Once a batch is formed, it goes to the batch process. If utilization is high and there is variability, there is likely to be a queue. The queue time can be computed by using the *VUT* equation to be

$$\text{CT}_q = \left(\frac{c_a^2/k + c_e^2}{2}\right)\left(\frac{u}{1-u}\right)t$$

Consequently, total cycle time is

$$\begin{aligned}\text{CT} &= \text{WTBT} + \text{CT}_q + t \\ &= \frac{k-1}{2r_a} + \left(\frac{c_a^2/k + c_e^2}{2}\right)\left(\frac{u}{1-u}\right)t + t \\ &= \frac{k-1}{2ku}t + \left(\frac{c_a^2/k + c_e^2}{2}\right)\left(\frac{u}{1-u}\right)t + t \end{aligned} \tag{9.6}$$

where the last equality follows from the fact that $u = r_a/(k/t)$ so $r_a = uk/t$.

Notice that Equation (9.6) implies that cycle time becomes large when u approaches zero, as well as when it approaches one. The reason is that when utilization is low, arrivals are slow relative to process times and hence the time to form a batch becomes long.

As we saw in Figure 9.6, the cycle time at a parallel batch operation is significantly impacted by the batch size. Depending on the capacity of the operation, it may be optimal to run less-than-full batches. To find the optimal batch size, we could implement the expressions from the above technical note in a spreadsheet and use trial and error. Alternatively, we could use an analytical approach, like that presented in Chapter 15.

9.4.3 Move Batching

On a tour of an assembly plant, our guide proudly displayed one of his recent accomplishments—a manufacturing cell. Castings arrived at this cell from the foundry and, in less than an hour, were drilled, machined, ground, and polished. From the cell, they went to a subassembly operation. Our guide indicated that by placing the various processes in close proximity to one another and focusing on streamlining flow within the cell, cycle times for this portion of the routing had been reduced from several days to one hour. We were impressed—until we discovered that castings were delivered to the cell and completed parts were moved to assembly by forklift in totes containing approximately 10,000 parts! The result was that the first part required only one hour to go through the cell, but had to wait for 9,999 other parts before it could move on to assembly. Since

the capacity of the cell was about 100 parts per hour, the tote sat waiting to be filled for 100 hours. Thus, although the cell had been designed to reduce WIP and cycle time, the actual performance was the closest we have ever seen to the worst case of Chapter 7.

The reason the plant had chosen to move parts in batches of 10,000 was the mistaken (but common) assumption that transfer batches should equal process batches. However, in most production environments, there is no compelling need for this to be the case. As we noted above, splitting of batches or lots can reduce cycle time tremendously. Of course, smaller lots also imply more material handling. For instance, if parts in the above cell were moved in lots of 1,000 (instead of 10,000), then a tote would need to be moved every 10 hours (instead of every 100 hours). Although the assembly plant was large and interprocess moves were lengthy, this additional material handling was clearly manageable and would have reduced WIP and cycle time in this portion of the line by a factor of 10.

The behavior underlying this example is summarized in the following law of factory physics.

Law (Move Batching): *Cycle times over a segment of a routing are roughly proportional to the transfer batch sizes used over that segment, provided there is no waiting for the conveyance device.*

This law suggests one of the easiest ways to reduce cycle times in some manufacturing systems—reduce transfer batches. In fact, it is sometimes so easy that management may overlook it. But because reducing transfer batches can be simple and inexpensive, it deserves consideration before moving on to more complex cycle time reduction strategies. Of course, smaller transfer batches will require *more* material handling, hence the caveat *provided there is no waiting for the conveyance device.* If the more often we move parts between stations, the longer they wait for the material handling device, then this additional queue time might cancel out the reduction in wait-to-batch time. Thus, the Move Batching Law describes the cycle time reduction that is possible through move batch reduction, *provided* there is sufficient material handling capacity to carry out the moves without delay.

To appreciate the relationship between cycle time and move batch size, note that the dynamics are identical to those of a parallel batch process in which the material handling device is the parallel batch operation. If batches are too small, utilization will grow and cause the queue waiting for the material handler to become excessive. We illustrate these mechanics more precisely by means of a mathematical model in the following technical note.

Technical Note—Transfer Batches

Consider the effects of batching in the simple two-station serial line shown in Figure 9.7. The first station receives single parts and processes them one at a time. Parts are then collected into transfer batches of size k before they are moved to the second station, where they are processed as a batch and sent downstream as single parts. For simplicity, we assume that the time to move between the stations is zero.

Letting r_a denote the arrival rate to the line and $t(1)$ and $c_e(1)$ represent the mean and CV, respectively, of processing time at the first station, we can compute the utilization as $u(1) = r_a t(1)$ and the expected waiting time in queue by using the *VUT* equation.

$$\mathrm{CT}_q(1) = \left(\frac{c_a^2(1) + c_e^2(1)}{2}\right)\left(\frac{u(1)}{1-u(1)}\right)t \tag{9.7}$$

The total time spent at the first station includes this queue time, the process time itself, and the time spent forming a batch. The average batching time is computed by observing

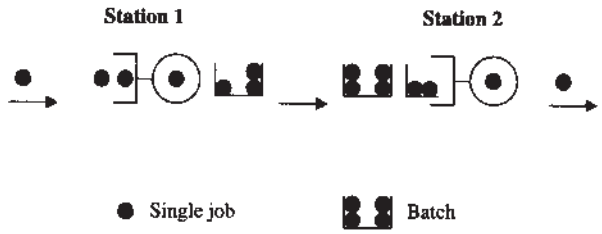


FIGURE 9.7
A batching and unbatching example

that the first part must wait for $k - 1$ other parts, while the last part does not wait at all. Since parts arrive to the batching process at the same rate as they arrive to the station itself r_a (remember conservation of flow), the average time spent forming a batch is the average between $(k-1)(1/r_a)$ and 0, which is $(k-1)/(2r_a)$. Since $u(1) = r_a t(1)$, we have

$$\text{average wait-to-batch-time} = \frac{k-1}{2r_a} = \frac{k-1}{2u(1)}t(1)$$

As we would expect, this quantity becomes zero if the batch size k is equal to one. We can now express the total time spent by a part at the first station CT(1) as

$$\text{CT}(1) = \text{CT}_q(1) + t(1) + \frac{k-1}{2u(1)}t(1) \tag{9.8}$$

To compute average cycle time at the second station, we can view it as a queue of whole batches, a queue of single parts (i.e., partial batch), and a server. We can compute the waiting time in the queue of whole batches $\text{CT}_q(2)$ by using Equation (9.7) with the values of $u(2)$, $c_a^2(2)$, $c_e^2(2)$, and $t(2)$ adjusted to represent batches. We do this by noting that interdeparture times for batches are equal to the sum of k interdeparture times for parts. Hence, because, as we saw in Chapter 8, adding k independent, identically distributed random variables with SCVs of c^2 results in a random variable with an SCV of c^2/k, the arrival SCV of batches to the second station is given by $c_a^2(1)/k = c_a^2(2)/k$. Similarly, since we must process k separate parts to process a batch, the SCV for the batch process times at the second station is $c_e^2(2)/k$, where $c_e^2(2)$ is the process SCV for individual parts at the second station. The effective average time to process a batch is $kt(2)$ and the average arrival rate of batches is r_a/k. Thus, as we would expect, utilization is

$$u(2) = \frac{r_a}{k}kt(2) = r_a t(2)$$

Hence, by the *VUT* equation, average cycle time at the second station is

$$\begin{aligned}\text{CT}_q(2) &= \left(\frac{c_a^2(2)/k) + (c_e^2(2)/k}{2}\right)\left(\frac{u(2)}{1-u(2)}\right)kt(2)\\ &= \left(\frac{c_a^2(2) + c_e^2(2)}{2}\right)\left(\frac{u(2)}{1-u(2)}\right)t(2)\end{aligned}$$

Interestingly, the waiting time in the queue of whole batches is the same as the waiting time we would have computed for single parts (because the k's cancel, leaving us with the usual *VUT* equation).

In addition to the queue of full batches, we must consider the queue of partial batches. We can compute this by considering how long a part spends in this partial queue. The first piece arriving in a batch to an idle machine does not have to wait at all, while the last piece in the batch has to wait for $k - 1$ other pieces to finish processing. Thus, the average time that parts in the batch have to wait is $(k-1)t(2)/2$.

The total cycle time of a part at the second station is the sum of the wait time in the queue of batches, the wait time in a partial batch, and the actual process time of the part:

$$\text{CT}(2) = \text{CT}_q(2) + \frac{k-1}{2}t(2) + t(2) \tag{9.9}$$

We can now express the total cycle time for the two-station system with batch size k as

$$\begin{aligned} \mathrm{CT}_{\mathrm{batch}} &= \mathrm{CT}(1) + \mathrm{CT}(2) \\ &= \mathrm{CT}_q(1) + t(1) + \frac{k-1}{2u(1)}t(1) + \mathrm{CT}_q(2) + \frac{k-1}{2}t(2) + t(2) \\ &= \mathrm{CT}_{\mathrm{single}} + \frac{k-1}{2u(1)}t(1) + \frac{k-1}{2}t(2) \end{aligned} \tag{9.10}$$

where $\mathrm{CT}_{\mathrm{single}}$ represents the cycle time of the system without batching (i.e., with $k = 1$).

Expression (9.10) quantitatively illustrates the Move Batching Law—cycle times increase proportionally with batch size. Notice, however, that the increase in cycle time that occurs when batch size k is increased has nothing to do with process or arrival variability (i.e., the terms in Equation (9.10) that involve k do not include any coefficients of variability). There *is* variability—some parts wait a long time due to batching while others do not wait at **all**—but it is variability caused by *bad control* or *bad design* (similar to the worst case in Chapter 7), rather than by process or flow uncertainty.

Finally, we note that the impact of transfer batching is largest when the utilization of the first station is low, because this causes the $(k-1)t(1)/[2u(1)]$ term in Equation (9.10) to become large. The reason for this is that when arrival rate is low relative to processing rate, it takes a long time to fill up a transfer batch. Hence, parts spend a great deal of time waiting in partial batches. This is very similar to what happens in parallel process batches (see Equation (9.6)). The only difference between Equations (9.6) and (9.10) is that in the former we did not model the move process as having limited capacity. If we had, the two situations would have been identical.

Cellular Manufacturing. The fundamental implication of the Move Batching Law is that large transfer batches directly inflate cycle times. Hence, reducing them can be a useful cycle time reduction strategy. One way to keep transfer batches small is through **cellular manufacturing,** which we discussed in the context of JIT in Chapter 4.

In theory, a cell positions all workstations needed to produce a family of parts in close physical proximity. Since material handling is minimized, it is feasible to move parts between stations in small batches, ideally in batches of one. If the cell truly processes only one family of parts, so there are no setups, the process batch can be one, infinity, or any number in between (essentially controlled by demand).

If the cell handles multiple families, so that there are significant setups, we know from our previous discussions that serial process batching is very important to the capacity and cycle time of the cell. Indeed, as we will see in Chapter 15, it may make sense to set the process batch size differently for different families and even vary these over time. Regardless of how process batching is done, however, it is an independent decision from move batching. Even if large process batches are required because of setups, we can use lot splitting to move material in small transfer batches and take advantage of the physical compactness of a cell.

9.5 Cycle Time

Having considered issues of utilization, variability, and batching, we now move to the more complicated performance measure, cycle time. First we consider the cycle time at a single station. Later we will describe how these station cycle times combine to form the cycle time for a line.

9.5.1 Cycle Time at a Single Station

We begin by breaking down cycle time at a single station into its components.

Definition (Station Cycle Time): *The average cycle time at a station is made up of the following components:*

$$\begin{aligned}\text{Cycle time} = {} & \text{move time} + \text{queue time} + \text{setup time} + \text{process time} \\ & + \text{wait-to-batch time} + \text{wait-in-batch time} \\ & + \text{wait-to-match time}\end{aligned} \tag{9.11}$$

Move time is the time jobs spend being moved from the previous workstation. **Queue time** is the time jobs spend waiting for processing at the station or to be moved to the next station. **Setup time** is the time a job spends waiting for the station to be set up. Note that this could actually be less than the station setup time if the setup is partially completed while the job is still being moved to the station. **Process time** is the time jobs are actually being worked on at the station. As we discussed in the context of batching, **wait-to-batch time** is the time jobs spend waiting to form a batch for either (parallel) processing or moving, and **wait-in-batch time** is the average time a part spends in a (process) batch waiting its turn on a machine. Finally, **wait-to-match time** occurs at assembly stations when components wait for their mates to allow the assembly operation to occur.

Notice that of these, only process time actually contributes to the manufacture of products. Move time could be viewed as a necessary evil, since no matter how close stations are to one another, some amount of move time will be necessary. But all the other terms are sheer inefficiency. Indeed, these times are often referred to as non-value-add time, waste, or *muda*. They are also commonly lumped together as delay time or queue time. But as we will see, these times are the consequence of very different causes and are therefore amenable to different cures. Since they frequently constitute the vast majority of cycle time, it is useful to distinguish between them in order to identify specific improvement policies.

We have already discussed the batching times, so now we deal with wait-to-match time before moving on to cycle times in a line.

9.5.2 Assembly Operations

Most manufacturing systems involve some kind of assembly. Electronic components are inserted into circuit boards. Body parts, engines, and other components are assembled into automobiles. Chemicals are combined in reactions to produce other chemicals. Any process that uses two or more inputs to produce its output is an assembly operation.

Assemblies complicate flows in production systems because they involve **matching.** In a matching operation, processing cannot start until all the necessary components are present. If an assembly operation is being fed by several fabrication lines that make the components, shortage of any one of the components can disrupt the assembly operation and thereby all the other fabrication lines as well. Because they are so influential to system performance, it is common to subordinate the scheduling and control of the fabrication lines to the assembly operations. This is done by specifying a **final assembly schedule** and working backward to schedule fabrication lines. We will discuss assembly operations from a quality standpoint in Chapter 12, from a shop floor control standpoint in Chapter 14, and from a scheduling standpoint in Chapter 15.

For now, we summarize the basic dynamics underlying the behavior of assembly operations in the following factory physics law.

Law (Assembly Operations): *The performance of an assembly station is degraded by increasing any of the following:*

1. *Number of components being assembled.*
2. *Variability of component arrivals.*
3. *Lack of coordination between component arrivals.*

Note that each of these could be considered an increase in variability. Thus, the Assembly Operations Law is a specific instance of the more general Variability Law. The reasoning and implications of this law are fairly intuitive. To put them in concrete terms, consider an operation that places components on a circuit board. All components are purchased according to an MRP schedule. If any component is out of stock, then the assembly cannot take place and the schedule is disrupted.

To appreciate the impact of the number of components on cycle time, suppose that a change is made in the bill of material that requires one more component in the final product. All other things being equal, the extra component can only inflate the cycle time, by being out of stock from time to time.

To understand the effect of variability of component arrivals, suppose the firm changes suppliers for one of the components and finds that the new supplier is much more variable than the old supplier. In the same fashion that arrival variability causes queueing at regular nonassembly stations, the added arrival variability will inflate the cycle time of the assembly station by causing the operation to wait for late deliveries.

Finally, to appreciate the impact of lack of coordination between component arrivals, suppose the firm currently purchases two components from the same supplier, who always delivers them at the same time. If the firm switches to a policy in which the two components are purchased from separate suppliers, then the components may not be delivered at the same time any longer. Even if the two suppliers have the same level of variability as before, the fact that deliveries are uncoordinated will lead to more delays. Of course, this neglects all other complicating factors, such as the fact that having two components to deliver may cause a supplier to be less reliable, or that certain suppliers may be better at delivering specific components. But all other things being equal, having the components arrive in synchronized fashion will reduce delays. We will discuss methods for synchronizing fabrication lines to assembly operations in Chapter 14.

9.5.3 Line Cycle Time

In the Penny Fab examples in Chapter 7, where all jobs were processed in batches of one and moves were instantaneous, cycle times were simply the sum of process times and queue times. But when batching and moving are considered, we cannot always compute the cycle time of the line as the sum of the cycle times at the stations. Since a batch may be processed at more than one station at a time (i.e., if lot splitting is used), we must account for overlapping time at stations. Thus, we define the cycle time in a line as follows.

Definition (Line Cycle Time): *The average cycle time in a line is equal to the sum of the cycle times at the individual stations less any time that overlaps two or more stations.*

To illustrate the impact of overlapping cycle times, we consider the two lines in Table 9.5. Lines 1 and 2 are both three-station lines with no process variability that experience (deterministic) arrivals of batches of $k = 6$ jobs every 35 hours. A setup is done for each batch, after which jobs are processed one at a time and are sent to the next station. The only difference is that the process and setup times are different in the two lines (line 2 is the reverse of line 1). Hence, in line 1 the utilizations of the stations are increasing, with station 1 at 49 percent, station 2 at 75 percent, and station 3 at 100 percent utilization. In line 2 these are reversed. For modeling purposes we use $t(i)$ and $s(i)$ to represent the unit process time and setup time, respectively, at station i.

Consider line 1. Since we are processing jobs in series on stations with setups and letting them go as they are finished, we can apply Equation (9.5) to compute the cycle time at each station. At station 1, this yields

$$\text{CT}(1) = \text{CT}q + s(1) + \frac{k+1}{2}t(1) = 0.0 + 5 + \frac{6+1}{2}(2) = 12$$

where the queue time is zero because there is no variability in the system.

For stations 2 and 3, we can do the same thing to get

$$\text{CT}(2) = \text{CT}q + s(2) + \frac{k+1}{2}t(2) = 0.0 + 8 + \frac{6+1}{2}(3) = 18.5$$

$$\text{CT}(3) = \text{CT}q + s(3) + \frac{k+1}{2}t(3) = 0.0 + 11 + \frac{6+1}{2}(4) = 25$$

which yields a total cycle time of

$$\text{CT} = \text{CT}(1) + \text{CT}(2) + \text{CT}(3) = 12 + 18.5 + 25 = 55.5$$

But this is not right. The first job in a batch at station 2 or 3 is already in process while the last job in the batch is still at the previous station. Therefore, the wait-in-batch time component of Equation (9.5) overestimates the total delay at stations 2 and 3 due to batching.

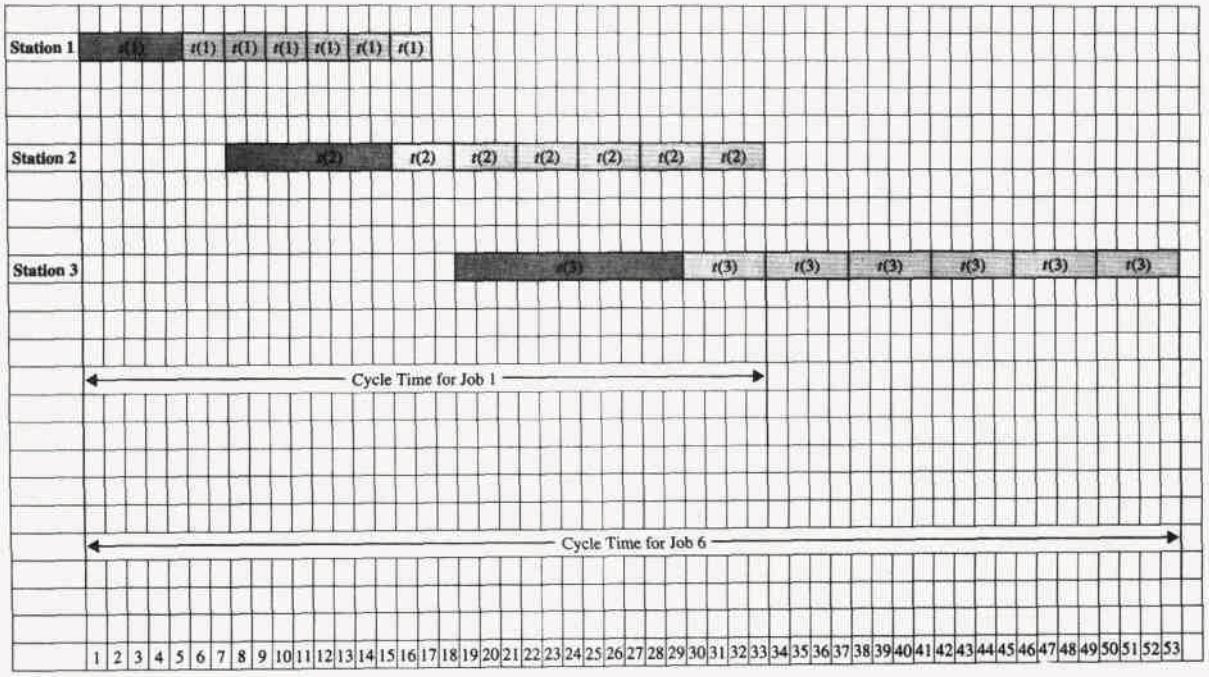
For this deterministic example, we can compute the cycle time by following the jobs in a batch one at a time through the station. As shown in Figure 9.8, the first job to arrive at station 2 has a cycle time of $s(2) + t(2)$. The second finishes at $s(2) + 2t(2)$ but arrived $t(1)$ hour later than the first job, so its cycle time at station 2 is $s(2) + 2t(2) - t(1)$. Likewise, the third has a cycle time of $s(2) + 3t(2) - 2t(1)$. This continues until the kth (last) job in the batch, which starts at $(k-1)t(1)$ and completes at $s(2) + kt(2)$ for a

TABLE 9.5 Examples Illustrating Cycle Time Overlap

	Station 1	Station 2	Station 3
	Line 1		
Setup time (hour)	5	8	11
Unit process time (hour)	2	3	4
	Line 2		
Setup time (hour)	11	8	5
Unit process time (hour)	4	3	2

FIGURE 9.8

Lot splitting: faster to slower



cycle time of $s(2) + kt(2) - (k-1)t(1)$. The average cycle time at station 2 is therefore

$$\begin{aligned} \text{CT}(2) &= \frac{1}{k}[ks(2) + (1+2+\cdots+k)t(1) - (1+2+\cdots+k-1)t(1)] \\ &= s(2) + \frac{k+1}{2}t(2) - \frac{k-1}{2}t(1) \\ &= 8 + 3.5(3) - 2.5(2) = 13.5 \end{aligned}$$

The term $[(k-1)/2]t(1) = 5$ hours represents the batch *overlap* time.

The situation at station 3 is similar to that at station 2 and leads to a cycle time at station 3 of

$$\begin{aligned} \text{CT}(3) &= s(3) + \frac{k+1}{2}t(3) - \frac{k-1}{2}t(2) \\ &= 11 + 3.5(4) - 2.5(3) = 17.5 \end{aligned}$$

Thus, the correct total time through line 1 is computed by adding the corrected versions of CT(1), CT(2) and CT(3), which yields

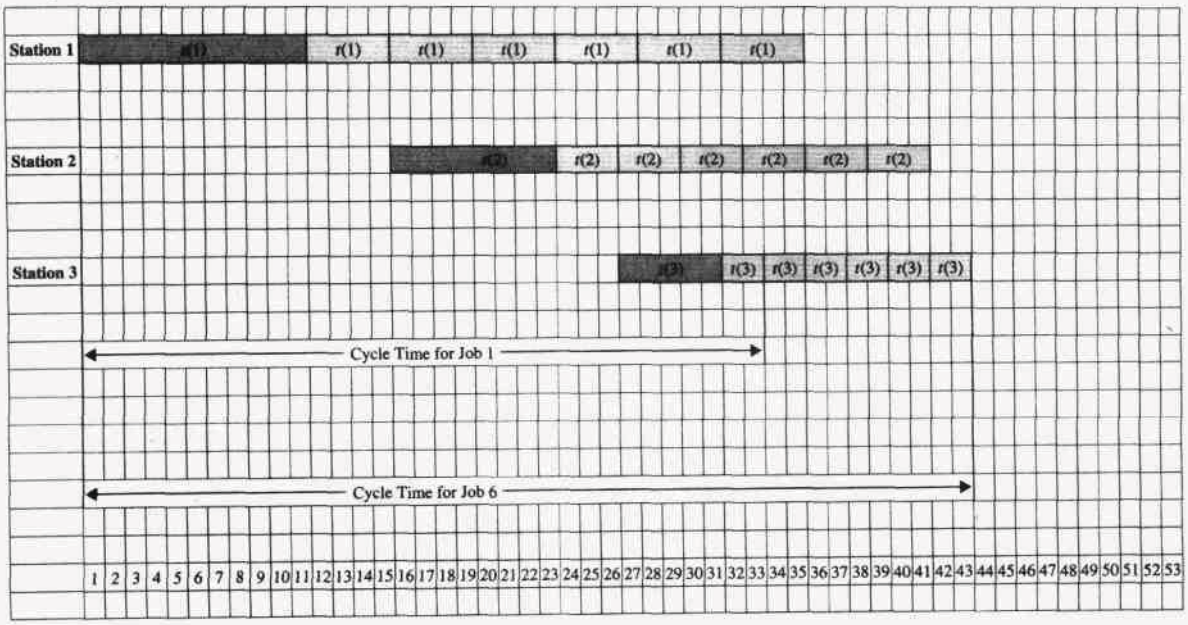
$$\text{CT(line)} = s(1) + s(2) + s(3) + t(1) + t(2) + \frac{k+1}{2}t(3) = 43 \text{ hours}$$

This is illustrated in Figure 9.8, which shows that the cycle time of the first job in the batch is 33 hours, while the cycle time of the sixth job is 53 hours, so the average cycle time is $(33+53)/2 = 434$ hours. Note that this is considerably less than the 55.5 hours arrived at by summing the cycle times at the stations.

If we were to compute the cycle time for line 2, using Equation (9.5) at each station, and add the results, we would get the same answer as for line 1, or 55.5 hours. The

FIGURE 9.9

Lot splitting: slower to faster



reason is that without variability the equation is unaffected by the order of the line. However, now if we work through the mechanics of the line directly, we find that the true average cycle time is 38 hours (see Figure 9.9, which shows that the cycle times of the first and sixth jobs are 33 hours and 43 hours respectively, so the average cycle time is $(33 + 43)/2 = 38$ hours). Again, this is considerably less than our initial estimate. It is also much less than the first case (there is more overlapping when slower processes are first). The point is that not only are overlapping cycle times important to determining the cycle time of a line, but also the mechanics are such that the order of the stations matters.

Although the behavior of lines with batching is complex, we can gain insight into the line cycle time by following a single job through the line. As in the above example, we assume that

1. Jobs arrive in batches.[8]
2. The first job in each batch sees a full setup at each station (i.e., we are not allowed to start setups before the first job in the batch arrives, although we do allow the case where all setup times at a station are zero).
3. Jobs are moved one at a time between stations.

Under these conditions, we develop upper and lower bounds on the cycle time of a line in the following technical note.

Technical Note—Cycle Time Bounds

We refer to nonqueueing (i.e., time in batch, setup time, and process time) time as *total in-process time*. We can bound the total in-process time by considering a line with no variability

[8]Since a full batch is committed to enter the line once the first job is released to the line, for the purposes of computing cycle time it is reasonable to assume that the entire batch arrives to the line simultaneously.

(and therefore no queueing) and examining the time it takes for the first job T_1 and time for the last job T_k of a batch to go through the line.[9] For a k-station line with $s(i)$ and $t(i)$ being the setup and process times, respectively, at station i, the first job will require a setup and a single process time at each station

$$T_1 = \sum_{i=1}^{K} s(i) + t(i)$$

The last job will require this time plus the time spent waiting behind the other jobs in the batch. The longest time this could possibly be occurs if the last job encountered all the $k-1$ other jobs at the process with the longest process time (see Figure 9.8). Thus,

$$T_k \leq T_1 + (k-1)t_b$$

where $t_b = \max_i\{t(i)\}$. An upper bound for the average total in-process time is the average of T_1 and T_k, which yields

$$\text{total in-process time} \leq \sum_{i=1}^{K}[s(i) + t(i)] + \frac{k-1}{2}t_b \tag{9.12}$$

Because all jobs arrive to the first station at one time, the last job will *always* finish after the other $k-1$ jobs at the last station. The smallest delay that can occur is seen if the last station has the *fastest* process time *and* there is no idle time at the last station (see Figure 9.9). So a lower bound on the average total in-process time can be computed by using $t_f = \min_i\{t(i)\}$ in place of t_b

$$T_k \geq T_1 + (k-1)t_f$$

and so

$$\text{total in-process time} \geq \sum_{i=1}^{K}[s(i) + t(i)] + \frac{k-1}{2}t_f \tag{9.13}$$

To get bounds on cycle time, we must consider queue time in addition to total in-process time. To do this, recall our discussion of batch moves. There, the total queue time did not depend on the batch size (remember how the k's "canceled out"). If we can assume that this is approximately true for the serial batching case, then a good approximation of the queue time can be made by using the *VUT* equation to compute the average time that *full batches* wait in queue at each station. At the first station, since arrivals occur in batches, this approximation is as accurate as the *VUT* equation itself. At other stations, where arrivals occur one at a time, more error is introduced by not really knowing c_a^2. Of course, this problem exists in systems without batching as well. Experience with a limited number of examples shows that the accuracy is no worse than the accuracy of the equations developed for single jobs (in Chapter 8).

Letting $\text{CT}_q^b(i)$ represent the average time that full batches wait at station i (which is computed by using the *VUT* equation in the usual way), we can express approximate upper and lower bounds on total cycle time in a line with serial batching as

$$\begin{aligned}\sum_{i=1}^{n}[\text{CT}_q^b(i) + s(i) + t(i)] + \frac{k-1}{2}t_f \leq \text{CT} \\ \leq \sum_{i=1}^{n}[\text{CT}_q^b(i) + s(i) + t(i)] + \frac{k-1}{2}t_b\end{aligned} \tag{9.14}$$

where $t_f = \min_i\{t(i)\}$, and $t_b = \max_i\{t(i)\}$.

[9] The authors would like to express their gratitude to Dr. Greg Diehl at Network Dynamics, Inc., for his assistance in the development of these equations.

Example: Bounding Cycle Time
Reconsider the two lines in Table 9.5. If there is no process or arrival variability, then the sum of the queue times is zero and the sum of the setup and process times is 33. Hence the cycle time bounds are

$$33 + \frac{6-1}{2}(2) \leq \text{CT} \leq 33 + \frac{6-1}{2}(4)$$
$$38 \leq \text{CT} \leq 43$$

For line 1, the upper bound is tight. For line 2, the lower bound is tight. However, if we switch things around so that the slowest station is at the front and the fastest station is in the middle, then it turns out that CT = 40.5, which is between the bounds. Likewise, if we place the slowest station in the middle and the fastest station at the end, CT = 39.5, which is also between the bounds. In these examples, no idle time occurs within batches (i.e., no machine goes idle between jobs of the same batch). However, this can occur and indeed does occur in this system if the slowest station is first and the fastest is second (see Problem 15).

The cycle time bounds in Equation (9.14) will be very close to one another for lines in which process times are similar (i.e., so that $t_f \approx t_b$). But for lines where the fastest machine is much faster than the slowest one (e.g., because it also has a very long setup time), these bounds can be quite far apart. Tighter bounds require more complex calculations (see Benjaafar and Sheikhzadeh 1997).

9.5.4 Cycle Time, Lead Time, and Service

In a manufacturing system with infinite capacity and absolutely no variability, the relation between cycle time and customer lead time is simple—they are the same. The lucky manager of such a system could simply quote a lead time to customers equal to the cycle time required to make the product and be assured of 100 percent service. Unfortunately, all real systems contain variability, and so perfect service is not possible and there is frequently confusion regarding the distinction between lead time, cycle time, and their relation to service level. Although we touched on these issues briefly in Chapters 3 and 7, we now define them more precisely and offer a law of factory physics that relates variability to lead time, cycle time, and service.

Definitions. Throughout this book we have used the terms *cycle time* and *average cycle time* interchangeably to denote the average time it takes a job to go through a line. To talk about lead times, however, we need to be a bit more precise in our terminology. Therefore, for the purposes of this section, we will define **cycle time** as a *random variable* that gives the time an individual job takes to traverse a routing. Specifically, we define T to be a random variable representing cycle time, with a mean of CT and a standard deviation of σ_{CT}.

Unlike cycle time, **lead time** is a *management constant* used to indicate the anticipated or maximum allowable cycle time for a job. There are two types of lead time: customer lead time and manufacturing lead time. **Customer lead time** is the amount of time allowed to fill a customer order from start to finish (i.e., multiple routings), while the **manufacturing lead time** is the time allowed on a particular routing.

In a **make-to-stock** environment, the customer lead time is zero. When the customer arrives, the product either is available or is not. If it is not, the service level (usually

called **fill rate** in such cases) suffers. In a **make-to-order** environment, the customer lead time is the time the customer allows the firm to produce and deliver an item. For this case, when variability is present, the lead time must generally be greater than the average cycle time in order to have acceptable service (defined as the percentage of on-time deliveries).

One way to reduce customer lead times is to build lower-level components to stock. Since customers only see the cycle time of the remaining operations, lead times can be significantly shorter. We discuss this type of **assemble-to-order** system in the context of push and pull production in Chapter 10.

Relations. With complex bills of material, computing suitable customer lead times can be difficult. One way to approach this problem is to use the manufacturing lead time that specifies the anticipated or maximum allowable cycle time for a job on a specific routing. We denote the manufacturing lead time for a specific routing with cycle time T as ℓ. Manufacturing lead time is often used to plan releases (e.g., in an MRP system) and to track service.

Service s can now be defined for routings operating in make-to-order mode as the probability that the cycle time is less than or equal to the specified lead time, so that

$$s = \Pr\{T \leq \ell\} \tag{9.15}$$

If T has distribution function F, then Equation (9.15) can be used to set ℓ as

$$s = F(\ell) \tag{9.16}$$

If cycle times are normally distributed, then for a service level of s

$$\ell = \mathrm{CT} + z_s \sigma_{\mathrm{CT}} \tag{9.17}$$

where z_s is the value in the standard normal table for which $\Phi(z_s) = s$. For instance, if cycle time on a given routing has a mean of eight days and a standard deviation of three days, the value for z_s for 95 percent is 1.645, so the required lead time is

$$\ell = 8 + 1.645(3) = 12.94 \approx 13 \text{ days}$$

Figure 9.10 shows both the distribution function F and its associated density function f for cycle time. The additional five days above the mean is called the **safety lead time.**

By specifying a high enough service level (to guarantee that jobs generally finish on time), we can compute customer lead times by simply adding the longest manufacturing lead times (when several routings come together in an assembly) for each level in the bill of material. For example, Figure 9.11 illustrates a system with two fabrication lines feeding an assembly operation followed by several more operations. The manufacturing lead time for assembly and the subsequent operations is four days for a service level of 95 percent. Since assembly represents level 0 in the bill of material (recall low-level codes from Chapter 3), we have that the level 0 lead time is four days. Similarly, the 95 percent manufacturing lead time is four days for the top fabrication line and six days for the bottom one, so that the lead time for level 1 is six days. Thus, total customer lead time is 10 days.

Unfortunately, the overall service level using a customer lead time of 10 days will be something less than 95 percent. This is because we did not consider the possibility of **wait-to-match time** in front of assembly. As we noted in the assembly operation law, wait-to-match time results when variability causes the fabrication lines to deliver product to assembly in an unsynchronized fashion. Because of this, whenever we have assembly operations, we must add some safety lead time.

We can now summarize the fundamental principle relating variability in cycle time to required lead times in the following law of factory physics.

FIGURE 9.10
Distribution function for cycle time and required lead time

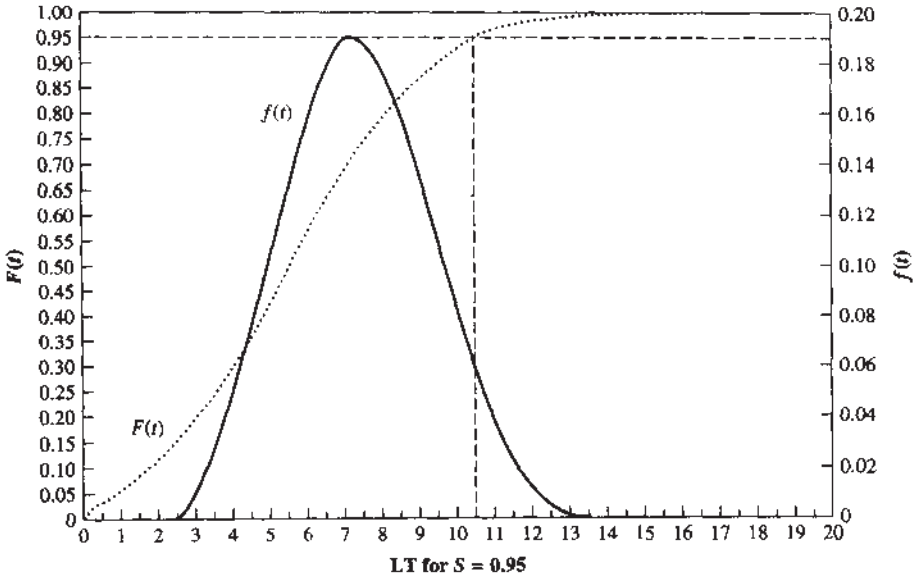
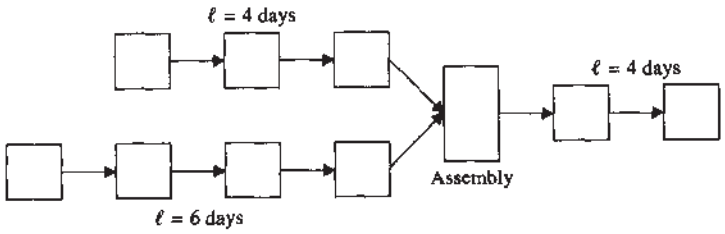


FIGURE 9.11
An assembly system



Law (Lead Time): *The manufacturing lead time for a routing that yields a given service level is an increasing function of both the mean and standard deviation of the cycle time of the routing.*

Intuitively, this law suggests that we view manufacturing lead times as given by the cycle time plus a "fudge factor" that depends on the cycle time standard deviation. The larger the cycle time standard deviation, the larger the fudge factor must be to achieve a given service level. In a make-to-order environment, where we want manufacturing lead times short in order to keep customer lead times short, we need to keep both the mean and the standard deviation of cycle time low.

The factors that inflate *mean* cycle time are generally the same as those that inflate the *standard deviation* of process time, as we noted in Chapter 8. These include operator variability, random outages, setups, rework, and the like. However, from a cycle time perspective, *rework* is particularly disruptive. Whenever there is a chance that a job will be required to go back through a portion of the line, the variability of cycle time increases dramatically. We will return to this and other issues related to cycle time variability when we discuss the impact of quality on logistics in Chapter 12.

9.6 Diagnostics and Improvements

The factory physics laws discussed describe fundamental aspects of the behavior of manufacturing systems and highlight key tradeoffs. However, by themselves they do not yield design and management policies. The reason is that the "optimal" operational structure depends on environmental constraints and strategic goals. A firm that competes on customer service needs to focus on swift and responsive deliveries, while a firm that competes on price needs to focus on equipment utilization and cost. Fortunately, the laws of factory physics can help identify areas of leverage and opportunities for improvement, regardless of the system specifics.

The following examples illustrate the use of the principles developed in this chapter to improve an existing system with regard to three key performance measures: throughput, cycle time, and customer service.

9.6.1 Increasing Throughput

Throughput of a line is given by

$$\text{TH} = \text{bottleneck utilization} \times \text{bottleneck rate}$$

Therefore, the two ways to increase throughput are to increase utilization of the bottleneck or increase its rate. It may sound blasphemous to talk of increasing utilization, since we know that increasing utilization increases cycle time. But different objectives call for different policies. In a system without restrictions on WIP, high utilization causes queueing and hence increases cycle time. But, as we saw in the pay-me-now-or-pay-me-later examples, in systems with constraints on WIP (finite buffers or logical limitations such as those imposed by kanban), blocking and starving will limit utilization of the bottleneck and hence degrade throughput.

A basic checklist of policies for increasing throughput is as follows.

1. **Increase bottleneck rate** by increasing the effective rate of the bottleneck. This can be done through equipment additions, staff additions or training, covering stations through breaks or lunches, use of flexible labor, quality improvements, product design changes to reduce time at the bottleneck, and so forth.
2. **Increase bottleneck utilization** by reducing blocking and starving of the bottleneck. There are two basic ways to do this:
   - *Buffer bottleneck with WIP.* This can be done by increasing the size of the buffers (or equivalently, the number of kanban cards) in the system. Most effective are buffer spaces immediately in front of the bottleneck (where allowing a queue to grow helps prevent starvation) and immediately after the bottleneck (where building a queue helps prevent blocking). Buffer space farther away from the bottleneck can still help, but will have a smaller effect than space close to it.
   - *Buffer bottleneck with capacity.* This can be done by increasing the effective rates of nonbottleneck stations. Faster stations upstream from the bottleneck make starving less frequent, while faster stations downstream make blocking less frequent. Adding capacity to the highest-utilization nonbottleneck stations will generally have the largest impact, since these are the stations most likely to cause blocking/starving. These can be made through the usual capacity enhancement policies, such as those listed above for increasing capacity of the bottleneck station.

Example: Throughput Enhancement

HAL Computer has a printed-circuit board plant that contains a line with two stations. The first station (resist apply) applies a photoresist material to circuit boards. The second station (expose) exposes the boards to ultra-violet light to produce a circuit pattern that is later etched onto the boards. Because the expose operation must take place in a clean room, space for WIP between the two processes is limited to 10 jobs. Capacity calculations show the bottleneck to be expose, which requires an average of 22 minutes to process a job, with an SCV of one. Resist apply requires 19 minutes per job, with an SCV of 0.25. In addition (and not included in the above process times), expose has a mean time to failure (MTTF) of $3\frac{1}{3}$ hours and a mean time to repair (MTTR) of 10 minutes, while resist apply has an MTTF of 48 hours and an MTTR of 8 hours. Jobs arrive to resist apply with a fair amount of variability, so we assume an arrival SCV c_a^2 of one. The desired throughput rate is 2.4 jobs per hour.

From past experience, HAL knows the line to be incapable of achieving the target throughput. To remedy this situation, the responsible engineers are in favor of installing a second expose machine. However, in addition to being expensive, a second machine would require expanding the clean room, which would add significantly to the cost and would result in substantial lost production during construction. The challenge, therefore, is to use factory physics to find a better solution.

The two principal tools at our disposal are the *VUT* equation for computing queue time

$$\mathrm{CT}_q = \left(\frac{c_a^2 + c_e^2}{2}\right)\left(\frac{u}{1-u}\right)t \tag{9.18}$$

and the linking equation

$$c_d^2 = u^2 c_e^2 + (1-u^2)c_a^2 \tag{9.19}$$

Using these in conjunction with the formulas presented in Chapter 8 for the effective squared coefficient of variation, we can analyze the reasons why the line is failing to meet its throughput target.

Formulas (9.18) and (9.19) (along with additional calculations to compute the average process times $t_e(1)$ and $t_e(2)$, and the process SCVs $c_e^2(1)$ and $c_e^2(2)$, which we will come back to later), we estimate the waiting time in queue station to be 645 minutes at resist apply and 887 minutes at expose, when the arrival rate is set at 2.4 jobs per hour. The average WIP levels are 25.8 and 35.5 jobs at stations 1 and 2, respectively.

This reveals why the system cannot make 2.4 jobs per hour, even though the utilization of the bottleneck (expose) is only 92.4 percent. Namely, the clean room can hold only 20 jobs, while the model predicts an average number in queue of 35.5 jobs. Since the real system cannot allow WIP in front of expose to reach this level, resist apply will occasionally become *blocked* (i.e., idled due to a lack of space in the downstream buffer to which to send completed parts). The resulting lost production at resist apply eventually causes expose to become *starved* (i.e., idled due to a lack of parts to work on). The result is that neither station can maintain the utilization necessary to produce 2.4 parts per hour.[10]

Thus, we conclude that the problem is rooted in the long queue at expose. By Little's Law, reducing average queue length is equivalent to reducing average queue time. So

[10]Note that we could also have analyzed this situation by using the blocking model of Section 8.7.2. The reader is invited to try Problem 13 to see how this more sophisticated tool can be used to obtain the same qualitative result, albeit with greater quantitative precision.

we now consider the queue time at expose more closely:

$$\begin{aligned} \mathrm{CT}_q(2) &= \left(\frac{c_a^2(2)+c_e^2(2)}{2}\right)\left(\frac{u(2)}{1-u(2)}\right)t_e(2) \\ &= (3.16)(12.15)(23.1 \text{ minutes}) \\ &= 887 \text{ minutes} \end{aligned}$$

The third term $t_e(2)$ is the effective process time at expose, which is simply raw process time divided by availability

$$\begin{aligned} t_e(2) &= \frac{t(2)}{A(2)} = \frac{t(2)}{m_f(2)/(m_f(2)+m_r(2))} \\ &= 22\left(\frac{31/3+1/6}{31/3}\right) \\ &= 23.1 \text{ minutes} \end{aligned}$$

Since this only slightly larger than the raw process time of 22 minutes, there is little room for improvement by increasing availability.

The second term in the expression for $\mathrm{CT}_q(2)$ is the utilization term $u(2)/(1-u(2))$. Although at first glance a value of 12.15 may appear large, it corresponds to a utilization of 92.4 percent, which is large but not excessive. Although increasing the capacity of this station would certainly reduce the queue time (and queue size), we have already noted that this is an expensive option.

So we look to the first term, the variability inflation factor $(c_a^2(2)+c_e^2(2))/2$. Recall that moderate variability in arrivals (that is, $c_a^2(2) = 1$) and moderate variability in process times (that is, $c_e^2(2) = 1$) result in a value of one for this term. Therefore, a value of 3.16 is unambiguously large in any system. To investigate why this occurs, we break it down into its constituent parts, which reveals

$$\begin{aligned} c_e^2(2) &= 1.04 \\ c_a^2(2) &= 5.27 \end{aligned}$$

Obviously, the arrival process is the dominant source of variability. This points to the problem lying upstream in the resist apply process. So we now investigate the cause of the large $c_a^2(2)$. Recall that $c_a^2(2) = c_d^2(1)$, which from Equation (9.19) is given by

$$\begin{aligned} c_d^2(1) &= u^2(1)c_e^2(1) + [1-u^2(1)]c_a^2(1) \\ &= (0.887^2)(6.437) + (1-0.887^2)(1.0) \\ &= 5.05 + 0.22 \\ &= 5.27 \end{aligned}$$

The component that makes $c_d^2(1)$ large is $c_e^2(1)$, the effective SCV of the resist apply machine. This coefficient is in turn made up of two components: a natural SCV, $c_0^2(1)$ and an inflation term due to machine failures. Using formulas from Chapter 8, we can break down $c_e^2(1)$ as follows:

$$A(1) = \frac{m_f(1)}{m_f(1)+m_r(1)} = \frac{48}{48+8} = 0.8571$$

$$t_e(1) = \frac{t(1)}{A(1)} = \frac{19}{0.8571} = 22.17 \text{ minutes}$$

$$c_e^2(1) = c_0^2(1) + \frac{2m_r(1)A(1)[1 - A(1)]}{t(1)}$$
$$= 0.25 + \frac{2(480)(0.8571)(0.1429)}{19} = 6.44$$

The lion's share of $c_e^2(1)$ is a result of the random outages. This suggests that an alternative to increasing capacity at expose is to improve the breakdown situation at resist apply. It is important to note that resist apply is the problem even though expose is the bottleneck. Because variability propagates through a line, a congestion problem at one station may actually be the result of a variability problem at an upstream station.

Various practical options might be available for mitigating the outage problem at resist apply. For instance, HAL could attempt to reduce the mean time to repair by holding "field-ready" spares for parts subject to failures. If such a policy could halve the MTTR, the resulting increase in effective capacity and reduction in departure SCV from resist apply would cause queue time to fall to 146 minutes at resist apply (less than one-fourth of the original) and 385 minutes at expose (less than one-half of the original).

Alternatively, HAL could perform more frequent preventive maintenance. Suppose we could avoid the long (eight-hour) failures by shutting down the machine every 30 minutes to perform a five-minute adjustment. The capacity will be the same as in the original case (i.e., because availability is unchanged), but because outages are more regular, queue time is reduced to 114 minutes at resist apply and 211 minutes at expose. Using Little's Law, this translates to an average of 8.4 jobs at expose, which is well within the space limit.

With either of the above improvements in place, it turns out to be feasible to run at (actually slightly above) the desired rate of 2.4 jobs per hour. Any other policy that would serve to reduce the variability of inter output times from resist apply would have a similar effect. Because improving the repair profile of resist apply is likely to be less expensive and disruptive than adding an expose machine, these alternatives deserve serious consideration.

9.6.2 Reducing Cycle Time

Combining the definitions of station and line cycle time, we can break down cycle times in a production system into the following:

1. Move time.
2. Queue time.
3. Setup time.
4. Process time.
5. Process batch time (wait-to-batch and wait-in-batch time).
6. Move batch time (wait-to-batch and wait-in-batch time).
7. Wait-to-match time.
8. *Minus* station overlap time.

In most production systems, actual process and move times are a small fraction (5 to 10 percent) of total cycle time (Bradt 1983). Indeed, lines for which these terms dominate are probably already very efficient with little opportunity for improvement. For inefficient lines, the major leverage lies in the other terms. The following is a brief checklist of generic policies for reducing each of these terms.

Queue time is caused by utilization and variability. Hence, the two categories of improvement policies are as follows:

1. *Reduce utilization* by increasing the effective rate at the bottleneck. This can be done by either increasing the bottleneck rate (by adding equipment, reducing setup times, decreasing time to repair, making process improvements, spelling operators through breaks and lunches, cross-training workers to take advantage of flexible capacity, etc.) or reducing flow into the bottleneck (by scheduling changes to route flow to nonbottlenecks, improving yield, or reducing rework).
2. *Reduce variability* in either process times or arrivals at any station, but particularly at high-utilization stations. Process variability can be reduced by reducing repair times, reducing setup times, improving quality to reduce rework or yield loss, reducing operator variability through better training, etc. Arrival variability can be reduced by decreasing process variability at upstream stations, by using better scheduling and shop floor control to smooth material flow, eliminating batch releases (i.e., releases of more than one job at a time), and installing a pull system (see Chapter 10).

Process batch time is driven by process batch size. The two basic means for reducing (serial or parallel) process batch size are as follows:

1. *Batching optimization* to better balance batch time with queue time due to high utilization. We gave some insight into this tradeoff earlier in this chapter. We pursue more detailed optimization in Chapter 15.
2. *Setup reduction* to allow smaller batch sizes without increasing utilization. Well-defined techniques exist for analyzing and reducing setups (Shingo 1985).

Wait-to-match time is caused by lack of synchronization of component arrivals to an assembly station. The main alternatives for improving synchronization are as follows:

1. *Fabrication variability reduction* to reduce the volatility of arrivals to the assembly. This can be accomplished by the same variability reduction techniques used to reduce queue time.
2. *Release synchronization* by using the shop floor control and/or scheduling systems to coordinate releases in the line to completions at assembly. We discuss shop floor mechanisms in Chapter 14 and scheduling procedures in Chapter 15.

Station overlap time. Unlike the other "times," we would like to *increase* station overlap time because it is subtracted from the total cycle time. It can be increased by the use of lot splitting where feasible. Streamlined material handling (e.g., through the use of cells) makes the use of smaller transfer batches possible and hence enhances the cycle time benefits of lot splitting.

Example: Cycle Time Reduction

SteadyEye, a maker of commercial camera mounts, sells its products in make-to-order fashion to the motion picture industry. Lately the company has become concerned that customer lead times are no longer competitive. SteadyEye offers 10-week lead times, quoted from the end of two-week order buckets. (For instance, if an order is received

anywhere in the two-week interval between September 5, 1999, and September 18, 1999, it is quoted a delivery date 10 weeks from September 18, 1999.) However, their major competitor is offering five-week lead times from the date of the order. Worse yet, SteadyEye's inventory levels are at record levels, average cycle time (currently nine weeks) is as long as it has ever been, and customer service (fraction of orders delivered on-time) is poor (less than 70 percent) and declining.

SteadyEye's process begins with the entry of customer orders, which is done by a clerk daily. Much to the clerk's frustration, it seems that most of the orders seem to come at the end of the two-week interval, which forces her to fall behind even though she puts in significant overtime every other weekend. Using the most recent customer orders, an ERP system generates a daily set of purchase orders and dispatch lists. These lists are sent to each process center but are especially important at the assembly area because that is where parts are matched to fill orders. Unfortunately, it is common for lists to be ignored because the requisite parts are not available.

SteadyEye manufactures legs, booms, and other structural components of its camera mounts, as well as gears and gearboxes that go into the control assembly. It purchases all motors and electronics from outside suppliers. Raw materials and subassemblies are received at the receiving dock. Bar stock is sawed to the correct lengths for the various gears and is then sent to the milling operation on a pallet carried by a forklift. Because of long changeover times at the mills, process batches are very large. Other operations include drilling, grinding, and polishing. The polisher is very fast, and so there is only one. Unfortunately, it is also difficult to adjust, and so downtimes are very long and generate a lot of parts that need to be scrapped. The heat treat operation takes three hours and involves a very large oven that can hold nearly 1,000 parts. Since most process batches are larger than those required by a single order, parts are returned to a crib inventory location after each operation.

The root of SteadyEye's problem is excessive cycle time, which from factory physics is a consequence of variability (arrival and process) and utilization. Thus, improvement policies must focus on these.

To begin, the arrival variability is being unnecessarily magnified by the order processing system. By establishing a two-week window within which all orders are quoted the same due date, the system encourages procrastination on the part of the customers and sales engineers. (Why get an order in before the end of the time window if it won't be shipped any earlier?) The resulting last-minute behavior creates a burst of arrivals to the system, thereby greatly increasing the effective c_a^2. Fortunately, this problem can be remedied by simply eliminating the order window. A better policy would have orders received on day t promised delivery on day $t + \ell$ (where ℓ is a lead time, which we hope to get down to five weeks or less). Orders can still be batched within the system by pulling in orders later on the master production schedule, but this can be transparent to customers.

Next, variability analysis of the effective process times shows that the polisher has an enormous c_e^2 of around seven. This is further aggravated by the fact that utilization of the polisher, after considering the various detractors, is greater than 90 percent. An attractive improvement policy, therefore, is to analyze the parameters affecting the polisher to find ways to reduce the time needed for adjustment. This will also reduce scrap and the need to expedite small jobs of parts to replace those that were scrapped. The net effect will be to reduce c_e^2 and u at a bottleneck operation, which will significantly reduce queueing, and hence average cycle time. Since these measures will also reduce cycle time variability, they will enable reduction of customer lead time by even more than the reduction in average cycle time.

Another large source of variability and cycle time in this system is batching, so we turn to it next. Batching is driven by both material handling and processing considerations. Move batches are large (typically a full pallet) because processes are far apart so that forklift capacity does not permit frequent transfers. An appealing policy therefore would be to organize processes into cells near the assembly lines. With this and some investment in material handling devices (e.g., conveyors) it may be practical to reduce move sizes to one. Process batches are large because of long setups. Hence, the logical improvement step is to implement a rigorous setup reduction program (e.g., using single minute exchange of die (SMED) techniques, see Shingo 1985). Since cutting setup times by a factor of four or more is not uncommon, such steps could enable SteadyEye to reduce process batch sizes by 75 percent or more.

In addition to these improvements in the processes themselves, there are some system changes that could further reduce cycle times. One would be to restrict use of the ERP system to providing purchase orders for outside parts and to generating "planned orders" but *not* for converting these to actual jobs. A separate module is needed to combine orders into jobs such that like orders of like families will be processed together (to share a setup at milling where setups are still significant) while still meeting due dates. The mechanics for such a module are given in Chapter 15.

Additionally, it may make sense to convert some commonly used components from make-to-order to make-to-stock parts. The crib that is now storing remnants of large batches of many parts would be converted to storage of stocks of these parts. Because batch sizes will be much smaller, all other parts will never enter the crib, but instead will be used as produced. Thus, even though stock levels of selected parts (common parts for which elimination of cycle time would appreciably reduce customer lead time) will increase, the overall stock level in the crib should be significantly less.

The net result of this battery of changes will be to substantially reduce cycle times. To go from an average cycle time of 10 weeks to less than two weeks is not an unreasonable expectation. If the company can pull it off, SteadyEye will transform its manufacturing operation from a competitive millstone to a strategic advantage.

For a more detailed example of cycle time reduction, the reader is referred to Chapter 19.

9.6.3 Improving Customer Service

In operational terms, satisfying customer needs is primarily about lead time (quick response) and service (on-time delivery). As we noted earlier, one way to radically reduce lead time is to move from a make-to-order system to a make-to-stock system, or to do this partially by making generic components to stock and assembling to order. We discuss this approach more fully in Chapter 10.

For the segment of the system that is make to order, the Lead Time Law implies

$$\begin{aligned}\text{lead time} &= \text{average cycle time} + \text{safety lead time}\\ &= \text{average cycle time} + z_s \times \text{standard deviation of cycle time}\end{aligned}$$

where z_s is a safety factor that increases in the desired level of service. Therefore, reducing lead time for a fixed service level (or improving service for a fixed lead time) requires reducing average cycle time and/or reducing standard deviation of cycle time. Policies for reducing average cycle time were noted above. Fortunately, these same policies are effective for reducing cycle time standard deviation. However, as we noted, some policies, such as reducing long rework loops, are particularly effective at reducing cycle time variability.

Example: Customer Service Enhancement

The focus of the SteadyEye example was on reducing mean cycle time. The underlying reason for this, of course, was the firm's concern about responsiveness to customers. But it makes no sense to address lead time without simultaneously considering service. Promising short lead times and then failing to meet them is hardly the way to improve customer service. Fortunately, the improvements we suggested can enable the system to both reduce lead time and improve service.

For example, recall that one proposed policy was to reduce scrap at the polisher, which in turn will reduce the need to expedite small jobs of parts to catch up with the rest of the batch at final assembly. Doing this will significantly reduce the *standard deviation* of cycle time, as well as mean cycle time. Therefore, even if we increase service (i.e., raise the safety factor z_s), total customer lead time can still be reduced. The other variability reduction measures will have similar impacts.

To illustrate this, suppose that the original mean cycle time was nine weeks with a standard deviation of three weeks. A lead time of 10 weeks allows for only about one-third of a standard deviation for safety lead time. Since $z_{0.33} = 0.63$, this results in service of only around 63 percent, which is consistent with what is being observed.

Suppose that after all the cycle time reduction steps have been implemented, average cycle time is reduced to seven shop days (1.4 weeks) and the standard deviation is reduced to one-half week. In this case, a two-week lead time represents a safety lead time of 0.6 week, or 1.2 standard deviations, which would result in 88 percent service. A (probably more reasonable) three-week lead time represents a safety lead time of 3.2 standard deviations, which would result in more than 99.9 percent service. The combination of significantly shorter lead times than the competition *and* reliable delivery would be a very strong competitive weapon for SteadyEye.

Finally, we point out that the benefits of variability and cycle time reduction are not limited to make-to-order systems. Recall that one of the improvement suggestions for cycle time reduction was to shift some parts to make-to-stock control. For instance, suppose SteadyEye stocks a common gear for which there is average demand of 500 per week with a standard deviation of 100. The cycle time to make the part is nine weeks with a standard deviation of three weeks. Thus, the mean demand during the replenishment time is 4,500, and the standard deviation is 1,530. If we produce $Q = 500$ at a time, then we can use the (Q, r) model of Chapter 2 to compute that a reorder point of $r = 7{,}800$ will be needed to ensure a 99 percent fill rate. This policy will result in an average on-hand inventory of 3,555 units. However, if the variability reduction measures suggested above reduced the cycle time to 1.4 weeks with a standard deviation of 0.4 week, the reorder point would fall to $r = 1{,}080$ and the average on-hand inventory would decrease to 631 units, a 92 percent reduction. This makes moving to the more responsive make-to-stock control for common parts an economically viable option.

9.7 Conclusions

The primary focus of this chapter is the effect of variability on the performance of production lines. The main points can be summarized as follows:

1. *Variability degrades performance.* If variability of any kind—process, flow, or batching—is increased, something has to give. Inventory will build up, throughput will decline, lead times will grow, or some other performance measure will get worse. As a result, almost all effective improvement campaigns involve at least some amount of variability reduction.

2. *Variability buffering is a fact of manufacturing life.* All systems buffer variability with inventory, capacity, and time. Hence, if you cannot reduce variability, you will have to live with one or more of the following:
   a. Long cycle times and high inventory levels
   b. Wasted capacity
   c. Lost throughput
   d. Long lead times and/or poor customer service
3. *Flexible buffers are more effective than fixed buffers.* Having capacity, inventory, or time that can be used in more than one way reduces the total amount of buffering required in a given system. This principle is behind much of the flexibility or agility emphasis in modern manufacturing practice.
4. *Material is conserved.* What flows into a workstation will flow out as either good product or scrap.
5. *Releases are* always *less than capacity in the long run.* The intent may be to run a process at 100 percent of capacity, but when true capacity, including overtime, outsourcing, etc., is considered, this can never occur. It is better to *plan* to reduce release rates before the system "blows up" and rates have to be reduced anyway.
6. *Variability early in a line is more disruptive than variability late in a line.* High process variability toward the front of a push line propagates downstream and causes queueing at later stations, while high process variability toward the end of the line affects only those stations. Therefore, there tends to be greater leverage from variability reduction applied to the front end of a line than to the back end.
7. *Cycle time increases nonlinearly in utilization.* As utilization approaches one, long-term WIP and cycle time approach infinity. This means that system performance is very sensitive to release rates at high utilization levels.
8. *Process batch sizes affect capacity.* The interaction between process batch size and setup time is subtle. Increasing batch sizes increases capacity and thereby reduces queueing. However, increasing batch sizes also increases wait-to-batch and wait-in-batch times. Therefore, the first focus in serial batching situations should be on setup time reduction, which will enable use of small, efficient batch sizes. If setup times cannot be reduced, cycle time may well be minimized at a batch size greater than one. Likewise, depending on the capacity and demand, the most efficient batch size in a parallel process may be in between one and the maximum number that will fit into the process.
9. *Cycle times increase proportionally with transfer batch size.* Waiting to batch and unbatch can be a large source of cycle time. Hence, reducing transfer batches is one of the simplest cycle time reduction measures available in many production environments.
10. *Matching can be an important source of delay in assembly systems.* Lack of synchronization, caused by variability, poor scheduling, or poor shop floor control, can cause significant buildup of WIP, and hence delay, wherever components are assembled.
11. *Diagnosis is an important role for factory physics.* The laws and concepts of factory physics are useful to trace the sources of performance problems in a manufacturing system. While the analytical formulas are certainly valuable in this regard, it is the intuition behind the formulas that is most critical in the diagnostic process.

Because variability is not well understood in manufacturing, the ideas in this chapter are among the most useful factory physics concepts presented in this book. We will rely heavily on them in Part III to address specific manufacturing management problems.

Study Questions

1. Under what conditions is it possible for a workstation to operate at 100 percent capacity over the long term and not be unstable (i.e., not have WIP grow to infinity)? Can this occur in practice?
2. In a line with large transfer batches, why is wait-for-batch time larger when utilization is low than when it is high? What assumption about releases is behind this, and why might it not be the case in practice?
3. In what way are variability reduction and capacity expansion analogous improvement options? What important differences are there between them?
4. Consider two adjacent stations in a line, labeled A and B. A worker at station A performs a set of tasks on a job and passes the job to station B, where a second worker performs another set of tasks. There is a finite amount of space for inventory between the two stations. Currently, A and B simply do their own tasks. When the buffer is full, A is blocked. When the buffer is empty, B is starved. However, a new policy has been proposed. The new policy designates a set of tasks, some from A's original set and others from B's set, as "shared tasks." When the buffer is more than half full, A does the shared tasks before putting jobs into the buffer. When the buffer is less than half full, A leaves the shared tasks for B to do. Assuming that the shared tasks can be done equally quickly by either A or B, comment on the effect that this policy will have on overall variability in the line. Do you think this policy might have merit?
5. The JIT literature is fond of the maxim "Variability is the root of all evil." The Variability Law of factory physics states that "variability degrades performance." However, in Chapter 7, we showed that the worst possible behavior for a line with a given r_b (bottleneck rate) and T_0 (raw process time) occurs when the system is completely deterministic (i.e., there is no variation in arrivals or process times). How can these be consistent?
6. Consider a one-station plant that consists of four machines in parallel. The machines have moderately variable random process times. Note that if the WIP level is fixed at four jobs, the plant will be able to maintain 100 percent utilization, minimum cycle time, and maximum throughput whether or not the process times are random. How do you explain this apparent "perfect" performance in light of the variability that is present? (*Hint:* Consider *all* the performance measures, including those for FGI and demand, when there is no variability at all. What happens to these measures when process times are made variable and demand is still constant?)

Intuition-Building Exercises

The purpose of these exercises is to build your intuition. They are in no way intended to be realistic problems.

1. You need to make 35 units of a product in one day. If you make more than 35 units, you must pay a carrying cost of \$1 per unit extra. If you make less than 35 units, you must pay a penalty cost of \$10 per unit.

   You can make the product in one of two workstations (you cannot use both). The first workstation (W1) contains a single machine capable of making 35 units per day, on average. The second workstation (W2) contains 10 machines, each capable of making 3.5 units per day, on average. Which workstation should you use?

Exercise: Simulate the output of W1 by rolling a single die and multiplying the number of spots by 10. Simulate the output of W2 by rolling the die 10 times and adding the total number of spots.

Perform five replications of the experiment. Compute the amount of penalty and carrying cost you would incur for each time. Which is the better workstation to use? What implications might this have for replacing of a group of old machines with a single "flexible manufacturing system"?

2. You market 20 different products and have a choice of two different processes. In process one (P1) you stock each of the 20, maintaining a stock of five for each of the products for a total of 100 units. In process two (P2) you stock only the basic component and then give each order "personality" when the order is received. The time to do this is, essentially, no greater than that for processing the order. For this process you stock 80 of the basic components.

   Every day you receive demand for each of the products. The demand is between one and six items with each level equally likely. Stock is refilled at the end of each day.
   Exercise: Which process do you think would have the better fill rate (i.e., probability of having stock for an order), P1 with 100 parts in inventory or P2 with only 80? Simulate each, using a roll of a die to represent the demand for each of the 20 products, and keep track of total demand and the total number of stockouts. Repeat the simulation at least five times, and compute the average fill rate.

3. Consider a line composed of five workstations in series. Each workstation has the *potential* to produce anywhere between one and six parts on any given day, with each outcome equally likely (note that this implies the average potential production of each station is 3.5 units per day). However, a workstation in the middle of the line cannot produce more on a day than the amount of WIP it starts the day with.
   Exercise 1: Perform an experiment using a separate roll of a die for the daily potential production at each station. Use matchsticks, toothpicks, poker chips, whatever, to represent WIP. Each time you roll the die, actual production at the station will be the lesser of the die roll and the available WIP.

   Since you start out empty, it will take five days to fill up the line. So begin recording the output at the sixth period. Plot the cumulative output and total WIP in the line versus time up to day 25.
   Exercise 2: Now reduce the WIP by employing a kanban mechanism. To do this, do not allow WIP to exceed four units at any buffer (after all, the production rate is 3.5 so we should be able to live with four). Do this by reducing the actual production at a station if it will ever cause WIP at the next station to exceed four. Repeat the above exercise under these conditions. What happens to throughput? What about WIP?
   Exercise 3: Now reduce variability. To do this, change the interpretation of roll. If a roll is three or less, potential production is three units. If it is four or more, potential production is four units. Note that the average is the same as before. Now repeat both the first exercise (without the kanban mechanism) and the second exercise (with kanban). Compare your results with those of the previous cases.
   Exercise 4: Finally, consider the situation where there are two types of machine in the line, one that is highly variable and another that is less variable. Should we have the more variable ones feed the less variable ones, or the other way round? Repeat the first exercise for a line where the first two machines are extremely variable (i.e., potential production is given by the number of spots on the die) and the last three are less variable (i.e., potential production is three if the roll is three or less and four if it is four or more). Repeat with a line where the last two machines are extremely variable and the first three are less variable. Compare the throughput and WIP for the two lines, and explain your results.

Problems

1. Consider a line that makes two different astronomical digital cameras. The TS-7 costs $2,000 while the TS-8, which uses a much larger chip, costs $7,000. Most of the cost of the cameras is due to the cost of the chip.

   In manufacturing, both go through the same three steps but take different amounts of time. The capacities for the TS-7 are seven, five, and six per day at workstations 1, 2, and 3, respectively (that is, if we run exclusively TS-7 product). Similarly, capacity for the TS-8 is six per day at all stations (again, assuming we run only TS-8). Five percent of TS-8 units must be reworked, which requires them to go back through all three stations a second time (process times are the same as those for the first pass). Reworked jobs never make a third pass through the line. There is no rework for the TS-7.

   Demand is three per day for the TS-7 and one per day for the TS-8. The average inventory level of chips is 20 for the TS-7 and five for the TS-8. Cycle time for both cameras is four days, while the raw process time with no detractors is one-half a day. Cameras are made to stock and sold from finished goods inventory. Average finished goods inventory is four units of the TS-7 and one unit of the TS-8, while the fill rate is 0.85 for both cameras.
   a. Compute throughput TH(i) for each station for each product.
   b. Compute utilization $u(i)$ at each station.
   c. Using dollars as the aggregate measure, compute RMI, WIP, and FGI.
   d. Compute the efficiencies E_{TH}, E_u, E_{inv}, E_{CT}, E_{LT}, E_s, and E_Q.
   e. Suppose the machine at workstation 1 costs $1 million and the machines at the second and third workstations cost $10,000 each. Suggest a different measure for E_u than that given in the text. Compute it and compare with the previous value.
2. Describe the types of buffer(s) (i.e., inventory, time, or capacity) you would expect to find in the following situations.
   a. A maker of custom cabinets
   b. A producer of automotive spare parts
   c. An emergency room
   d. Wal-Mart
   e. Amazon.com
   f. A government contractor that builds submarines
   g. A bulk producer of chemical intermediates such as acetic acid
   h. A maker of lawn mowers for K-Mart, Sam's Club, and Target
   i. A freeway
   j. The space shuttle (i.e., as a delivery system for advanced experiments)
   k. A business school
3. Compute the capacity (jobs per day) for the following situations.
   a. A single machine with a mean process time of two and one-half hours and an SCV of 1.0. There are eight work hours per day.
   b. A single machine with a mean process time of two and one-half hours and an SCV of 0.5. There are eight work hours per day.
   c. A workstation consisting of 10 machines in parallel, each having a mean process time of two and one-half hours. There are two eight-hour shifts. Lunch and breaks take one and one-fourth hours per shift.
   d. A workstation with 10 machines in parallel, each having a mean process time of two and one-half hours. There are two eight-hour shifts. Lunch and breaks take one and one-fourth hours per shift. The machines have a mean time to failure of 100 hours with a mean time to repair of four hours.
   e. A workstation with 10 machines in parallel, each having a mean process time of two and one-half hours. There are two eight-hour shifts. Lunch and breaks take one and one-fourth hours per shift. The machines have a mean time to failure of 100 hours with a mean time to repair of four hours. The machines are set up every 10 jobs, and the mean setup time is three hours.

f. A workstation with 10 machines in parallel, each having a mean process time of two and one-half hours. There are two eight-hour shifts. Lunch and breaks take one and one-fourth hours per shift. The machines have a mean time to failure of 100 hours with a mean time to repair of four hours. The machines are set up every 10 jobs, and the mean setup time is three hours. Because the operators have to attend training meetings and the like, we cannot plan more than 85 percent utilization of the workers operating the machines.

4. Jobs arrive to a two-station serial line at a rate of two jobs per hour with deterministic interarrival times. Station 1 has one machine which requires exactly 29 minutes to process a job. Station 2 has one machine which requires exactly 26 minutes to process a job, provided it is up, but is subject to failures where the mean time to failure is 10 hours and the mean time to repair is one hour.
   a. What is the SCV c_a^2 of arrivals to station 1?
   b. What is the effective SCV $c_e^2(1)$ of process times at station 1?
   c. What is the utilization of station 1?
   d. What is the cycle time in queue at station 1?
   e. What is the total cycle time at station 1?
   f. What is the SCV of arrivals to station 2?
   g. What is the utilization of station 2?
   h. What is the effective SCV $c_e^2(2)$ of process times at station 2?
   i. What is the cycle time in queue at station 2?
   j. What is the total cycle time at station 2?
5. A punch press takes in coils of sheet metal and can make five different electrical breaker boxes, denoted by B1, B2, B3, B4, and B5. Each box takes exactly one minute to produce. To switch the process from one type of box to another takes four hours. There is demand of 1,800, 1,000, 600, 350, and 200 units per month for boxes B1, B2, B3, B4, and B5, respectively. The plant works one shift, five days per week. After lunch, breaks, etc., there is seven hours available per shift. Assume 52 weeks per year.
   a. What is r_a in boxes per hour?
   b. What would utilization be if there were no setups? (Note that utilization will approach this as batch sizes approach infinity.)
   c. Suppose the SCV of the press is 0.2 no matter what the batch sizes are. What is the average cycle time for the optimal batch sizes (assume $c_a^2 = 1$)?
   d. Use trial and error to find a set of batch sizes that minimizes cycle time.
   e. On average, how many times per month do we make each type of box if we use the batch sizes computed in part *d*?
6. A heat treat operation takes six hours to process a batch of parts with a standard deviation of three hours. The maximum that the oven can hold is 125 parts. Currently there is demand for 160 parts per day (16-hour day). These arrive to the heat treat operation one at a time according to a Poisson stream (i.e., with $c_a = 1$).
   a. What is the maximum capacity (parts per day) of the heat treat operation?
   b. If we were to use the maximum batch size, what would be the average cycle time through the operation?
   c. What is the minimum batch size that will meet demand?
   d. If we were to use the minimum feasible batch size, what would be the average cycle time through the operation?
   e. Find the batch size that minimizes cycle time. What is the resulting average cycle time?
7. Consider a balanced line, having five identical stations in series, each consisting of a single machine with low-variability process times and an infinite buffer. Suppose the arrival rate is r_a, utilization of all machines is 85 percent, and the arrival SCV is $c_a^2 = 1$. What happens to WIP, CT, and TH if we do the following?
   a. Decrease the arrival rate.
   b. Increase the variability of one station.
   c. Increase the capacity at one station.
   d. Decrease the capacity of all stations.

8. Consider a two-station line. The first station pulls from an infinite supply of raw materials. Between the two stations there is a buffer with room for five jobs. The second station can always push to finished goods inventory. However, if the buffer is full when the first station finishes, it must wait until there is room in the buffer before it can start another job. Both stations take 10 minutes per job and have exponential process times ($c_e = 1$).
   a. What are TH, CT, and WIP for the line?
   b. What are TH, CT, and WIP if we increase the buffer to seven jobs?
   c. What are TH, CT, and WIP if we slow down the second machine to take 12 minutes per job?
   d. What are TH, CT, and WIP if we slow down the first machine to take 12 minutes per job?
   e. What happens to TH if we decrease the variability of the second machine so that the effective SCV is a $\frac{1}{4}$?
9. Consider a single station that processes two items, A and B. Item A arrives at a rate of 30 per hour. Setup times are five hours, and the time it takes to process one part is one minute. Item B arrives at a rate of 20 per hour. The setup time is four hours, and the unit process time is two minutes. Arrival and process variability is moderate (that is, $c_a = c_e = 1$) regardless of the batch size (just assume they are).
   a. What is the minimum lot size for A for which the system is stable (assume B has an infinite lot size)?
   b. Make a spreadsheet and find the lot sizes for A and B that minimize average cycle time.
10. Consider a balanced and stable line with moderate variability and large buffers between stations. The line uses a push protocol, so that releases to the line are independent of line status. The capacity of the line is r_b, and the utilization is fairly high. What happens to throughput and cycle time when we do the following?
   a. Reduce the buffer sizes and allow blocking at all stations except the first where jobs balk if the buffer is full (i.e., they go away if there is no room).
   b. Reduce the variability in all process times.
   c. Unbalance the line, but do not change r_b.
   d. Increase the variability in the process times.
   e. Decrease the arrival rate.
   f. Decrease the variability in the process times *and* reduce the buffer sizes as in *a*. Compare to the situation in *a*.
11. A particular workstation has a capacity of 1,000 units per day and variability is moderate, such that $V = (c_a^2 + c_e^2)/2 = 1$. Demand is currently 900 units per day. Suppose management has decided that cycle times should be no longer than one and one-half times raw process time.
   a. What is the current cycle time in multiples of the raw process time?
   b. If variability is not changed, what would the capacity have to be in order to meet the cycle time and demand requirements? What percentage increase does this represent?
   c. If capacity is not changed, what value would be needed for V in order to meet the cycle time and demand requirements? What percentage decrease does this represent (compare CVs, not SCVs)?
   d. Discuss a realistic strategy for achieving management's goal.
12. Consider two stations in series. Each is composed of a single machine that requires a rather lengthy setup. Large batches are used to maintain capacity. The result is an effective process time of one hour per job and an effective CV of 3 (that is, $t_e = 1.0$ and $c_e^2 = 9.0$). Jobs arrive in a steady stream at a rate of 0.9 job per hour, and they come from all over the plant, so $c_a = 1.0$ is a reasonable assumption (see the discussion in Chapter 8).

    Now, suppose a flexible machine is available with the same capacity but less effective variability (that is, $t_e = 1.0$ and $c_e^2 = 0.25$) and can be used to replace the machine at either station. At which station should we replace the existing machine with the new one to get the largest reduction in cycle time? (*Hint:* Use the equation $c_d^2 = u^2 c_e^2 + (1 - u^2)c_a^2$ along with the cycle time equations.)
13. Recall the throughput enhancement example in Section 9.6.1. Assuming there is an unlimited amount of raw material for the coater, answer the following.

a. Compute t_e and c_e^2, using the data given in Section 9.6.1 for both the coater and the expose operation.
b. Use the general blocking model of Section 8.7.2 to compute the throughput for the line, assuming there is room for 10 jobs in between the two stations (that is, $b = 12$). Will the resulting throughput meet demand?
c. Reduce the MTTR from eight to four hours, and recompute throughput. Now does the throughput meet demand?

14. Table 9.6 gives the speed (in parts per hour), the CV, and the cost for a set of tools for a circuit-board line. Jobs go through the line in totes that hold 50 parts each (this cannot be changed). The CVs represent the *effective* process times and thus include the effects of downtime, setups, etc.

The desired average cycle time through this line is 1.0 day. The maximum demand is 1,000 parts per day.
a. What is the least-cost configuration that meets demand requirements?
b. How many possible configurations are there?
c. Find a good configuration.

15. Consider line 1 in Table 9.5. Assume batches of six jobs arrive every 35 hours with no variability in the arrivals, the setup times, or the process times. Construct a Gantt chart (i.e., time line) like that in Figure 9.8 for the system when the stations are permuted from the original order (1, 2, 3) as follows:
a. 1, 3, 2
b. 2, 1, 3
c. 2, 3, 1
d. 3, 1, 2
Check to see if the cycle times fall within the bounds given in Section 9.5.3.

16. Suppose parts arrive in batches of 12 every 396 minutes to a three-station line having no variability. The first station has a setup time of 15 minutes and a unit process time of seven minutes, the second sets up in eight minutes and processes one part every three minutes, the third requires 12 and four minutes for setup and unit processing, respectively.
a. What is the utilization of each station? Which is the bottleneck?
b. What is the cycle time if parts are moved 12 at a time?
c. What is the cycle time for the first part if parts are moved one at a time?
d. What is the range of cycle time for the 12th part if parts are moved one at a time?
e. What is the range of average cycle times if parts are moved one at a time?
f. Perform a Penny Fab–like experiment to determine the average cycle time. Let 12 parts arrive each 396 minutes, and then move them one at a time.
g. Double the arrival rate (i.e., batches of 12 arrive every 198 minutes). What happens to cycle time if parts are moved 12 at a time? What happens to cycle time if parts are moved one at a time?
h. Now let the arrivals be Poisson with the same average time between arrivals (396 minutes). What is the added queue time at each station?
i. Now double the Poisson arrival rate. What happens to cycle time?

TABLE 9.6 Possible Machines to Purchase for Each Work Center

	Possible Machines (speed [parts/hour], CV, cost [$000])			
Station	**Type 1**	**Type 2**	**Type 3**	**Type 4**
MMOD	42, 2.0, 50	42, 1.0, 85	50, 2.0, 65	10, 2.0, 110.5
SIP	42, 2.0, 50	42, 1.0, 85	50, 2.0, 65	10, 2.0, 110.5
ROBOT	25, 1.0, 100	25, 0.7, 120	—	—
HDBLD	5, 0.75, 20	5.5, 0.75, 22	6, 0.75, 24	—

CHAPTER

10 PUSH AND PULL PRODUCTION SYSTEMS

You say yes.
I say no.
You say stop,
And I say go, go, go!

John Lennon, Paul McCartney

10.1 Introduction

Virtually all descriptions of just-in-time make use of the terms *push* and *pull* production systems. However, these terms are not always precisely defined and, as a result, may have contributed to some confusion surrounding JIT in America.

In this chapter, we offer a formal definition of push and pull at the conceptual level. By separating the concepts of push and pull from their specific implementations, we observe that most real-world systems are actually hybrids or mixtures of push and pull. Furthermore, by contrasting the extremes of "pure push" and "pure pull" production systems, we gain insight into the factors that make pull systems effective. This insight suggests that there are many different ways to achieve the benefits of pull. Which is best depends on a variety of environmental considerations, as we discuss in this chapter and pursue further in Part III.

10.2 Definitions

The father of JIT, Taiichi Ohno, used the term *pull* only in a very general sense (Ohno 1988, xiv):

> Manufacturers and workplaces can no longer base production on desktop planning alone and then distribute, or *push*, them onto the market. It has become a matter of course for customers, or users, each with a different value system, to stand in the frontline of the marketplace and, so to speak, *pull* the goods they need, in the amount and at the time they need them.

Hall (1983, 39), in one of the most prominent American texts on JIT, was more specific, defining pull systems by the fact that "material is drawn or sent for by the

users of the material as needed." Although he acknowledged that different types of pull systems are possible, the only one he described in detail was the Toyota kanban system, which we discussed in Chapter 4.[1] Schonberger (1982), in the other major American JIT book, referred to pull systems strictly in the context of the Toyota-style kanban system. Hence, it is hardly surprising that the term *pull* is frequently viewed as synonymous with *kanban*.

However, we do not feel that such a narrow interpretation was Ohno's intent. In our view, limiting *pull* to mean *kanban* is downright counterproductive: It obscures the essence of pull by assigning it too much specificity. It mixes a concept (pull) with its implementation (kanban). In order for us to discuss the concept of pull from a factory physics perspective, it is important to give a general, but simple, definition of push and pull systems.

10.2.1 The Key Difference between Push and Pull

What distinguishes push from pull is the mechanism that triggers the movement of work in the system. Fundamentally, the trigger for work releases comes from *outside* a push system but from *inside* a pull system. More formally we define push and pull systems as follows:

Definition: *A* **push** *system* schedules *the release of work based on demand, while a* **pull** *system* authorizes *the release of work based on system status.*

The contrast between push and pull systems is depicted schematically in Figure 10.1. Strictly speaking, a push system releases a job into a production process (factory, line, or workstation) precisely when called to do so by an exogenous schedule, and the release time is not modified according to what is happening in the process itself. In contrast, a pull system only allows a job onto the floor when a signal generated by a change in line status calls for it. Typically, as in the Toyota kanban system, these authorization signals are the result of the completion of work at some point in the line. Notice that this definition has nothing to do with who actually moves the job. If an operator from a downstream process comes and gets work from an upstream process, but does it according to an exogenous schedule, then the process is push. If an upstream operator delivers work to the downstream process, but does so in response to status changes in the downstream process, then this is pull.

Another useful way to think about the distinction between push and pull systems is that push systems are inherently *make-to-order* while pull systems are *make-to-stock*. That is, the schedule that drives a push system is driven by orders (or forecasts), but not by system status. The signals that authorize releases in a pull system are voids in a stock level somewhere in the system. Viewed in this way, the base stock model, which triggers orders when stock drops below a specified level, is a pull system. An MRP system, which releases order into the system according to a schedule based on customer orders, is a push system.

Of course, most real-world systems have aspects of both push and pull. For instance, if a job is scheduled to be released by MRP, but is held out because the line is

[1] Hall also presented as a pull system the *broadcast* system, in which the final assembly schedule (FAS) is broadcast to all starting points in the line in order to trigger work releases. However, he noted that because the FAS is generated externally, this system does not place a restriction on the total inventory in the system. He distinguished the control in a broadcast system from that in a kanban system by referring to the FAS signals as *loose pull signals*. Because of its failure to limit WIP, we are not convinced that this system should be termed a pull system at all.

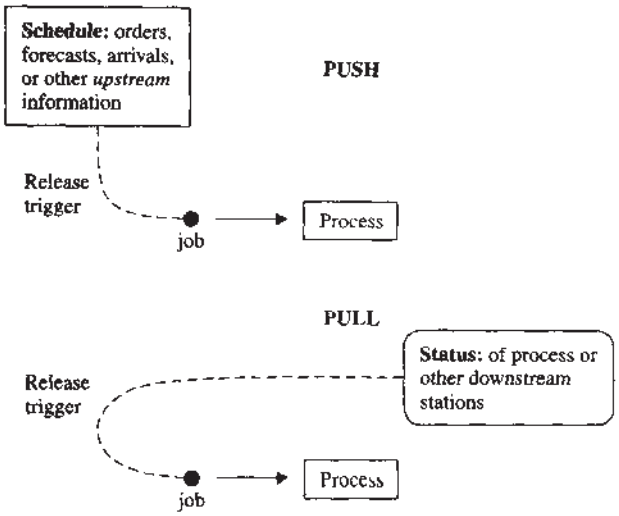


FIGURE 10.1
Release triggers in push and pull production systems

considered too congested, then the effect is a hybrid push-pull system. Conversely, if a kanban system generates a card authorizing production but the actual work release is delayed because of anticipated lack of demand for the part (i.e., it is not called for in the master production schedule), then this, too, is a hybrid system. There have been various attempts to formally combine push and pull into hybrid systems (e.g., see Wight 1970, Deelersnyder et al. 1988, and Suri 1998). We will discuss the virtues of hybrid systems and present an approach in Part III.

Our purpose in setting up a sharp distinction between push and pull is *not* to suggest that users must rigidly choose one or the other. Rather, in the spirit of factory physics, we use our definition to isolate the benefits of pull systems and trace their root causes. In a sense, we are taking a similar approach to that of (nonfactory) physics in which mechanical systems are frequently considered in frictionless environments. It is not that frictionless environments are common, but rather that the concepts of gravitation, acceleration, velocity, and so forth are clearer in this pristine framework. Just as the frictionless insights of classical mechanics underlie analysis of realistic physical systems, our observations about pure push and pure pull systems provide a foundation for analysis of realistic production systems.

10.2.2 The Push-Pull Interface

The questions of whether and how to use pull are only part of the picture; *where* to use pull is also important. Even in an individual production system, it is possible to run only part of it as a pull system. A useful concept for thinking about placement of pull mechanisms is the *push-pull interface,* which divides a production process into push and pull segments.[2] Choosing the location of this interface wisely can enable a system to take strategic advantage of the benefits of pull, while still retaining the customer-driven character of push.

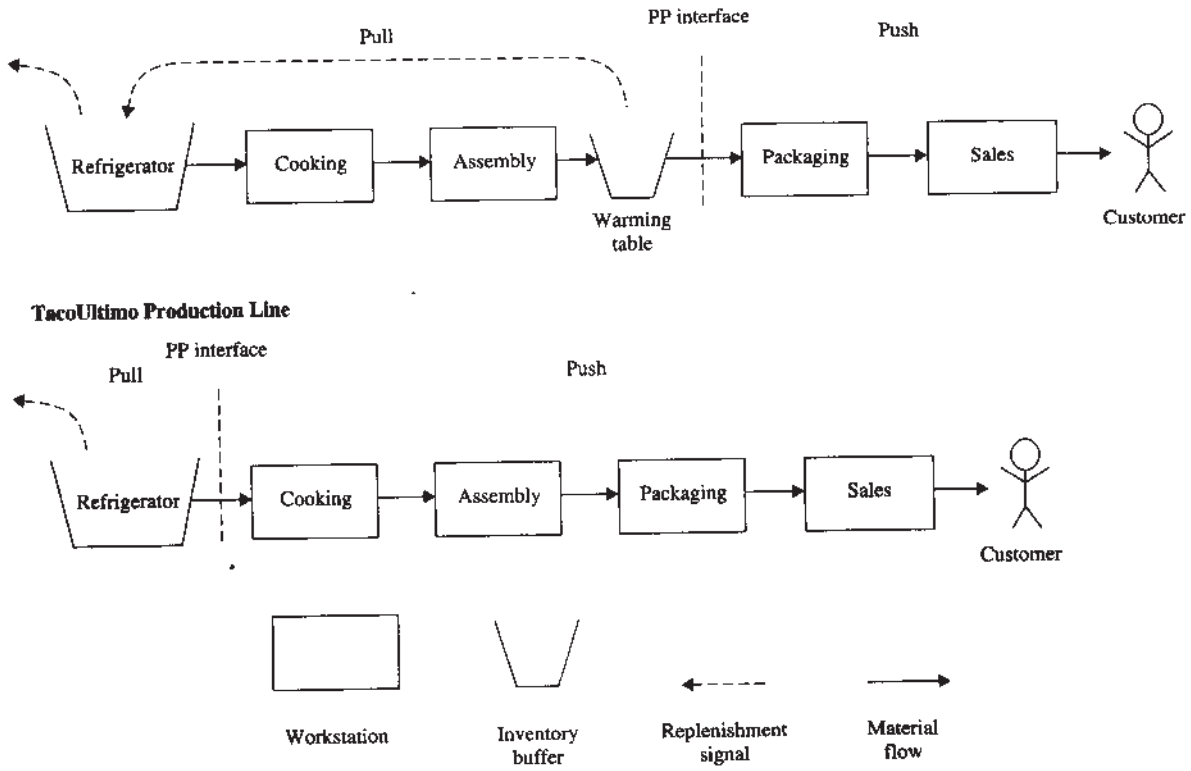
[2]We are indebted to Corey Billington of Hewlett-Packard (HP) for the term *push-pull interface*, which was coined to help describe practices developed as part of their "design for supply chain management" efforts. See Lee and Billington (1995) for an overview of HP supply chain initiatives.

To understand the concept of the push-pull interface, it is convenient to think in terms of push being defined as make to order and pull being defined as make to stock. To see how similar lines can be divided differently into push and pull segments, consider the two systems depicted in Figure 10.2. In the front part of the QuickTaco line, tacos are produced to stock, to maintain specified inventory levels at the warming table, which makes this portion of the line behave as a pull system. The back of the line moves product (tacos) only when triggered by customer orders, and hence it acts as a push system. The push-pull interface lies at the warming table. In contrast, the movement of tacos in the TacoUltimo line is triggered solely by customer orders, so it is entirely a push system. The push-pull interface lies at the refrigerator, where raw materials are stocked according to inventory targets.

By contrasting the relative advantages of the QuickTaco and TacoUltimo lines, we can gain insight into the tradeoffs involved in positioning the push-pull interface. The TacoUltimo line, because it is entirely order-driven and holds inventory almost exclusively in the form of raw materials, has the advantage of being very flexible (i.e., it can produce virtually any taco a customer wants). The QuickTaco line, because it holds finished tacos in stock, has the advantage of being responsive (i.e., it offers shorter lead times to the customer). Hence, the tradeoff is between speed and flexibility. By moving the push-pull interface closer to the customer, we can reduce lead times, but only at the expense of reducing flexibility.

FIGURE 10.2

Illustration of push-pull interface placement



So how does one choose the location of the push-pull interface for a given system? Since it depends on both customer preferences and the physical details of the production process, this is not a simple question. But we can offer some observations and some real-world examples.

First, note that the primary reason for moving the push-pull interface closer to the customer is speed. So it only makes sense to do it when the additional speed will produce a noticeable improvement in service from the perspective of the customer. For instance, in a production system with two-hour cycle times within the line but which makes end-of-day shipments, customers might not see any difference in lead times by shortening cycle time in the line through a push-pull interface shift. Even in the fast-food industry, where speed is clearly critical, there are restaurants that make use of a TacoUltimo type of line. They do this by making sure that the cycle time of the entire line is sufficiently short to enable the system to meet customer expectations. However, during rush hour, when the pressure for speed is especially great, many TacoUltimo-type fast-food restaurants shift to the QuickTaco mode.

Second, observe that the options for positioning the push-pull interface are strongly affected by the process itself. For instance, in the taco line, we could propose a push-pull interface somewhere in the middle of assembly. That is, cook the tortilla shell and fill it with meat, but leave it open, waiting for toppings. However, this would present storage and quality problems (e.g., partially assembled tacos falling apart) and hence is probably infeasible.

Third, notice that the economics of push-pull interface placement are affected by how the product is customized as it progresses through the system. In a system with very few end items (e.g., a plywood mill that takes a few raw materials like logs and glue and produces a few different thicknesses of plywood), it may be perfectly sensible to set the push-pull interface at finished goods. However in a system with many end items (e.g., a PC assembly plant, where components can be combined into a wide range of finished computers), holding inventory at the finished goods level would be very expensive (see the safety stock aggregation example in Section 8.8.2). For example, in the taco system, locating the push-pull interface after packaging is probably a bad idea, since it would require stocking bags of tacos in all needed sizes and combinations.

Finally, note that the issue of customization is closely related to the issue of variability pooling, which we introduced in Chapter 8. In a system in which the product becomes increasingly customized as it progresses down the line, moving the push-pull interface upstream can reduce the amount of safety stock that needs to be carried as protection against demand variability. For example, Benetton made use of a system in which undyed sweaters were produced to stock and then "dyed to order." That is, they moved the push-pull interface from behind the dying process to in front of it. In doing so, they were able to pool the safety stock for the various colors of sweaters and thereby reduce inventory costs of achieving a given level of customer service.

Some other real-world examples in which the push-pull interface was relocated to improve overall system performance include these:

1. *IBM* had a printed-circuit board plant that produced more than 150 different boards from fiberglass and a few thicknesses of copper. The front part of the line produced *core blanks*—laminates of copper and fiberglass from which all circuit boards are made. There were only about eight different core blanks, which were produced in an inherently batch lamination process that was difficult to match to customer orders. Management elected to stock core blanks (i.e., move the push-pull interface from raw materials to a stock point beyond the lamination process). The result was the elimination

of a day or two of cycle time from the lead time perceived by customers at the cost of very little additional inventory.

2. *General Motors* introduced a new vehicle delivery system, starting with Cadillac in Florida, in which popular configurations were stocked at regional distribution centers (*Wall Street Journal*, October 21, 1996, A1). The goal was to provide 24-hour delivery to buyers of these "pop cons" from any dealership. Lead times for other configurations would remain at the traditional level of several weeks. So, unlike in a traditional system, in which the push-pull interface is located at the assembly plant (for build-to-order vehicles) and at the dealerships (for build-to-stock vehicles), this new system places the push-pull interface at the regional distribution centers. The hope is that by pooling inventory across dealerships, General Motors will be able to provide quick delivery for a high percentage of sales with lower total inventory costs. Note that this example illustrates that it is possible, even desirable, to have different locations for the push-pull interface for different products in the same system.

3. *Hewlett-Packard* produced a variety of printers for the European market. However, because of varying voltage and plug conventions, printers required different power supplies for different countries. By modifying the production process to leave off the power supplies, Hewlett-Packard was able to ship generic printers to Europe. There, in the distribution centers, power supplies were installed to customize the printers for particular countries (see Lee, Billington, and Carter 1993 for a discussion of this system). By locating the push-pull interface at the Europe-based distribution center instead of at the American-based factory, the entire shipping cycle time was eliminated from the customer lead time. At the same time, by delaying customization of the printers in terms of power supply, Hewlett-Packard was able to pool inventory across countries. This is an example of **postponement,** in which the product and production process are designed to allow late customization. Postponement can be used to facilitate rapid customer response in a highly customized manufacturing environment, a technique sometimes referred to as **mass customization** (Feitzinger and Lee 1997).

10.3 The Magic of Pull

What makes Japanese manufacturing systems so good? We hope that the reader gathered from Chapter 4 that there is no simple answer to this question. The success of several high-profile Japanese companies in the 1980s was the result of a variety of practices, ranging from setup reduction to quality control to rapid product introduction. Moreover, these companies operated in a cultural, geographic, and economic environment very different from that in America. If we are to understand the essence of the success of JIT, we must narrow our focus.

At a macro level, the Japanese success was premised on an ability to bring quality products to market in a timely fashion at a competitive cost and in a responsive mix. At a micro level, this was achieved via an effective production control system, which facilitated low-cost manufacture by promoting high throughput, low inventory, and little rework. It fostered high external quality by engendering high internal quality. It enabled good customer service by maintaining a steady, predictable output stream. And it allowed responsiveness to a changing demand profile by being flexible enough to accommodate product mix changes (as long as they were not too rapid or pronounced).

What is the key to all these desirable features that made the Japanese production control system such an attractive basis for a business strategy? The American JIT

literature seems to suggest that the act of pulling is fundamental. Hall (1983, 39) cited a General Motors foreman who described the essence of pull as "You don't never make nothin' and *send* it no place. Somebody has to come get it."

We disagree. Our view, which we will expand upon in this chapter, is that the pulling of parts into workstations is merely a means to an end. The true underlying cause of the key benefits of a pull system is that *there is a limit on the maximum amount of inventory in the system.* In a (one-card) kanban system, the number of containers is bounded by the number of production cards. No matter what happens on the plant floor, the WIP level *cannot* exceed a prespecified limit. But this effect is not limited to kanban systems. Because a pull system authorizes releases on the basis of voids in stock levels, or equivalently is a make-to-stock system, any true pull system will establish an upper bound on the WIP level. As we discuss in the following subsections, the major benefits of JIT can be attributed to the existence of this **WIP cap,** no matter how it is achieved. The magic was in the WIP cap, not the pulling process.

10.3.1 Reducing Manufacturing Costs

If WIP is capped, then disruptions in the line (e.g., machine failures, shutdowns due to quality problems, slowdowns due to product mix changes) do not cause WIP to grow beyond a predetermined level. Note that in a pure push system, no such limit exists. If an MRP-generated schedule is followed literally (i.e., without adjustment for plant conditions), then the schedule could get arbitrarily far ahead of production and thereby bury the plant in WIP, causing a **WIP explosion.**

Of course, we never observe real-world plants with infinite amounts of WIP. Eventually, when things get bad enough, management does something. It schedules overtime. It hires temporary workers to increase capacity. It pushes out due dates and limits releases to the plant—in other words, management stops using a pure push system. And eventually things return to normal...until the next WIP explosion (see Chapter 9 for a discussion of the overtime vicious cycle). The key point here is that in a push environment, corrective action is not taken until *after* there is a problem and WIP has already spiraled out of control.

In a pull system that establishes a WIP cap, releases are choked off *before* the system has become overloaded. Output will fall off, to be sure, but this would happen regardless of whether the WIP level were allowed to soar. For example, if a key machine is down, then all the WIP in the world in front of it cannot make it produce more. But by holding WIP out of the system, the WIP cap retains a degree of flexibility that would be lost if it were released to the floor. As long as jobs exist only as orders on paper, they can accommodate engineering or scheduling priority changes relatively easily. But once the jobs are on the floor, and given "personality" (e.g., a printed-circuit board receives its circuitry), changes in scheduling priority require costly and disruptive expediting, and engineering changes may be almost impossible. Thus, a WIP cap reduces manufacturing costs by reducing costs due to expediting and engineering changes.

In addition to improving flexibility, a pull system promotes better timing of work releases. To see this, observe that a pure push system periodically allows too much work into the system (i.e., at times when congestion will prevent the new work from being processed quickly). This serves to inflate the average WIP level without improving throughput. A WIP cap, regardless of the type of pull mechanism used to achieve it, will reduce the average WIP level required to achieve a given level of throughput. This will directly reduce the manufacturing costs associated with holding inventory.

10.3.2 Reducing Variability

The key to keeping customer service high is a predictable flow through the line. In particular, we need low **cycle time variability.** If cycle time variability is low, then we know with a high degree of precision how long it will take a job to get through the plant. This allows us to quote accurate due dates to customers, and meet them. Low cycle time variability also helps us quote shorter lead times to customers. If cycle time is 10 days plus or minus 6 days, then we will have to quote a 16-day lead time to ensure a high service level. On the other hand, if cycle time is 10 days plus or minus 1 day, then a quote of 11 days will suffice.

Kanban achieves less variable cycle times than does a pure push system. Since cycle time increases with WIP level (by Little's law), and kanban prevents WIP explosions, it also prevents cycle time explosions. However, note that the reason for this, again, is the WIP cap—not the pulling at each station. Hence, any system that caps WIP will prevent the wild gyrations in WIP, and hence cycle time, that can occur in a pure push system.

Kanban is also often credited with reducing variability directly at workstations. This is the JIT "reduce the water level to expose the rocks" analogy. Essentially, kanban limits the WIP in the system, making it much more vulnerable to variability and thereby putting pressure on management to continually improve.

We illustrate the intuition behind this analogy by means of the simple example shown in Figure 10.3. The system consists of two machines, and machine 1 feeds machine 2. Machine 1 is extremely fast, producing parts at a rate of one per second, while machine 2 is slow, producing at a rate of one per hour. Suppose a (one-card) kanban system is in use, which limits the WIP between machines to five jobs. Because machine 1 is so fast, this buffer will virtually always be full whenever machine 1 is running.

However, suppose that machine 1 is subject to periodic failures. If a failure lasts longer than five hours, then machine 2, the bottleneck, will starve. Thus, depending on the frequency and duration of failures of machine 1, machine 2 could be starved a significant fraction of time, despite the tremendous speed of machine 1.

Clearly, if the buffer size (number of kanban cards) were increased, the level of starvation of machine 2 would decrease. For instance, if the buffer were increased to 10 jobs, only failures in excess of 10 hours would cause starvation. In effect, the extra WIP insulates the system from the disruptive effects of failures. But as we noted previously, a pure push system requires higher average WIP levels to attain a given throughput level. A push system will tend to mask the effects of machine 1 failures in precisely this way. The

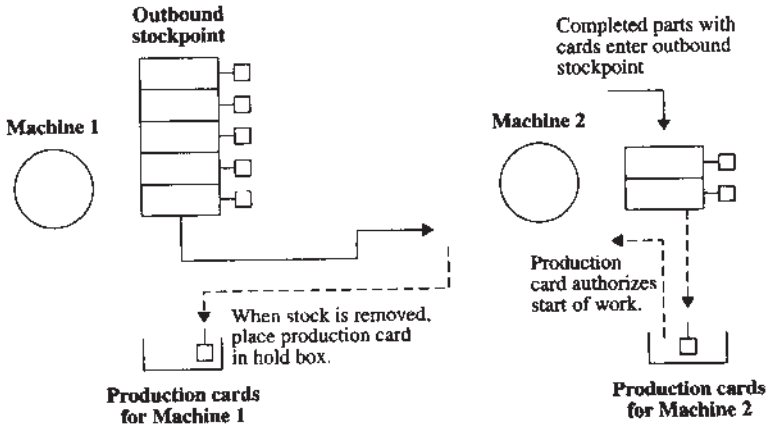


FIGURE 10.3
Workstations connected by a finite buffer

push system will have higher WIP levels throughout the system, and therefore failures will be less disruptive. As long as management is willing to live with high WIP levels, there is little pressure to improve the reliability of machine 1.

As the JIT literature correctly points out, if one wants to maintain high levels of throughput with *low* WIP levels (and short cycle times), one must reduce these disruptive sources of variability (failures, setups, recycle, etc.). We note that, again, the source of this pressure is the limited WIP level, not the mechanism of pulling at each station. To be sure, pulling at each station controls the WIP level at every point in the process, which would not necessarily be the case with a general WIP cap. However, reducing overall WIP level via a WIP cap *will* reduce the WIP between various workstations on average and thereby will apply the pressure that promotes continual improvement. Whether or not a general WIP cap will distribute WIP properly in the line is a question we will take up later.

10.3.3 Improving Quality

Quality is generally considered to be both a precondition for JIT and a benefit of JIT. As such, JIT promotes higher levels of quality out of sheer necessity and also establishes conditions under which high quality is easier to achieve.

As Chapter 4 observed, quality is a basic component of the JIT philosophy. The reason is that if WIP levels are low, then a workstation will effectively be starved for parts whenever the parts in its inbound buffer (stockpoint) do not meet quality standards. From a logistics standpoint, the effect of this is very similar to that of machine failures; once WIP levels become sufficiently low, the percentage of good parts in the system *must* be high in order to maintain reasonable throughput levels. To ensure this, kanban systems are usually accompanied by statistical process control (SPC), quality-oriented worker training, quality-at-the-source procedures, and other techniques for monitoring and improving quality levels throughout the system. Since the higher the quality, the lower the WIP levels can be, continual efforts at WIP reduction practiced in a JIT system will demand continual quality improvement.

Beyond this simple pressure for better quality, JIT can also directly facilitate improved quality because inspection is more effective in a low-WIP environment. If WIP levels are high and queues are long, a quality assurance (QA) inspection may not identify a process problem until a large batch of defective parts has already been produced. If WIP levels are low, so that the queue in front of QA is short, then defects can be detected in time to correct a process before it produces many bad parts. This, of course, is the goal of SPC, which monitors the quality of a process in real time. However, where immediate inspection is not possible, say, in a circuit-board plant where boards must be optically or electronically tested to determine quality, then low WIP levels can significantly amplify the power of a quality control program.

Notice that, once again, the benefits we are ascribing to kanban or JIT are really the consequence of WIP reduction. Hence, a simple WIP cap will serve to provide the same pressure for quality improvement and the same queue reduction for facilitating QA provided by kanban.

However, there is one further quality-related benefit that is often attributed directly to the pulling activity of kanban. The basic argument is that if workers from downstream workstations must go to an upstream workstation to get parts, then they will be able to inspect them. If the parts are not of acceptable quality, the worker can reject them immediately. The result will be quicker detection of quality problems and less likelihood of moving and working on bad parts.

This argument is not very convincing when the material handling is carried out by a separate worker, say, a forklift driver. Whether forklift drivers are "pushing" parts to the next station because they are finished or "pulling" them from the previous station because they are authorized to do so by a kanban makes little difference to their ability to conduct a quality inspection.

The argument is more persuasive when parts are small and workstations close, so that operators can move their own parts. Then, presumably, if the downstream operators go and get the parts, they will be more likely to check them for quality than if the upstream operator simply drops them off. But this reasoning unnecessarily combines two separate issues.

The first issue is whether the downstream operators inspect all parts that they receive (pushed or pulled). We have seen implementations in industry, not necessarily pull systems, in which operators had to approve material transfers by signing a routing form. Implicit in this approval was an inspection for quality.

It is a second and wholly separate issue whether to limit the WIP between two adjacent workstations. We will take up this issue later in this chapter. For now, we simply point out that the quality assurance benefits of pulling at each station can be attained via inspection transactions independently of the mechanism used for achieving the needed limit on WIP.

10.3.4 Maintaining Flexibility

A pure push system can release work to a very congested line, only to have the work get stuck somewhere in the middle. The result will be a loss of flexibility in several ways. First, parts that have been partially completed cannot easily incorporate engineering (e.g., design) changes. Second, high WIP levels impede priority or scheduling changes, as parts may have to be moved out of the line to make way for a high-priority part. And finally, if WIP levels are high, parts must be released to the plant floor well in advance of their due dates. Because customer orders become less certain as the planning horizon is increased, the system may have to rely on forecasts of future demand to determine releases. And since forecasts are never as accurate as one would like, this reliance serves to further degrade performance of the system.

A pull system that establishes a WIP cap can prevent these negative effects and thereby enhance the overall flexibility of the system. By preventing release of parts when the factory is overly congested, the pull system will keep orders on paper as long as possible. This will facilitate engineering and priority/scheduling changes. Also, releasing work as late as possible will ensure that releases are based on firm customer orders to the greatest extent possible. The net effect will be an increased ability to provide responsive customer service.

The analogy we like to use to illustrate the flexibility benefits of pull systems is that of air traffic control. When we fly from Austin, Texas, to Chicago, Illinois, we frequently wind up waiting on the ground in Austin past our scheduled departure due to what the airlines call *flow control*. What they mean is that O'Hare Airport in Chicago is overloaded (or will be by the time we get there). Even if we left Austin on time, we would only wind up circling over Lake Michigan, waiting for an opportunity to land. Therefore, air traffic control wisely (albeit maddeningly) keeps the plane on the ground in Austin until the congestion at O'Hare has cleared (or will clear by the time we get there). The net result is that we land at exactly the same time (late, that is!) as if we had left on schedule, but we use less fuel and reduce the risk of an accident. Importantly, we

also keep other options open, such as that of canceling the flight if the weather becomes too dangerous.

10.3.5 Facilitating Work Ahead

The preceding discussion implies that pull systems maintain flexibility by coordinating releases with the current situation in the line (i.e., by not releasing when the line is too congested). The benefits of coordination can also extend to the situation in which plant status is favorable. If we strictly follow a pull mechanism and release work into the system whenever WIP falls below the WIP cap, then we may "work ahead" of schedule when things go well. For instance, if we experience an interval of no machine failures, staffing problems, materials shortages, and so on, we may be able to produce more than we had anticipated. A pure push system cannot exploit this stretch of good luck because releases are made according to a schedule without regard to plant status.

Of course, in practice there is generally a limit to how far we should work ahead in a pull system. If we begin working on jobs whose due dates are so far into the future that they represent speculative forecasts, then completing them now may be risky. Changes in demand or engineering changes could well negate the value of early completion. Therefore, once we have given ourselves a comfortable cushion relative to demand, it makes sense to reduce the work pace. We will discuss mechanisms for doing this in Part III.

10.4 CONWIP

The simplest way we can think of to establish a WIP cap is to *just do it!* That is, for a given production line, establish a limit on the WIP in the line and simply do not allow releases into the line whenever the WIP is at or above the limit. We call the protocol under which a new job is introduced to the line each time a job departs **CONWIP** (*con*stant *w*ork *i*n *p*rocess) because it results in a WIP level that is very nearly constant.

Recall that in Chapter 7 we made use of the CONWIP protocol to control WIP so that we could determine the relationships among WIP, cycle time, and throughput. We now offer it as the basis of a practical WIP cap mechanism. First we describe it qualitatively, and then we give a quantitative model for analyzing the performance of a CONWIP line.

10.4.1 Basic Mechanics

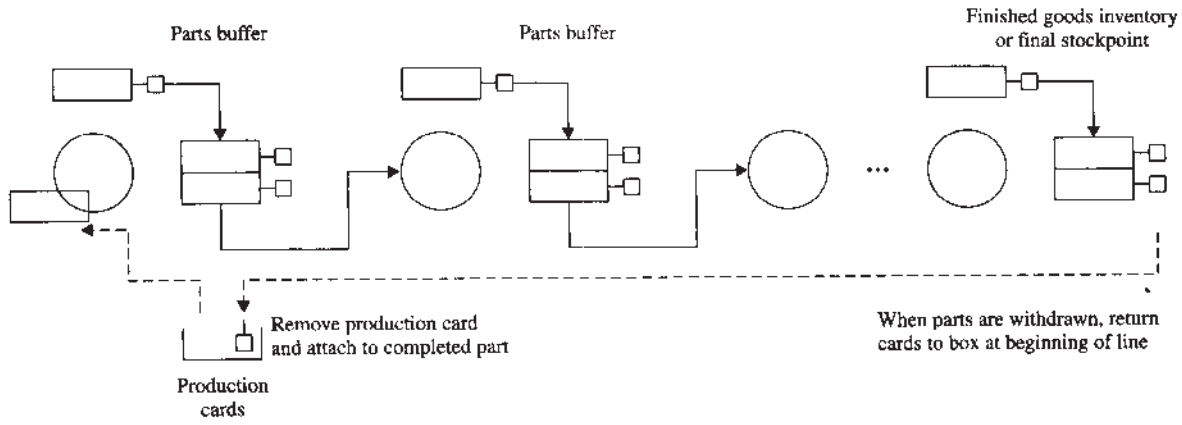
We can envision a CONWIP line operating as depicted in Figure 10.4, in which departing jobs send production cards back to the beginning of the line to authorize release of new jobs. Note that this way of describing CONWIP implicitly assumes two things:

1. The production line consists of a single routing, along which all parts flow.
2. Jobs are identical, so that WIP can be reasonably measured in units (i.e., number of jobs or parts in the line).

If the facility contains multiple routings that share workstations, or if different jobs require substantially different amounts of processing on the machines, then things are not so simple. There are, however, ways to address these complicating factors. For instance, we could establish CONWIP levels along different routings. We could also state the CONWIP levels in units of "standardized jobs," which are adjusted according to the amount of processing they require on critical resources. We address these types

FIGURE 10.4

A CONWIP production line



of implementation issues in Part III. For now, we focus on the single-product, single-routing production line in order to examine the essential differences between CONWIP, kanban, and MRP systems.

From a modeling perspective, a CONWIP system looks like a **closed queueing network,** in which customers (jobs) never leave the system, but instead circulate around the network indefinitely, as shown in Figure 10.5. Of course, in reality, the entering jobs are different from the departing jobs. But for modeling purposes, this makes no difference, because of the assumption that all jobs are identical.

In contrast, a pure push, or MRP, system behaves as an **open queueing network,** in which jobs enter the line and depart after one pass (also shown in Figure 10.5). Releases into the line are triggered by the material requirements plan without regard to the number of jobs in the line. Therefore, unlike in a closed queueing network, the number of jobs can vary over time.

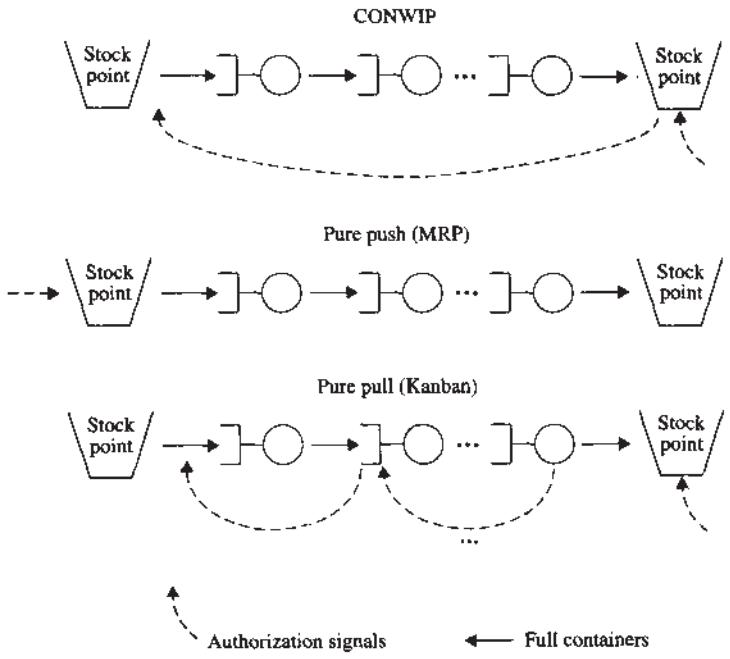
Finally, Figure 10.5 depicts a (one-card) kanban system as a **closed queueing network with blocking.** As in the closed queueing network model of a CONWIP system, jobs circulate around the network indefinitely. However, unlike the CONWIP system, the kanban system limits the number of jobs that can be at each station, since the number of production cards at a station establishes a maximum WIP level for that station. Each production card acts exactly like a space in a finite buffer in front of the workstation. If this buffer gets full, the upstream workstation becomes blocked.

10.4.2 Mean-Value Analysis Model

To analyze CONWIP lines and make comparisons with push systems, it is useful to have a quantitative model of closed (CONWIP) systems, similar to Kingman's equation model we developed for open (push) systems in Chapter 8. For the case in which all stations consist of single machines, we can do this by using a technique known as **mean-value analysis (MVA).**[3] This approach, which we used without specifically identifying it in

[3]Unfortunately, MVA is not valid for the multimachine case. We can approximate a station with parallel machines with a single fast machine (i.e., so the capacity is the same). But as we know from Chapter 7, parallel machines tend to outperform single machines, given the same capacity. Therefore, we would expect this approximation to underestimate the performance of a CONWIP line with parallel machine stations.

FIGURE 10.5
CONWIP, pure push, and kanban systems



Chapter 7 to develop the throughput and cycle time curves for the practical worst case, is an iterative procedure that develops the measures of the line with WIP level w in terms of those for WIP level $w-1$. The basic idea is that a job arriving to a station in a system with w jobs in it sees the other $w-1$ jobs distributed according to the average behavior of a system with $w-1$ jobs in it. This is exactly true for the case in which process times are exponential ($c_e = 1$). For general process times, it is only approximately true. As such, it gives us an approximate model, much like Kingman's model of open systems.

Using the following notation to describe an n-station CONWIP line

$u_j(w)$ = utilization of station j in CONWIP line with WIP level w
$\mathrm{CT}_j(w)$ = cycle time at station j in CONWIP line with WIP level w
$\mathrm{CT}(w) = \sum_{j=1}^{n} \mathrm{CT}_j(w)$ = cycle time of CONWIP line with WIP level w
$\mathrm{TH}(w)$ = throughput of CONWIP line with WIP level w
$\mathrm{WIP}_j(w)$ = average WIP level at station j in CONWIP line with WIP level w

we develop an MVA model for computing each of the above quantities as functions of the WIP level w. We give the details in the following technical note.

Technical Note

As was the case with Kingman's model of open systems, the basic modeling challenge in developing the MVA model of a closed system is to compute the average cycle time at a single station. We do this by treating stations as if they behaved as $M/G/1$ queues—that is, were single-machine stations with Poisson arrivals and general (random) processing times. Three key results for the $M/G/1$ queue are as follows:

1. The long-run average probability that the server is busy is

$$P(\text{busy}) = u$$

where u is the utilization of the station.